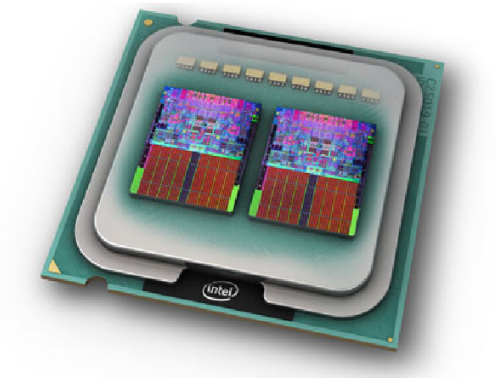
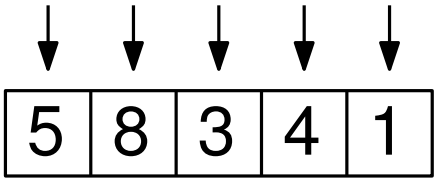
INFORMATION & COMPUTER SCIENCES
UNIVERSITY of HAWAI'I at MĀNOA

AlgoPARC

# ICS 643: Advanced Parallel Algorithms

Prof. Nodari Sitchinava






# Lecture 1: Introduction

# Welcome to ICS 643

Course Webpage:

`https://www2.hawaii.edu/~nodari/teaching/s22-643/`

# But don’t take my word for it...

From students’ Course Evaluations:

*“Brushing up on run time analysis, proof of correctness (induction), mergesort, and linked lists/trees could be handy.”*

ICS 443, Fall 2017

*“Also do NOT cheat in this class. Nodari means it when he says that he doesn’t take cheating lightly and unfortunately class mates of mine have been reported to the office of judicial affairs for academic dishonesty this semester.”*

ICS 311, Spring 2020

*“Go to office hours when possible AFTER reviewing the HW and ask for clarifications.”*

ICS 311, Spring 2020

# Take this course, if...

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- You enjoyed ICS 311

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- You enjoyed ICS 311
- You want to improve your algorithm design and analysis skills
  - Good algorithmic skills make better programmers

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- You enjoyed ICS 311
- You want to improve your algorithm design and analysis skills
  - Good algorithmic skills make better programmers
- Be one of the few people who know how to solve problems by properly taking advantage of available parallelism in modern and future systems

# This course is probably not for you if...

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- You hated ICS 311

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- You just want to pass to earn 3 units of credit

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- If you are looking for a parallel programming course, take ICS 432/632 instead

# Design an algorithm...

**m:** A finite sequence of well-defined (simple) instructions to complete a task.

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**procedure** DIRECTIONS(*A*, *B*)
    **if** *A* == *ICS* **and** *B* == *Waikiki* **then**
        Head east for 374 ft
        Turn right, proceed for 341 ft
        Turn left, proceed for 0.6 mi
        Turn right, proceed for 0.1 mi
        Turn right, proceed for 0.1 mi
        Continue for 1.3 mi
        Turn left, proceed for 0.2 mi
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        Continue straight for 0.2 mi

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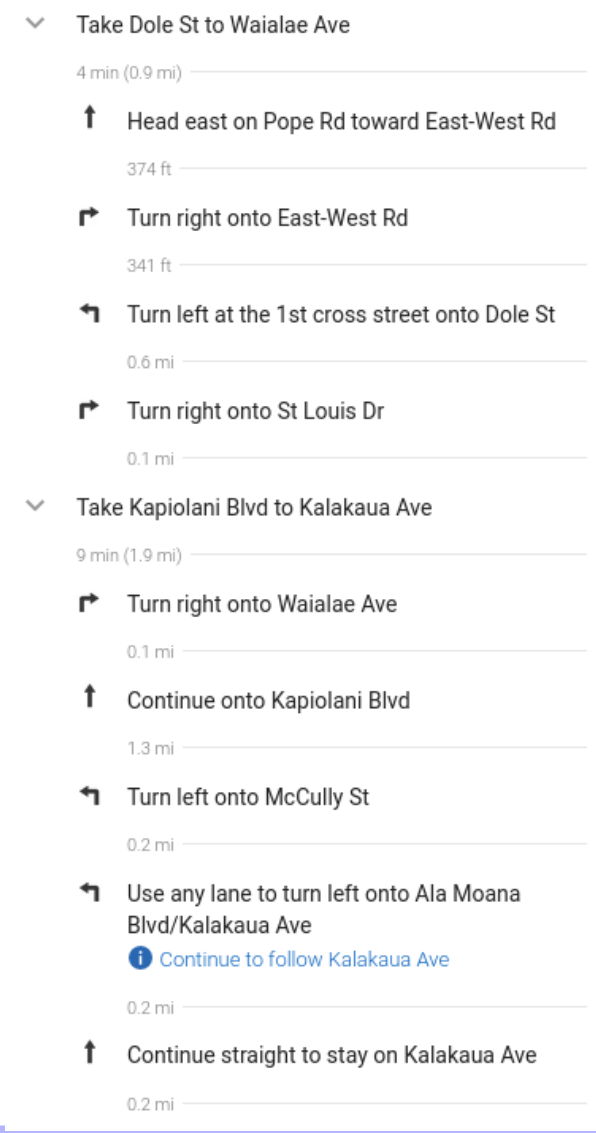

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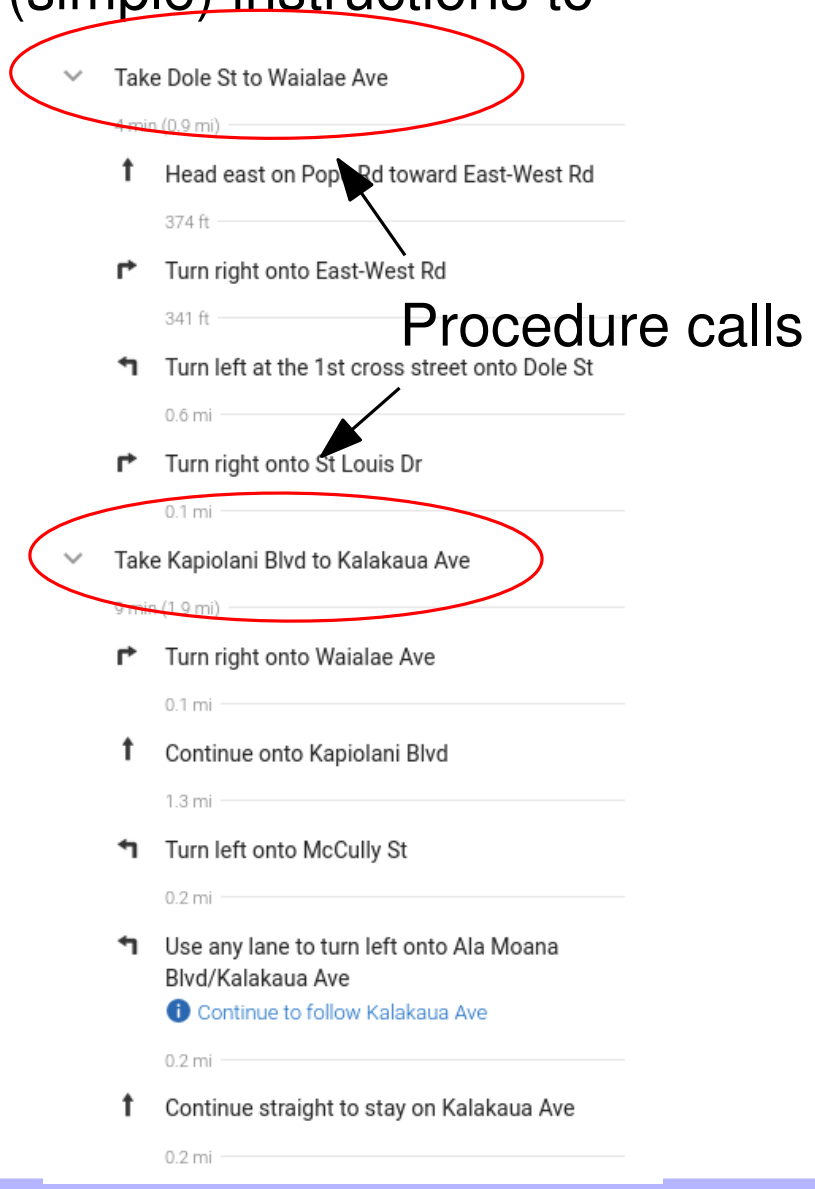


Procedure calls

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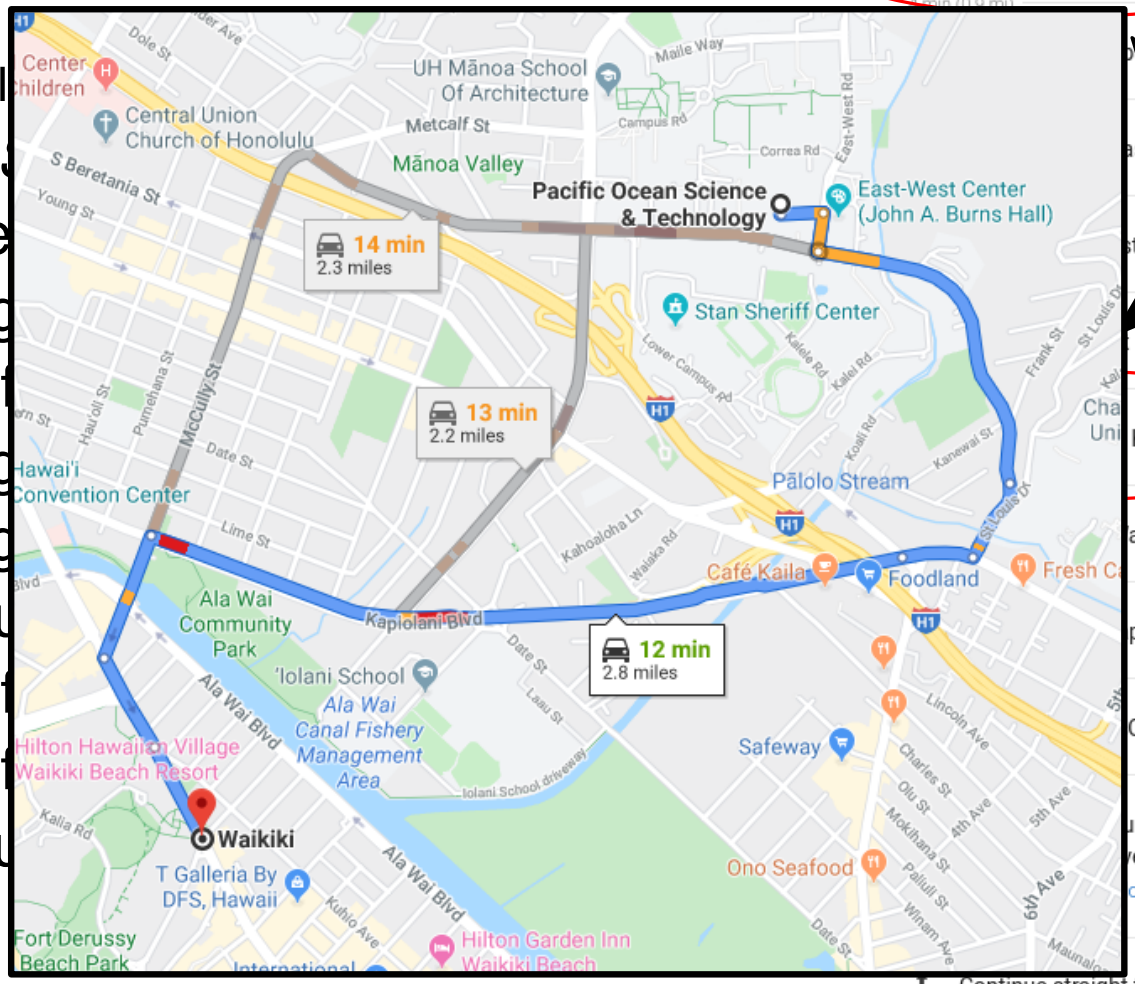


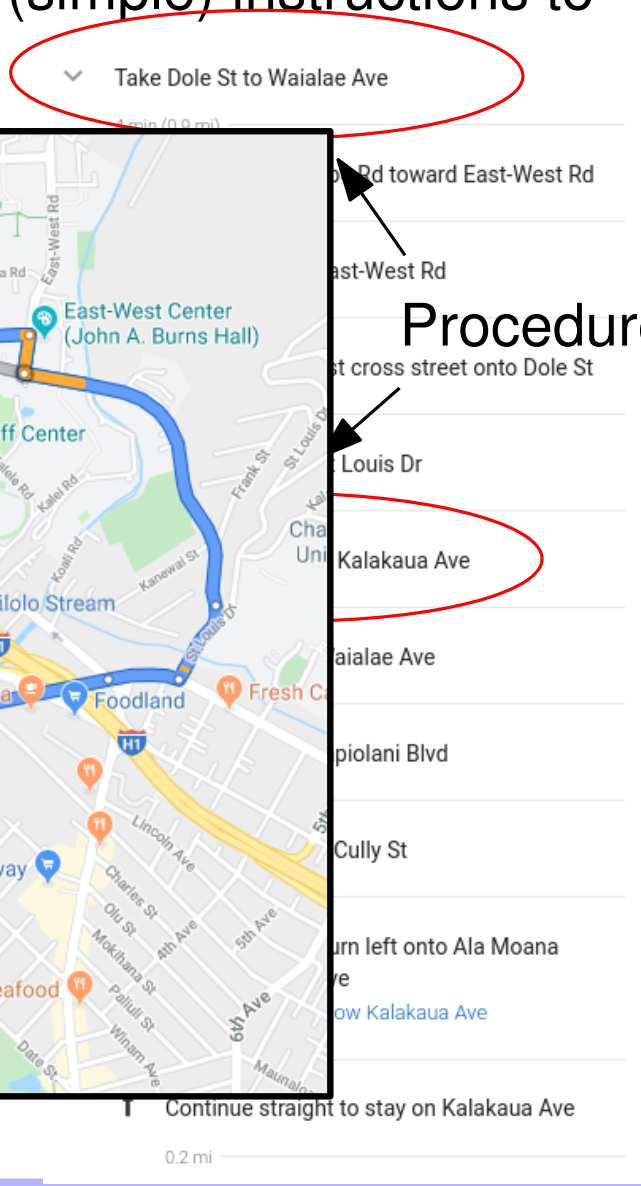


Procedure calls

# Design an algorithm...

When asked to “design an algorithm”, you should:

# Design an algorithm...

When asked to “design an algorithm”, you should:

1. Describe/explain your algorithm in plain English
   - Draw pictures to aid the explanation
2. Write down the pseudocode
3. Prove its correctness
4. Analyze its running time

# Parallel Algorithms

# Understanding Runtime

```
procedure FOO()
    a = 7
    b = 20
    c = 14
    d = 27
    e = 15
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Runtime = 5 steps

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procedure FOO()
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```

Runtime = 5 steps

```
procedure PARALLELFOO()
    a = 7
    spawn {
        b = 20
        c = 14
    }
    d = 27
    e = 15
    sync
```

# Understanding Runtime

**procedure** FOO()

a = 7

b = 20

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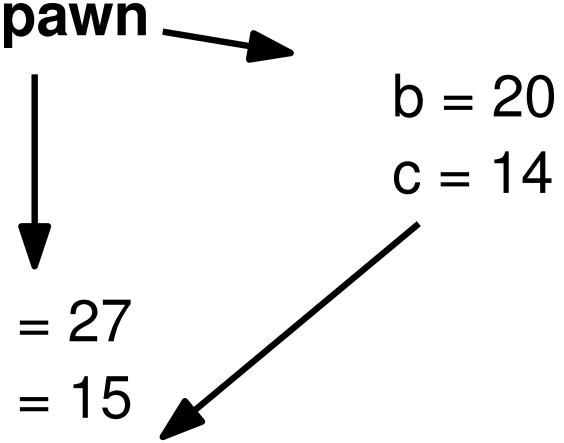
Runtime = 5 steps

**procedure** PARALLELFOO()

a = 7

**spawn**

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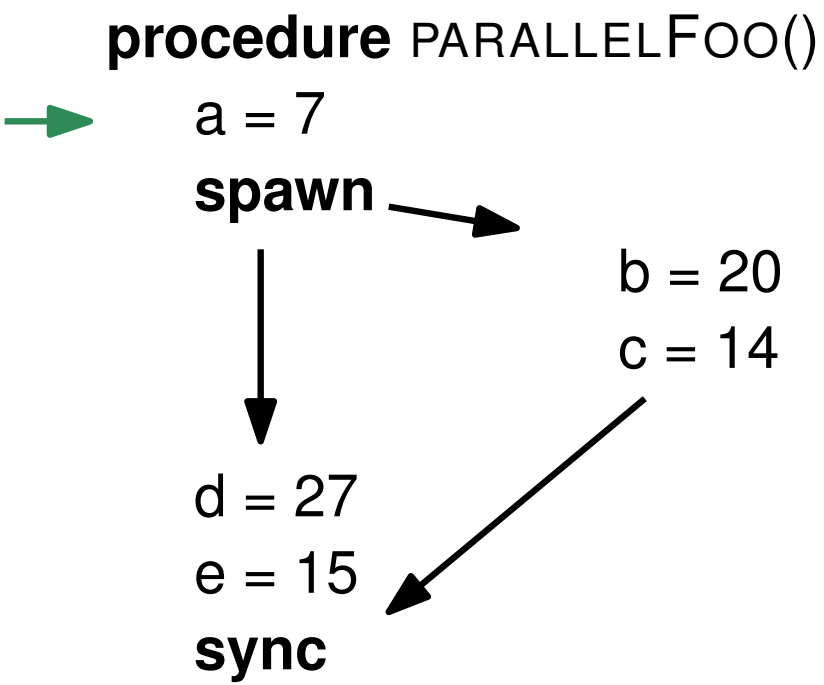

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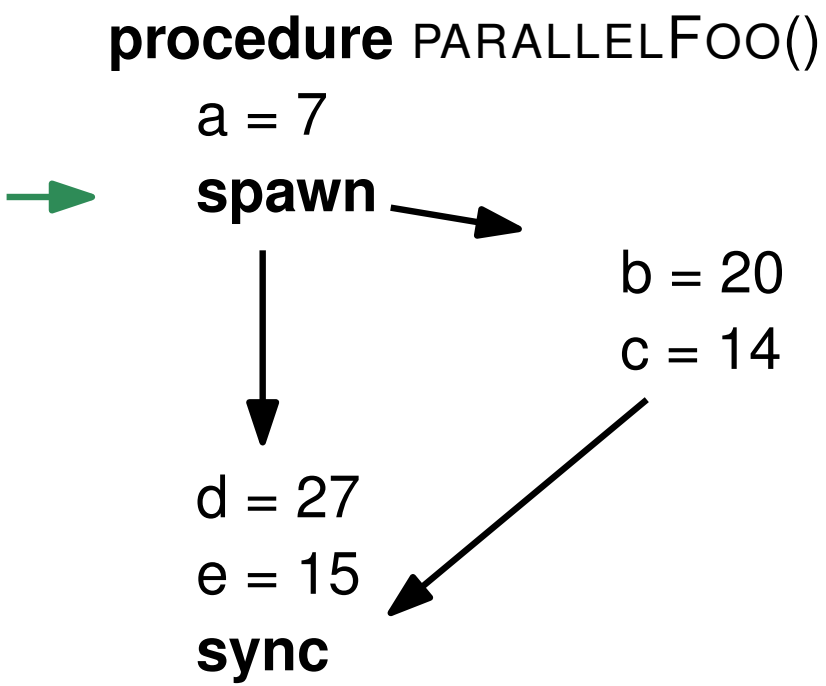

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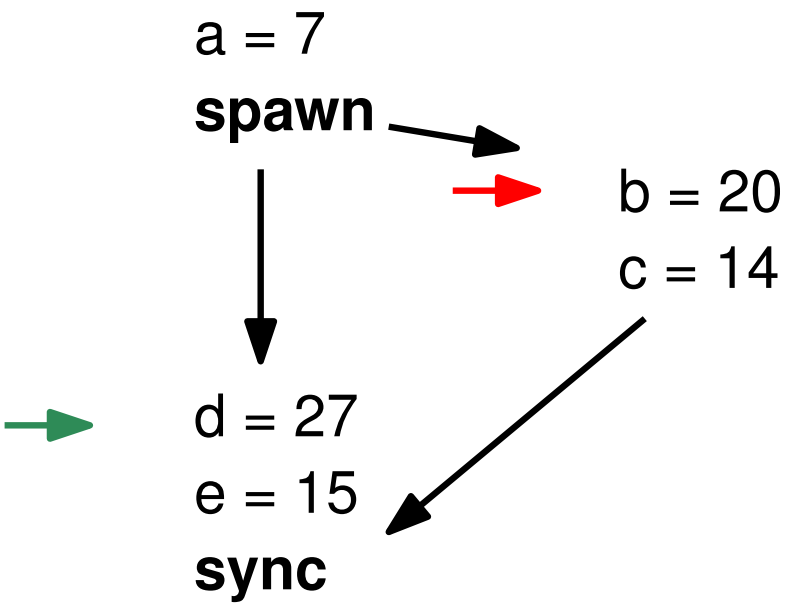

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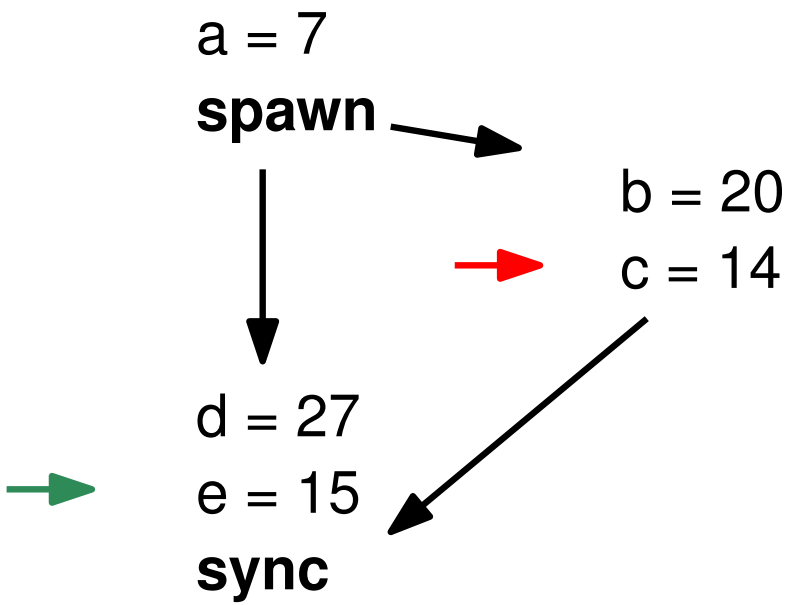

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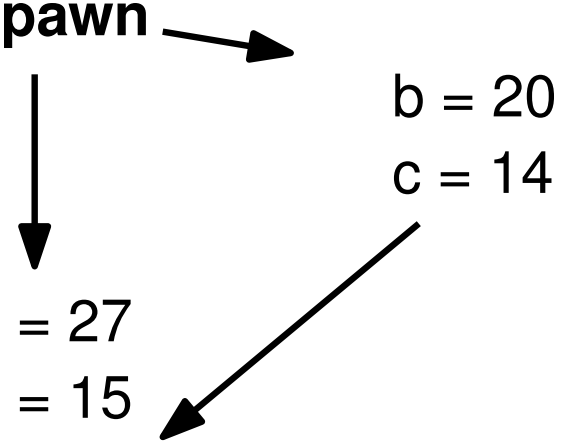


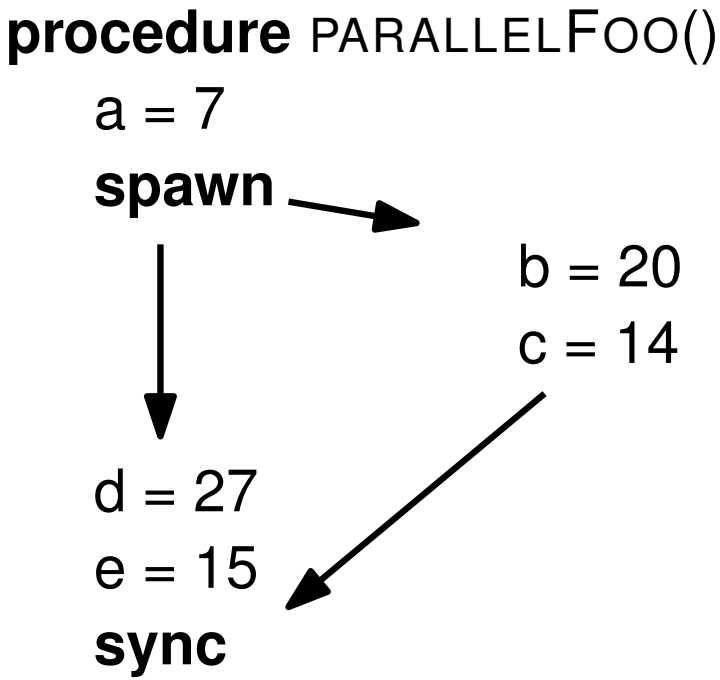

d = 27

e = 15

**sync**

Runtime = 5 steps

# Parallel Execution as a DAG



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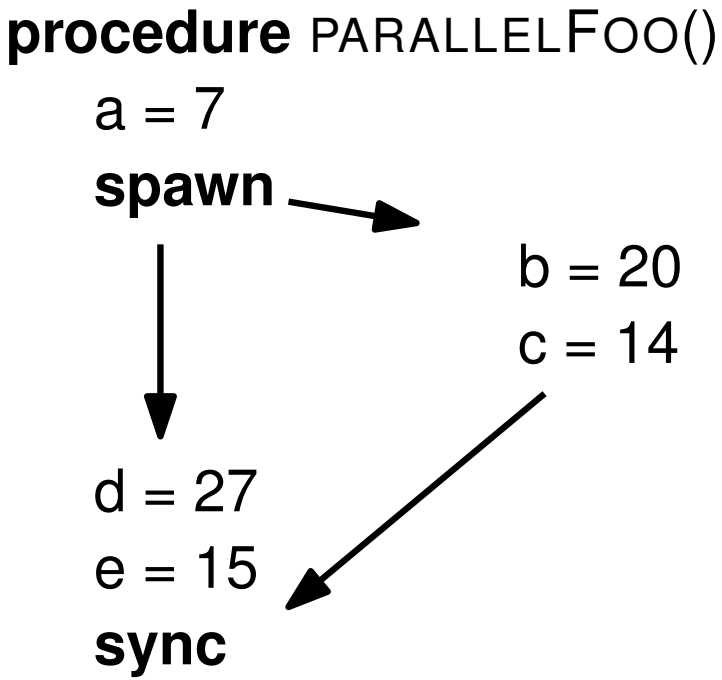
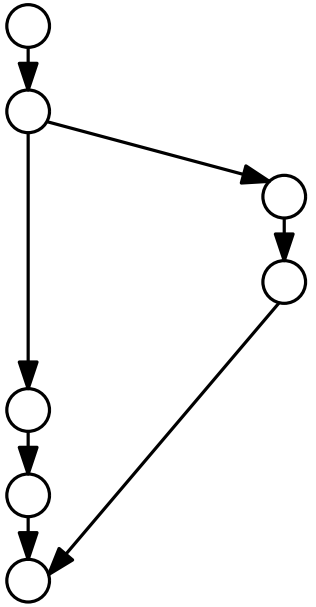



Dependency graph



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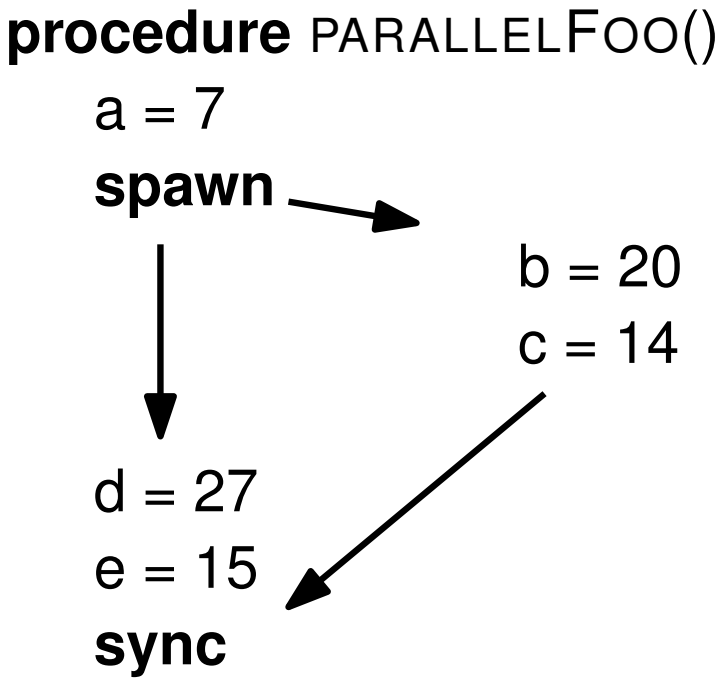


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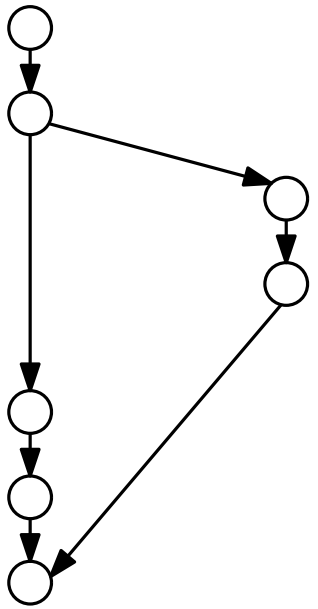
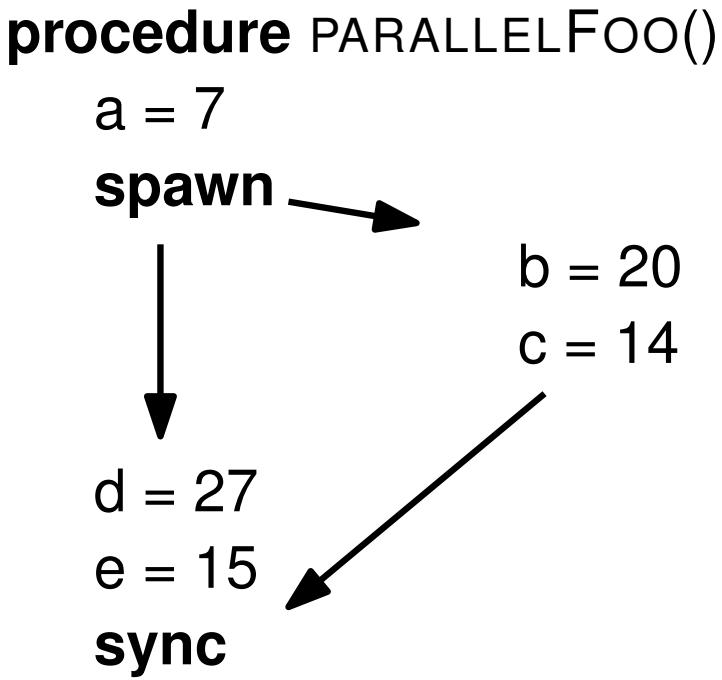


Parallel runtime = longest path length in the dependency graph

# Understanding Parallel Runtime



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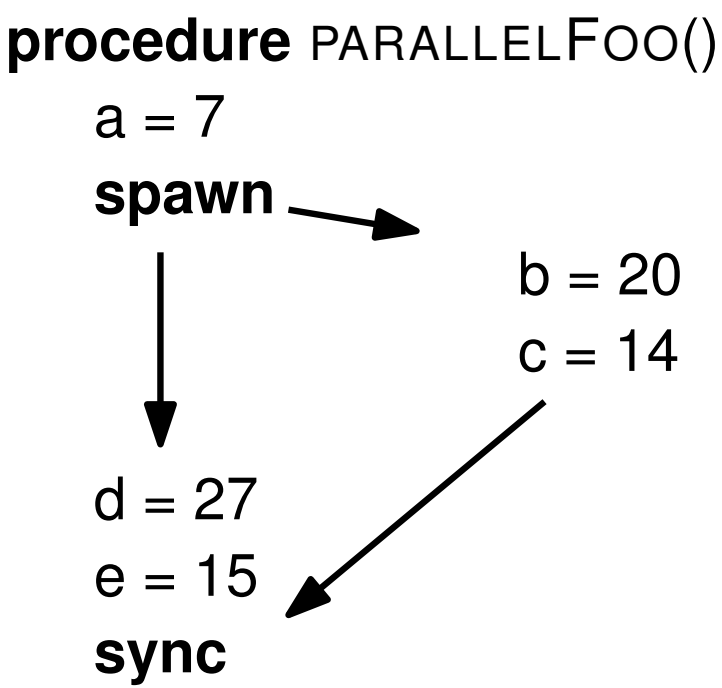
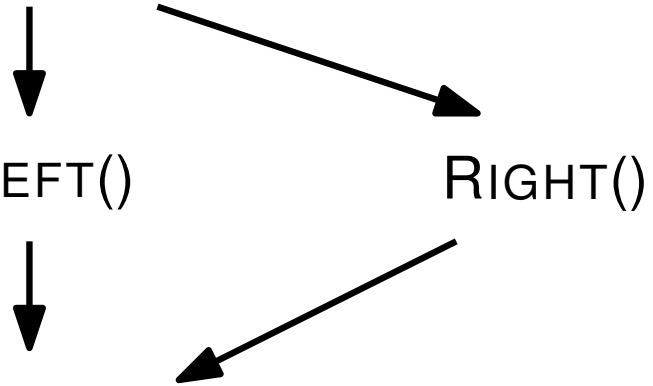



**procedure** ABOVE()
    a = 7
**procedure** LEFT()
    d = 27
    e = 15
**procedure** RIGHT()
    b = 20
    c = 14
**procedure** BELOW()

# Understanding Parallel Runtime

**procedure** PARALLELFOO()
    ABOVE()
    **spawn**



    LEFT()
    RIGHT()
    **sync**
    BELOW()

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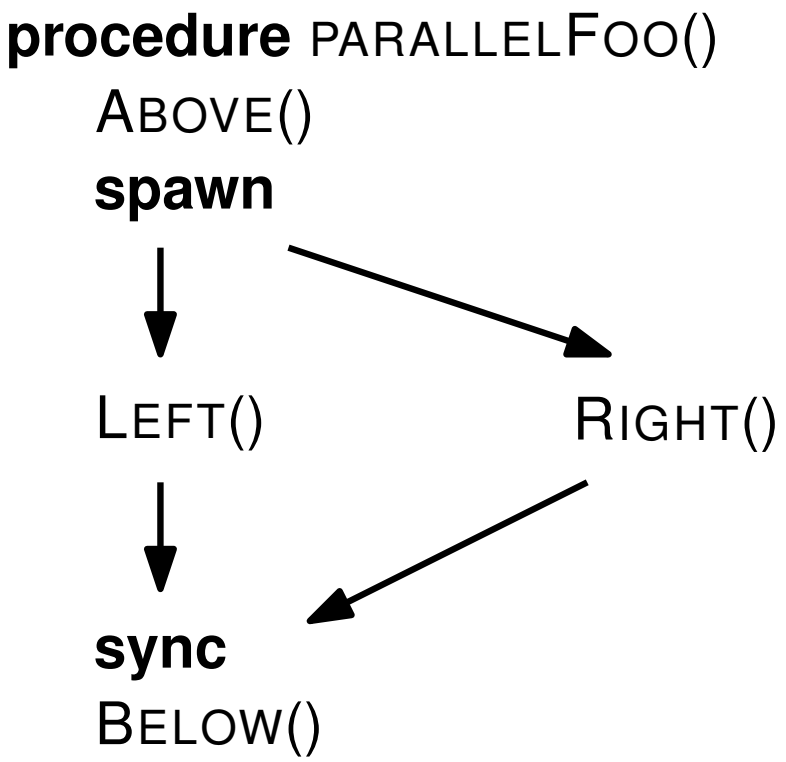



```
procedure ABOVE()
    a = 7
procedure LEFT()
    d = 27
    e = 15
procedure RIGHT()
    b = 20
    c = 14
procedure BELOW()
```

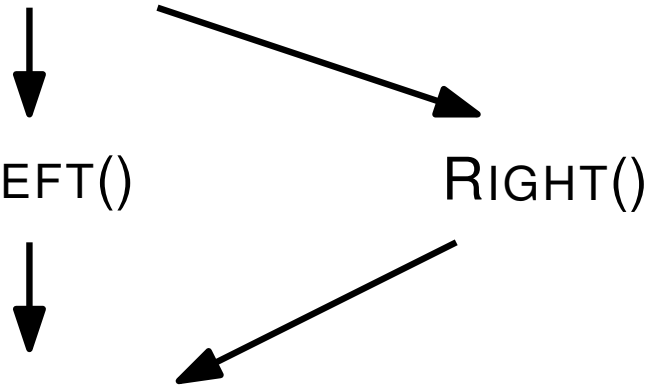
## Parallel Runtime

$$T(\text{ABOVE}) + 1 + \max(T(\text{LEFT}), T(\text{RIGHT})) + 1 + T(\text{BELOW})$$

$$= T(\text{ABOVE}) + \max \left\{ \begin{array}{c} T(\text{LEFT}) \\ T(\text{RIGHT}) \end{array} \right\} + T(\text{BELOW}) + O(1)$$

# Understanding Parallel Runtime

**procedure** PARALLELFOO()
    ABOVE()
    **spawn**
    LEFT()
    RIGHT()
    **sync**
    BELOW()



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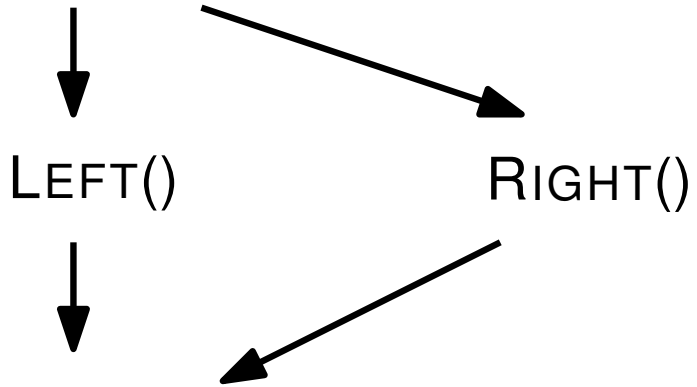
$$= T(\text{ABOVE}) + \max\left\{\begin{array}{c} T(\text{LEFT}) \\ T(\text{RIGHT}) \end{array}\right\} + T(\text{BELOW})$$

# Proper Pseudocode

**procedure** PARALLELFOO()
    ABOVE()
    **spawn**

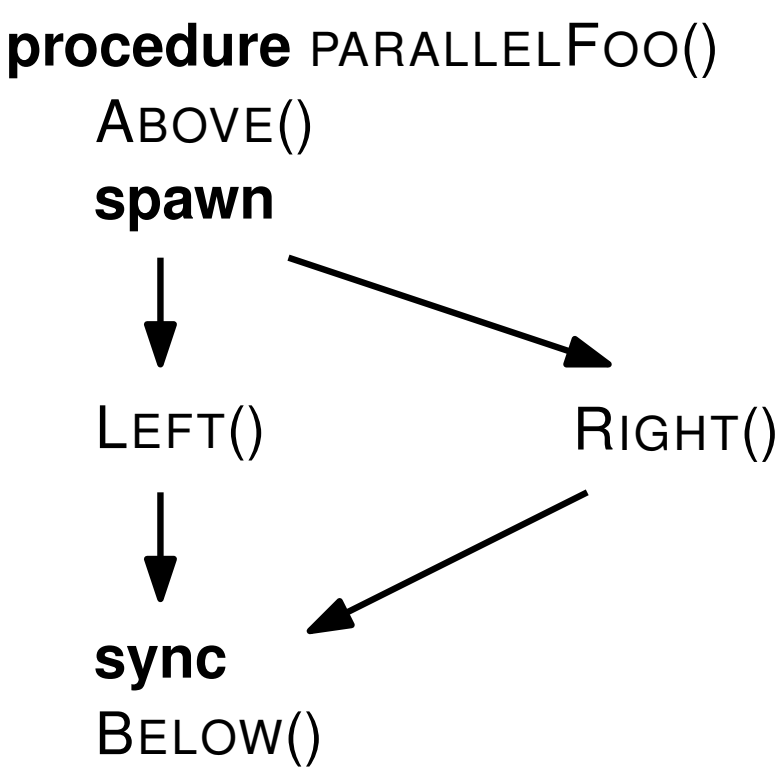


LEFT()  RIGHT()



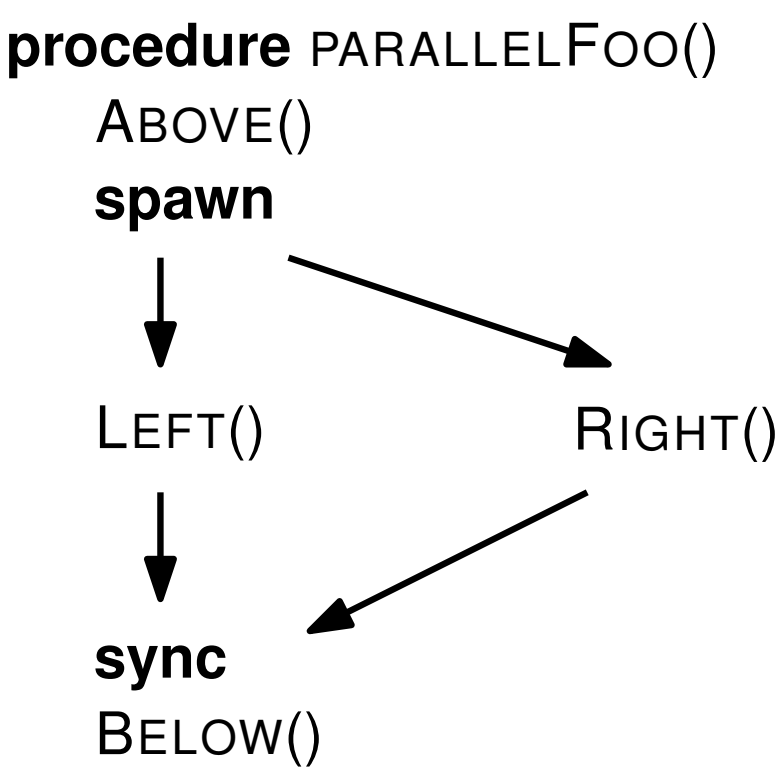
    **sync**
    BELOW()

# Proper Pseudocode




```
procedure PARALLELFOO()
    ABOVE()
    spawn RIGHT()
    LEFT()
    sync
    BELOW()
```

# Proper Pseudocode




```
procedure PARALLELFOO()
    ABOVE()
    spawn RIGHT()
    LEFT()
    sync
    BELOW()
```

```
procedure PARALLELFOO()
    ABOVE()
    in parallel do
        RIGHT()
        LEFT()
    BELOW()
```

# Taking a step further

```
procedure PARALLELFOO()
    ABOVE()
    in parallel do
        RIGHT()
        LEFT()
    BELOW()
```

# Taking a step further

```
procedure PARALLELFOO()
    ABOVE()
    in parallel do
        RIGHT()
        LEFT()
    BELOW()
```

```
procedure PARALLELFOO()
    ABOVE()
    in parallel do
        RIGHT()
        MIDDLE()
        LEFT()
    BELOW()
```

# Taking a step further

```
procedure PARALLELFOO()
    ABOVE()
    in parallel do
        RIGHT()
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procedure PARALLELFOO()
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procedure PARALLELFOO()
    ABOVE()
    spawn
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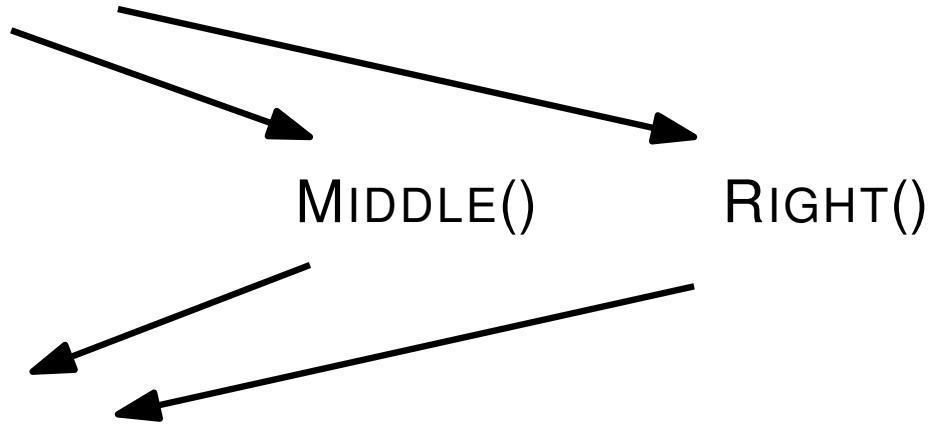
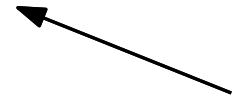


# Parallel **for** loop

**for** $i = 1$ to $n$ **in parallel do**

$a[i] = a[i] + 1$



Spawn $n$ threads $t_1, t_2, \ldots t_n$
Each thread $t_i$ (where $i = 1, 2, \ldots, n$) **do**:
$a[i] = a[i] + 1$
Synchronize all $n$ threads

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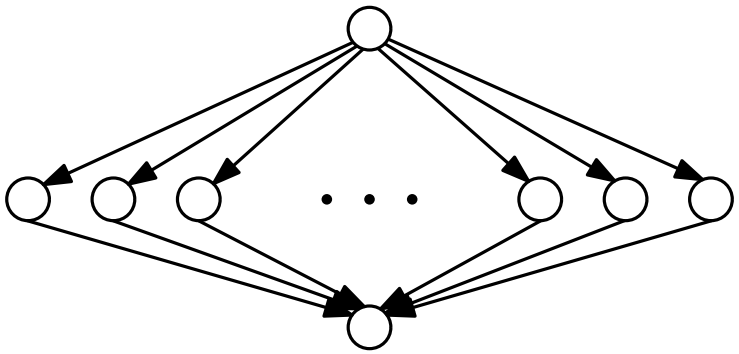

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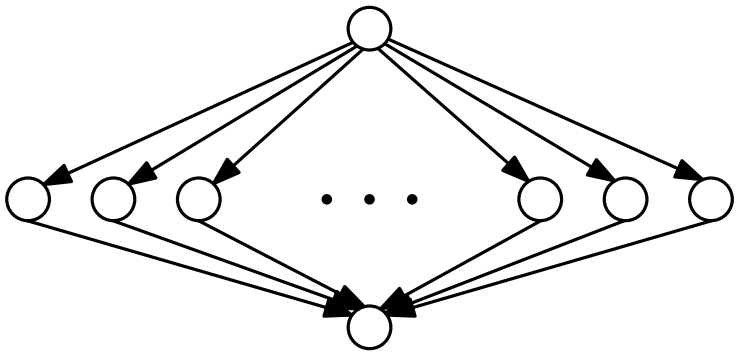

Parallel Runtime: $O(1)$

# Simple example

**for** $i = 1$ to $n$ **do**
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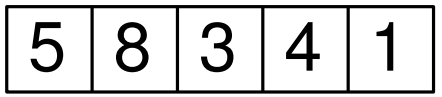

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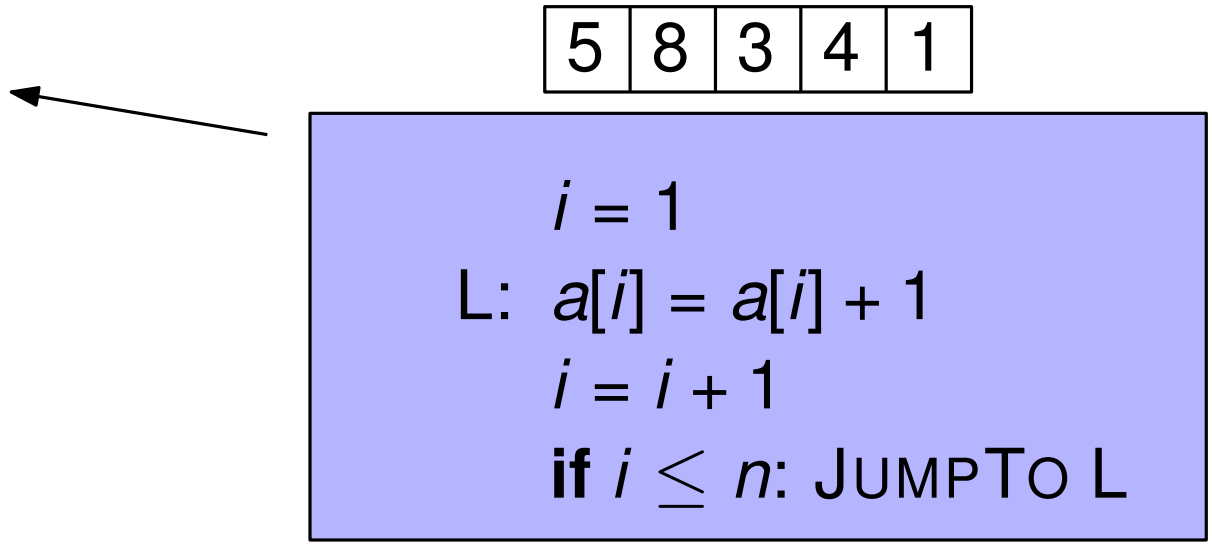

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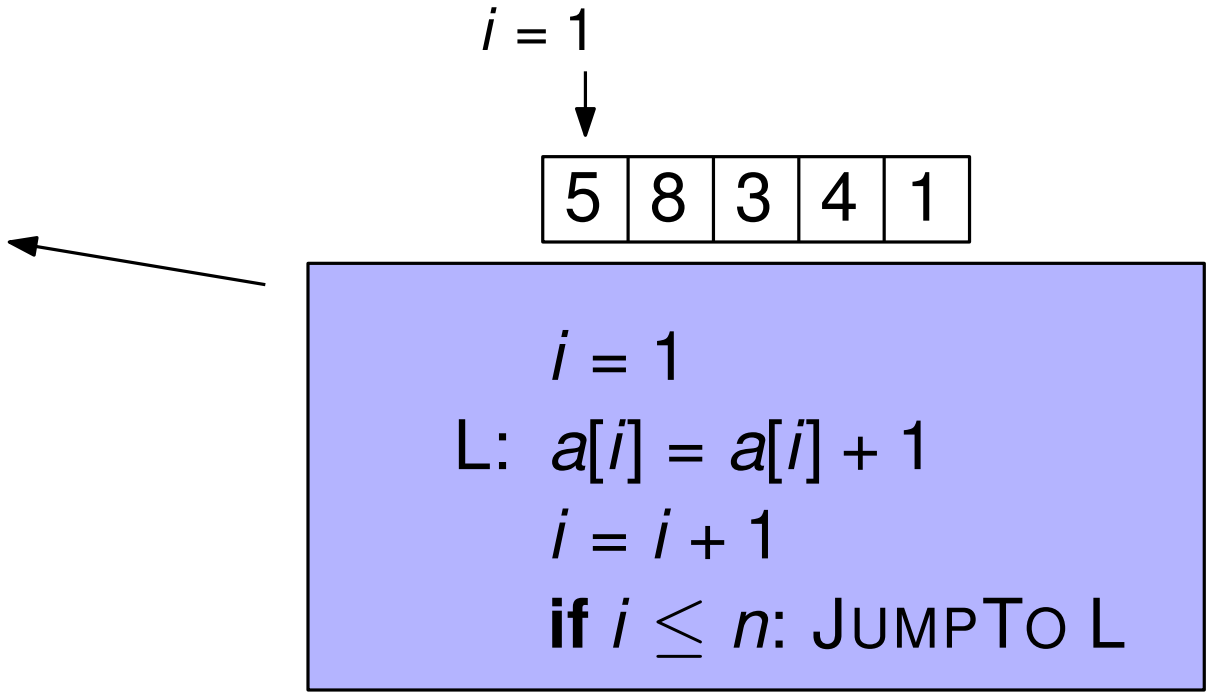

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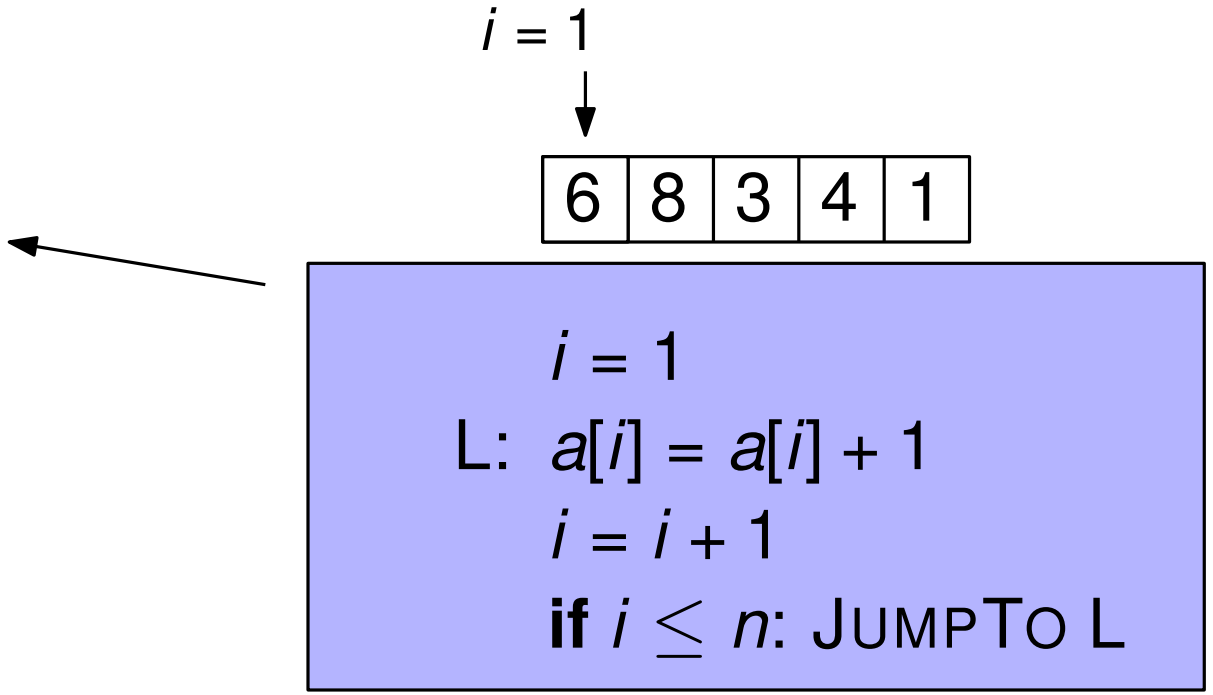

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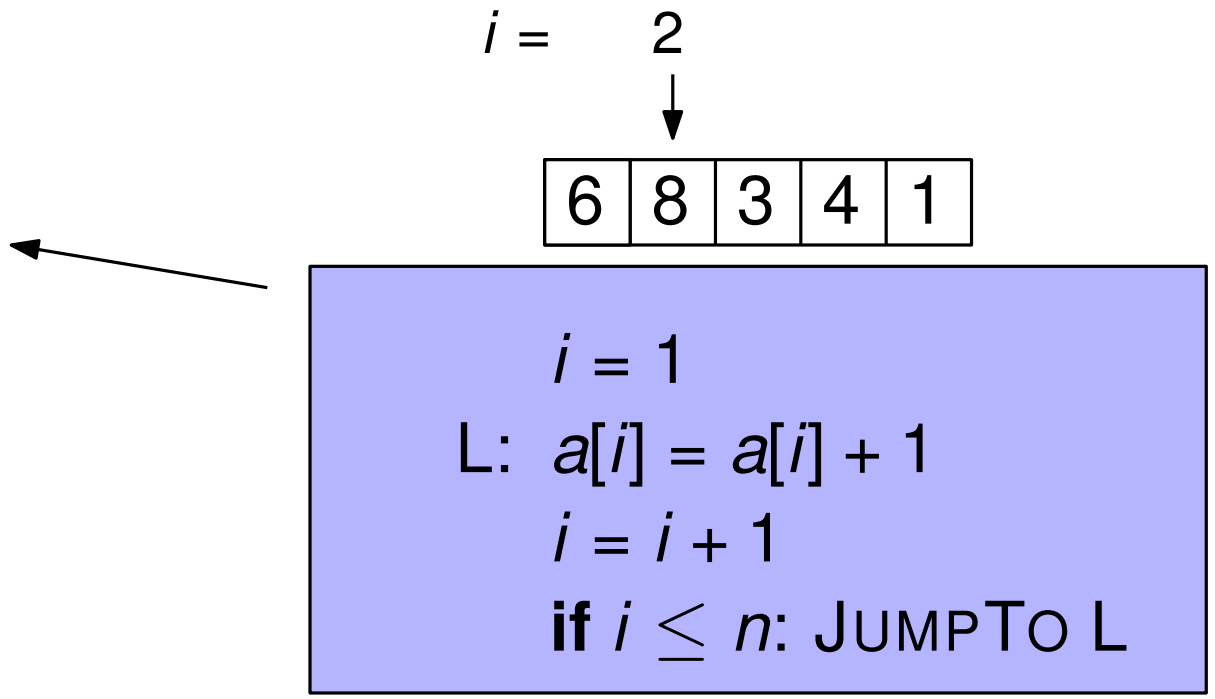

# Simple example

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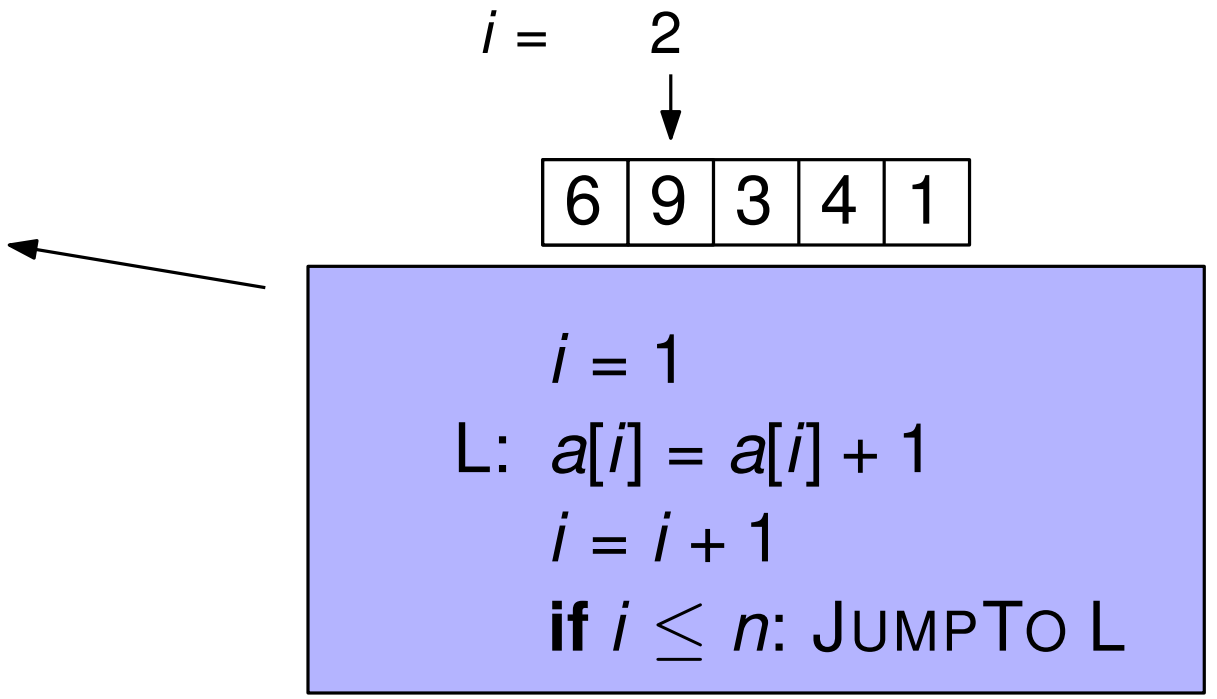
    $a[i] = a[i] + 1$



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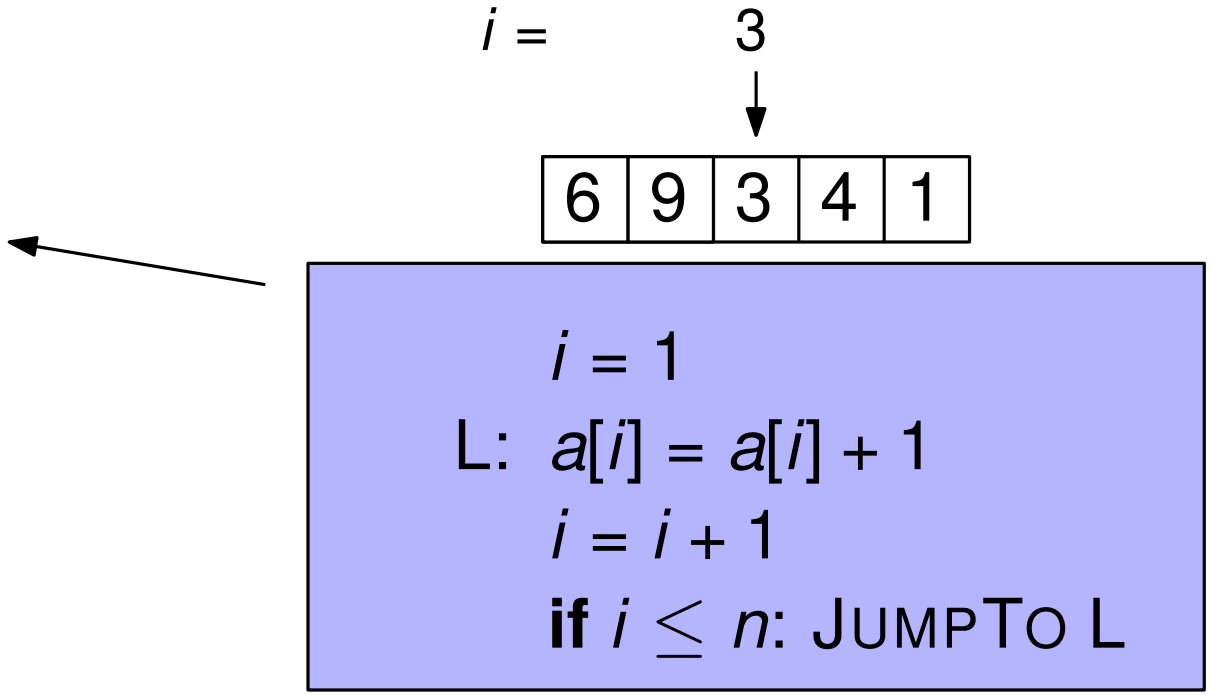

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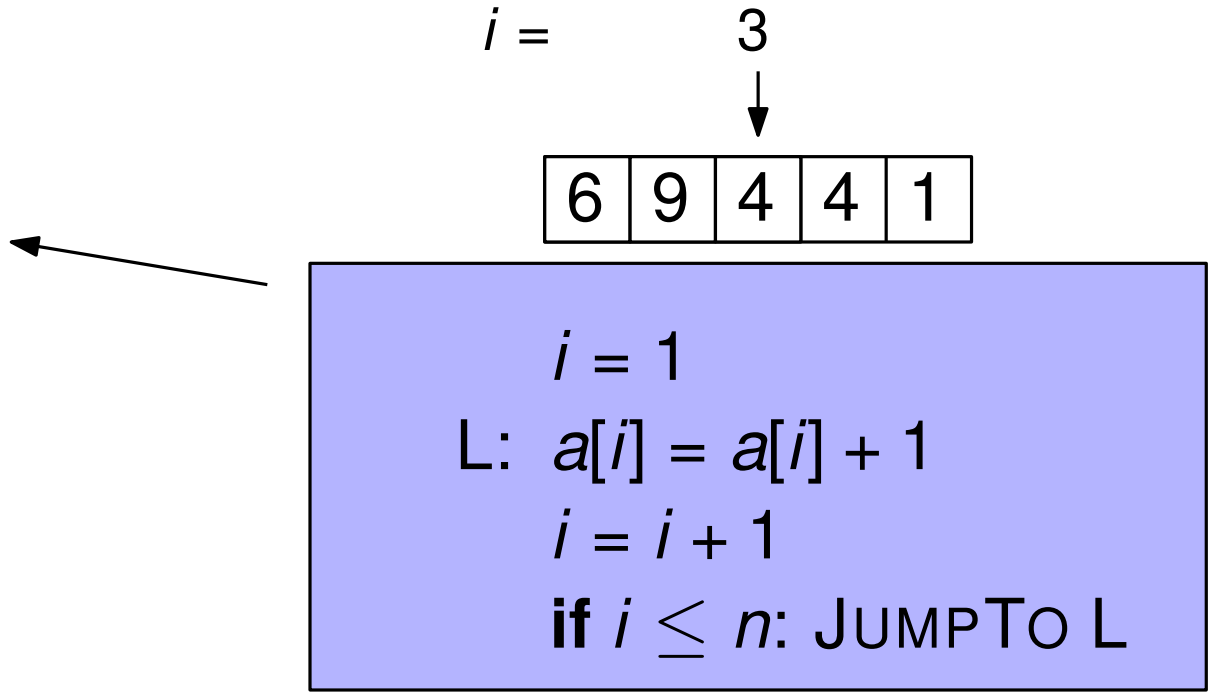

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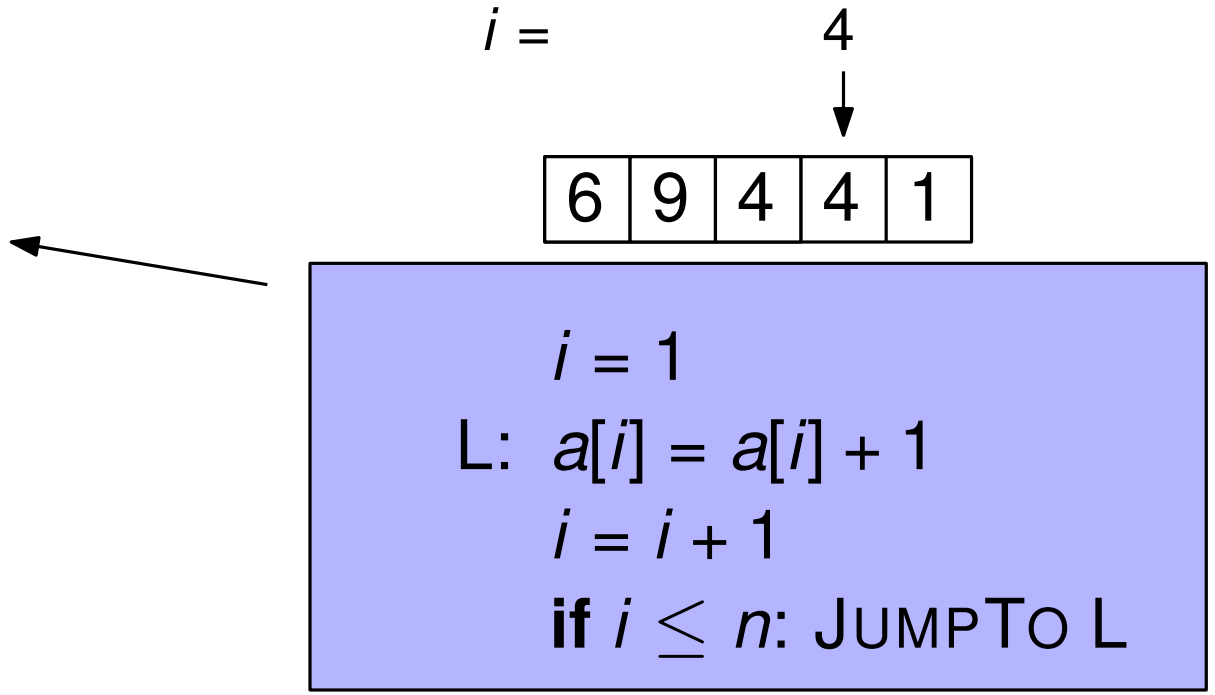

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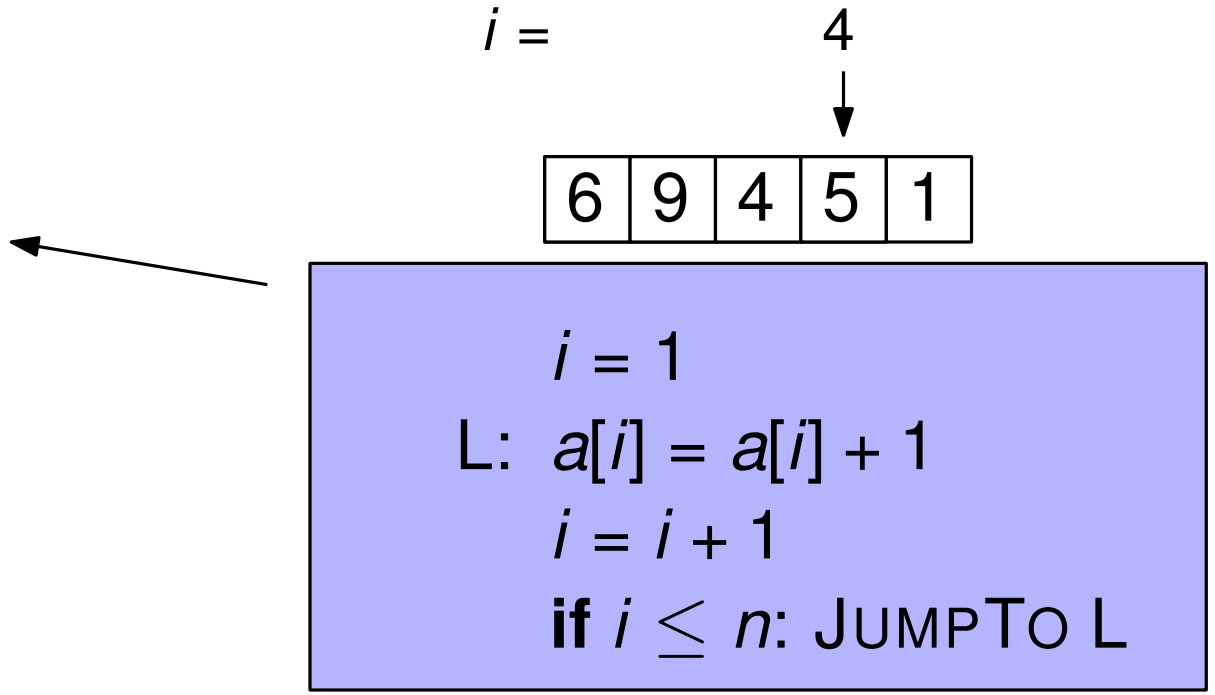

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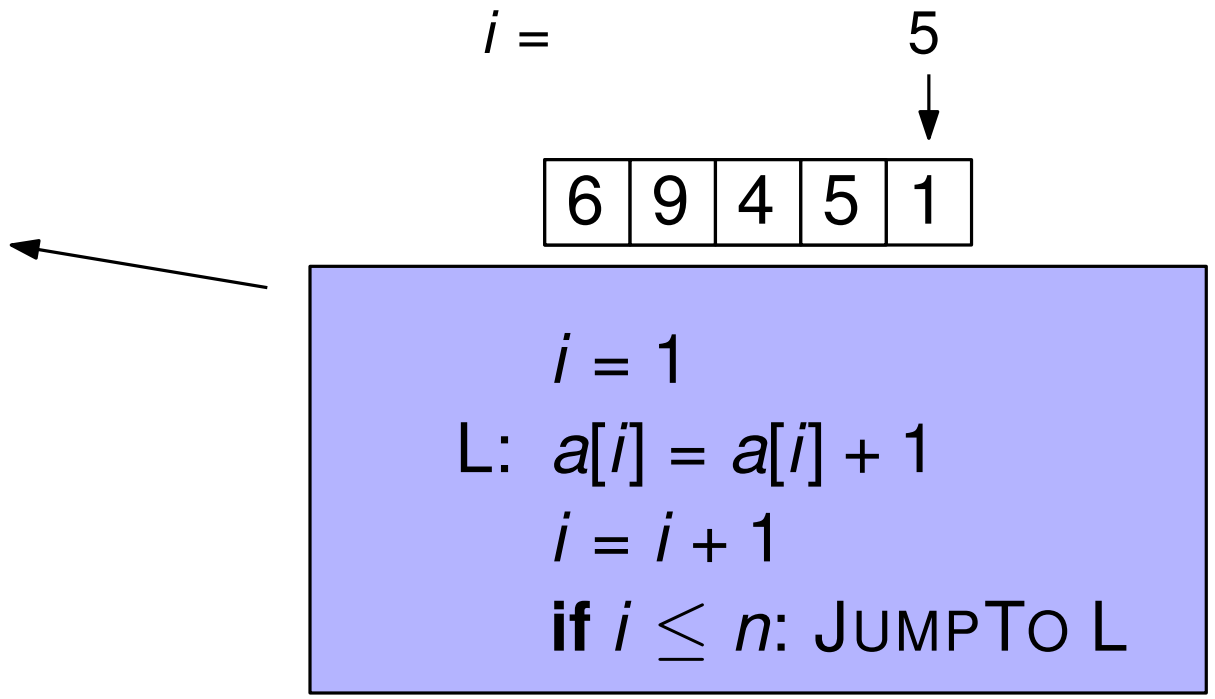

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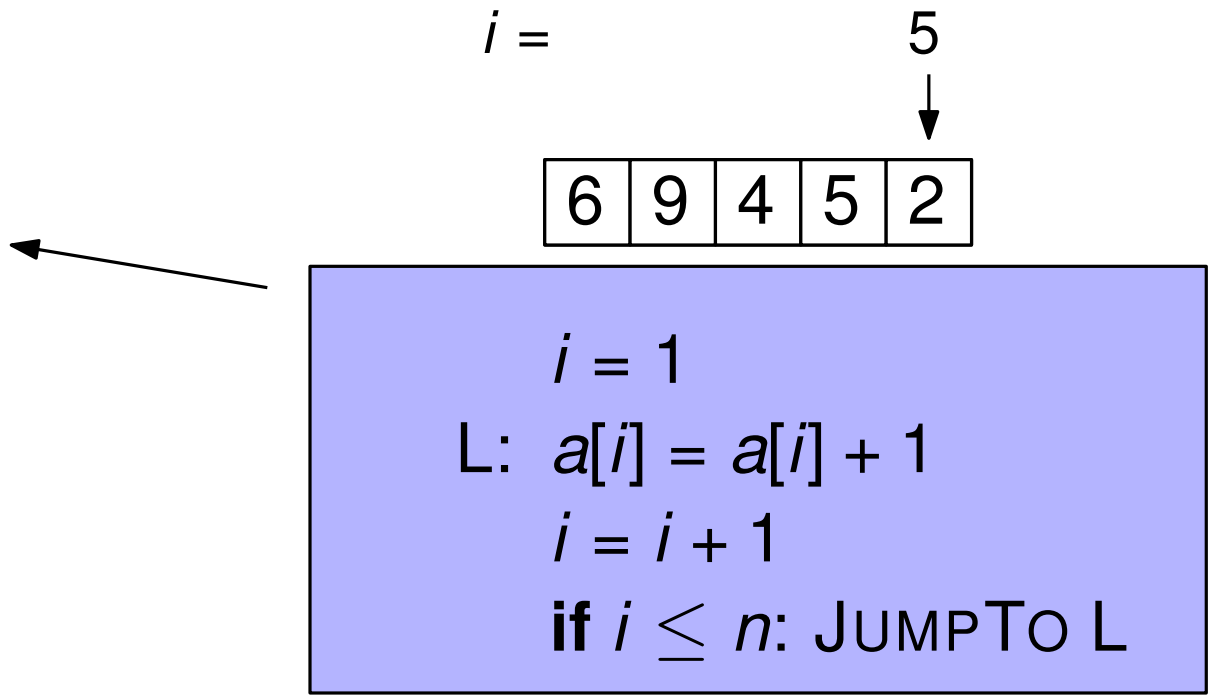

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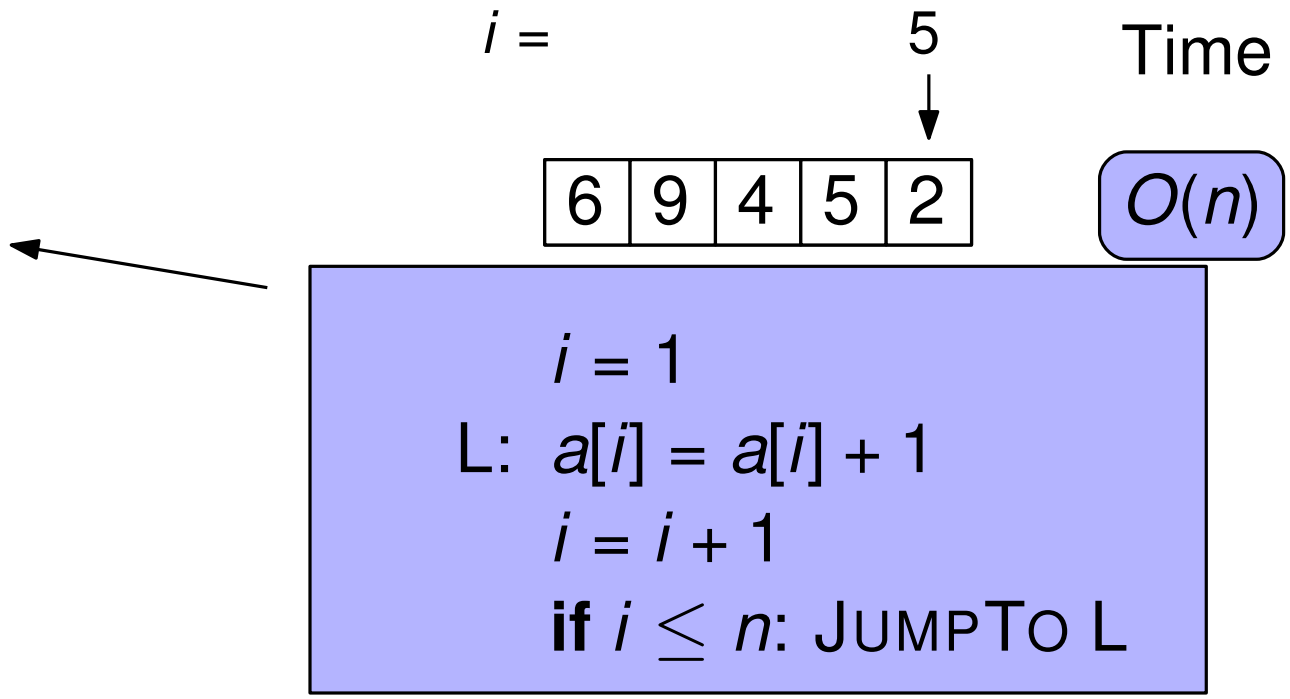

# Simple example

Time

**for** $i = 1$ to $n$ **do**
    $a[i] = a[i] + 1$


$O(n)$

**for** $i = 1$ to $n$ **in parallel do**
    $a[i] = a[i] + 1$

# Simple example

Time

**for** $i = 1$ to $n$ **do**
    $a[i] = a[i] + 1$

| 6 | 9 | 4 | 5 | 2 |
|---|---|---|---|---|

$O(n)$

**for** $i = 1$ to $n$ **in parallel do**
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Start $n$ threads $t_1, t_2, \ldots t_n$
Each thread $t_i$ (where $i = 1, 2, \ldots, n$) **do**:
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# Simple example

Time

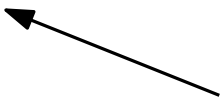
**for** $i = 1$ to $n$ **do**
    $a[i] = a[i] + 1$

| 6 | 9 | 4 | 5 | 2 |
|---|---|---|---|---|

$O(n)$

**for** $i = 1$ to $n$ **in parallel do**
    $a[i] = a[i] + 1$

| 5 | 8 | 3 | 4 | 1 |
|---|---|---|---|---|



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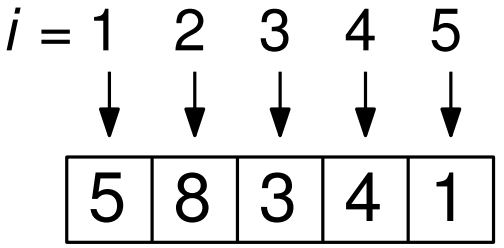


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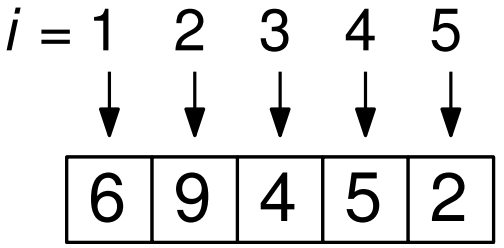


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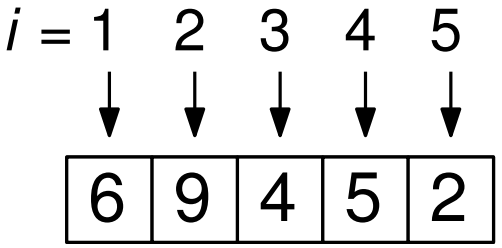


$O(1)$

Start $n$ threads $t_1, t_2, \ldots t_n$
Each thread $t_i$ (where $i = 1, 2, \ldots, n$) **do**:
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# Simple example

Time

**for** $i = 1$ to $n$ **do**
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$O(n)$

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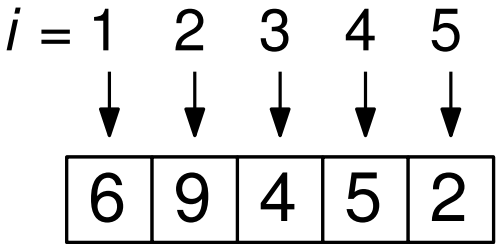


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Parallel Time = time of the slowest thread