| **ICS 621: Analysis of Algorithms**	Spring 2024

Problem Set 0

Prof. Nodari Sitchinava — *Due: OPTIONAL - DO NOT SUBMIT*

You may discuss the problems with your classmates, however **you must write up the solutions on your own** and **list the names** of every person with whom you discussed each problem.
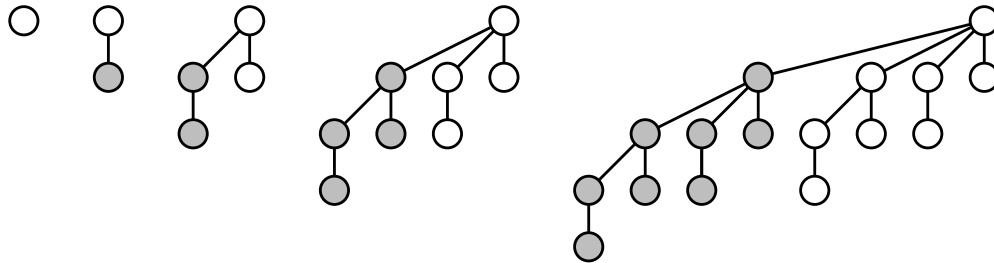
1   Binomial Trees (OPTIONAL - 0 pts)

A *binomial tree* B_k of order k is defined recursively as follows:

- A binomial tree B_0 of order 0 is a single node.

- For all $k > 0$, a binomial tree B_k of order k consists of two binomial trees of order $k-1$, with the root of one tree connected as a new *leftmost* child of the root of the other.
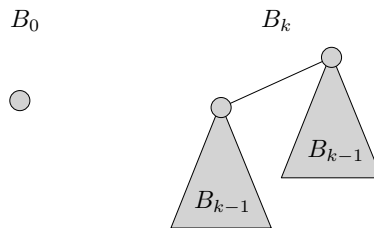
Binomial trees B_0 through B_4 look as follows:

Prove that for the binomial tree B_k,

1. there are 2^k nodes,

2. the height of the tree is k,

3. there are exactly $\binom{k}{i}$ nodes at depth i for $i = 0, 1, \ldots, k$, and

4. the root has degree k, which is greater than that of any other node; moreover, as the Figure below shows, if we number the children of the root from left to right by $k-1, k-2, \ldots, 2, 1, 0$, then child i is the root of a subtree B_i.

Hint: use induction.