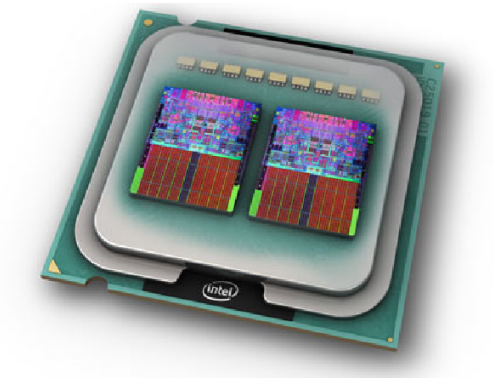
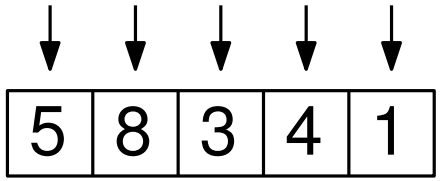
ICS 621: Analysis of Algorithms

Prof. Nodari Sitchinava



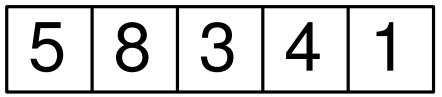


Parallel Algorithms

Course Evaluations

Simple example

for $i = 1$ to n **do**
    $a[i] = a[i] + 1$



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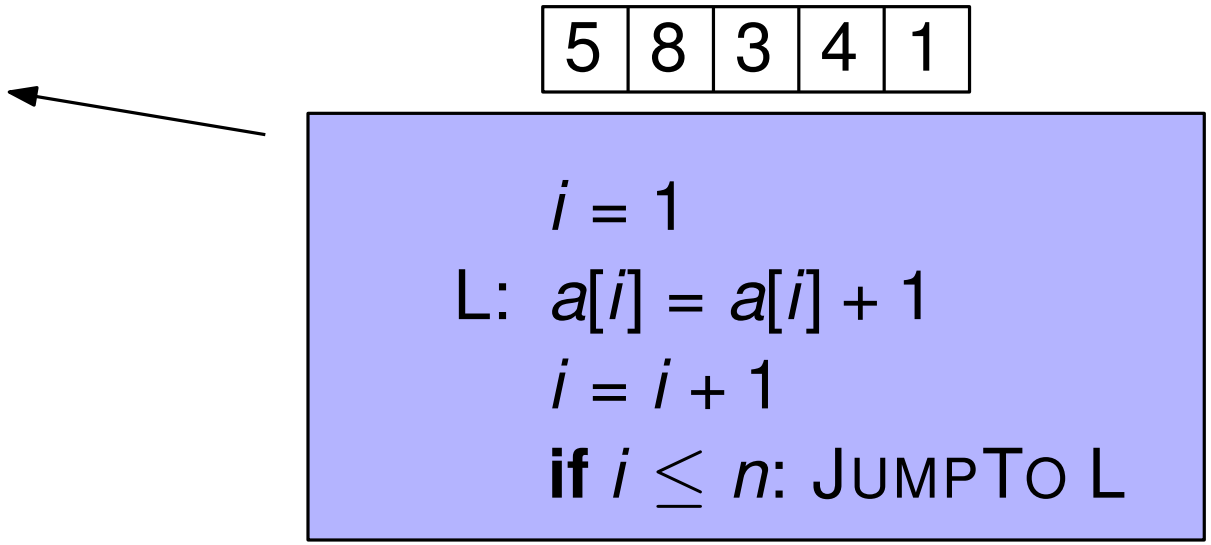

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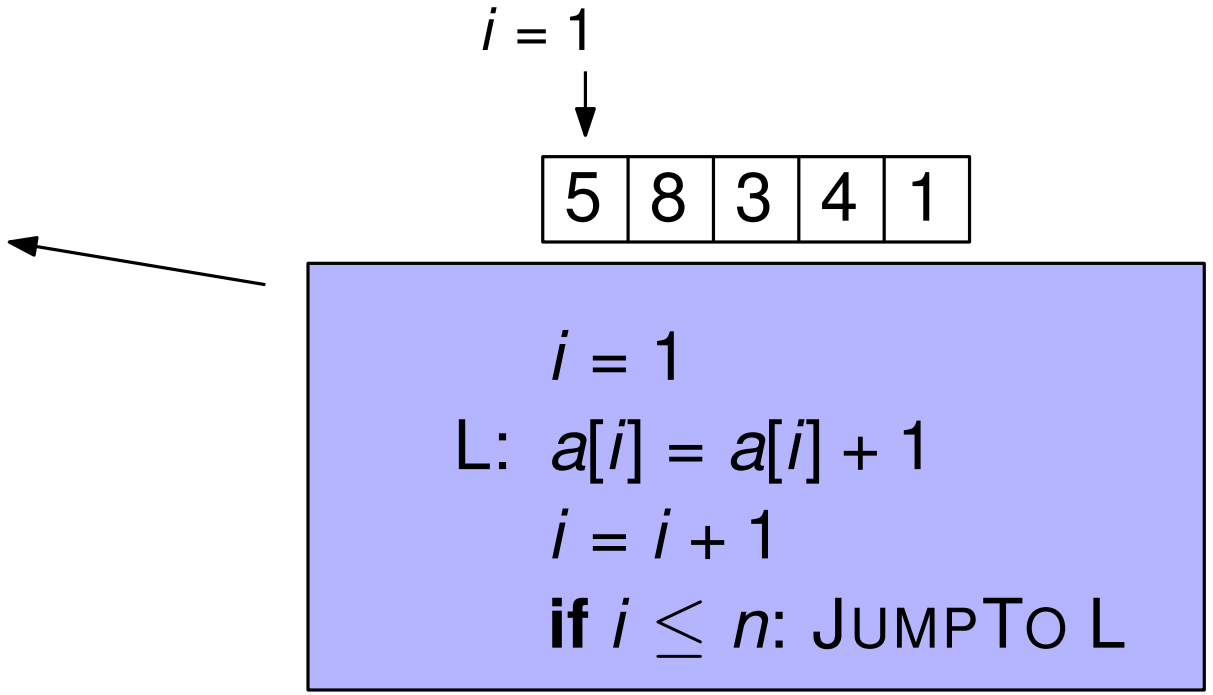

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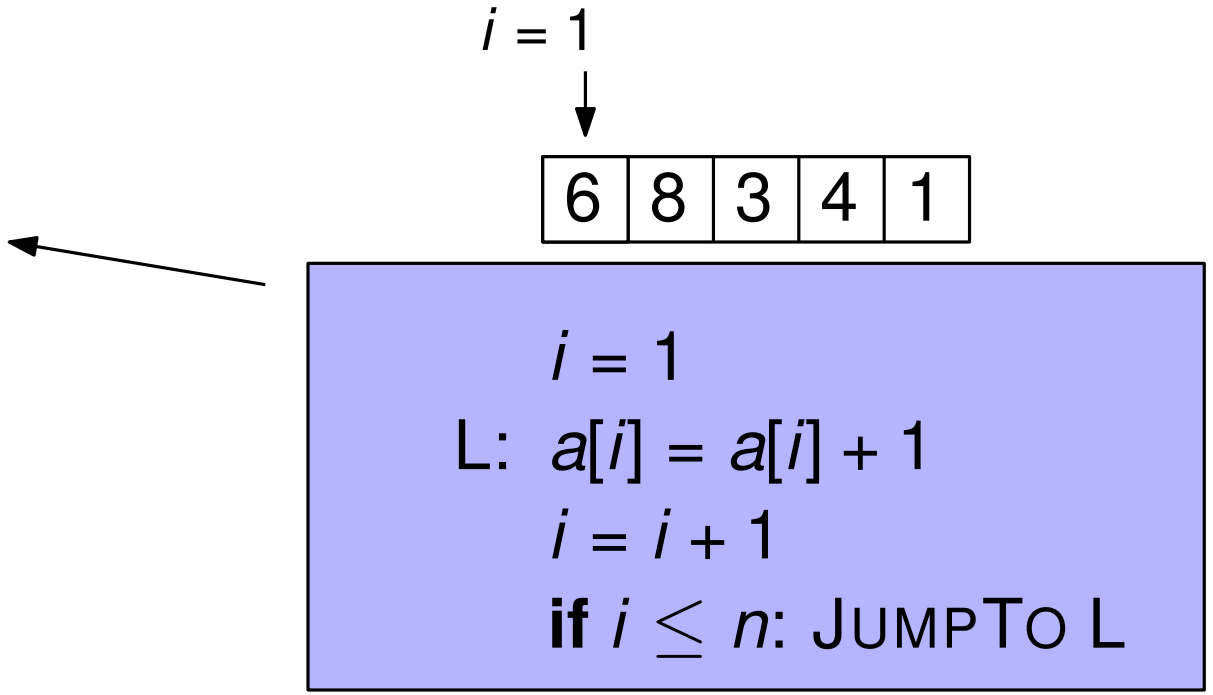

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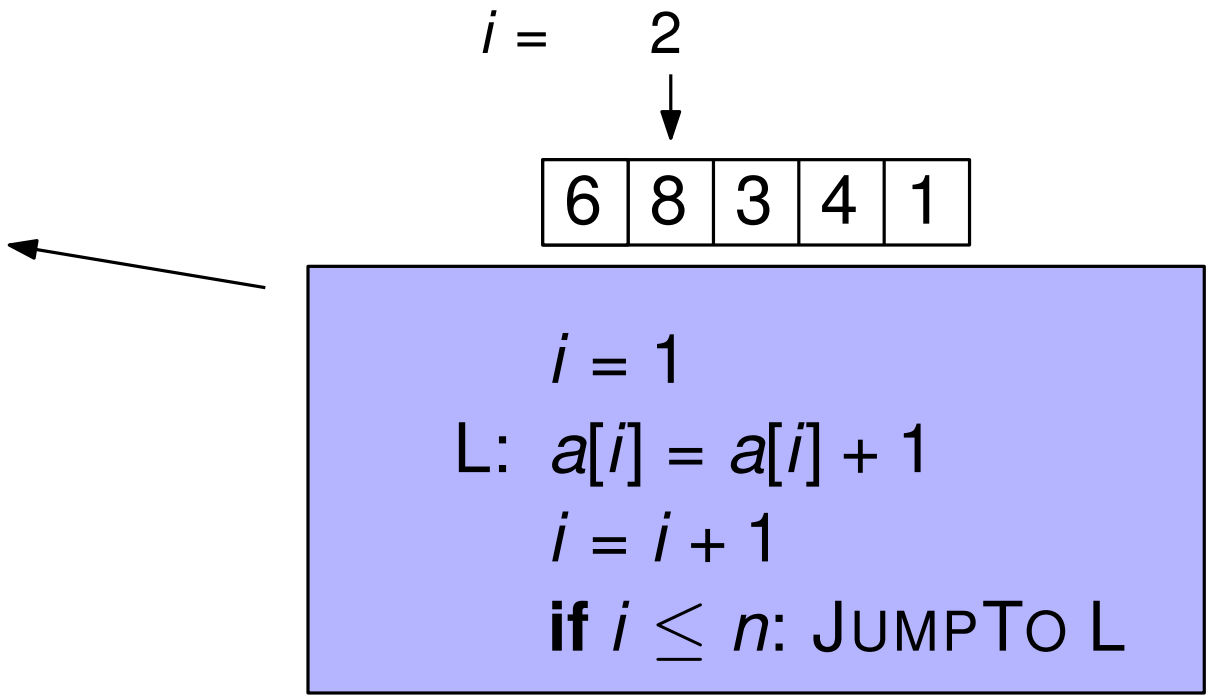

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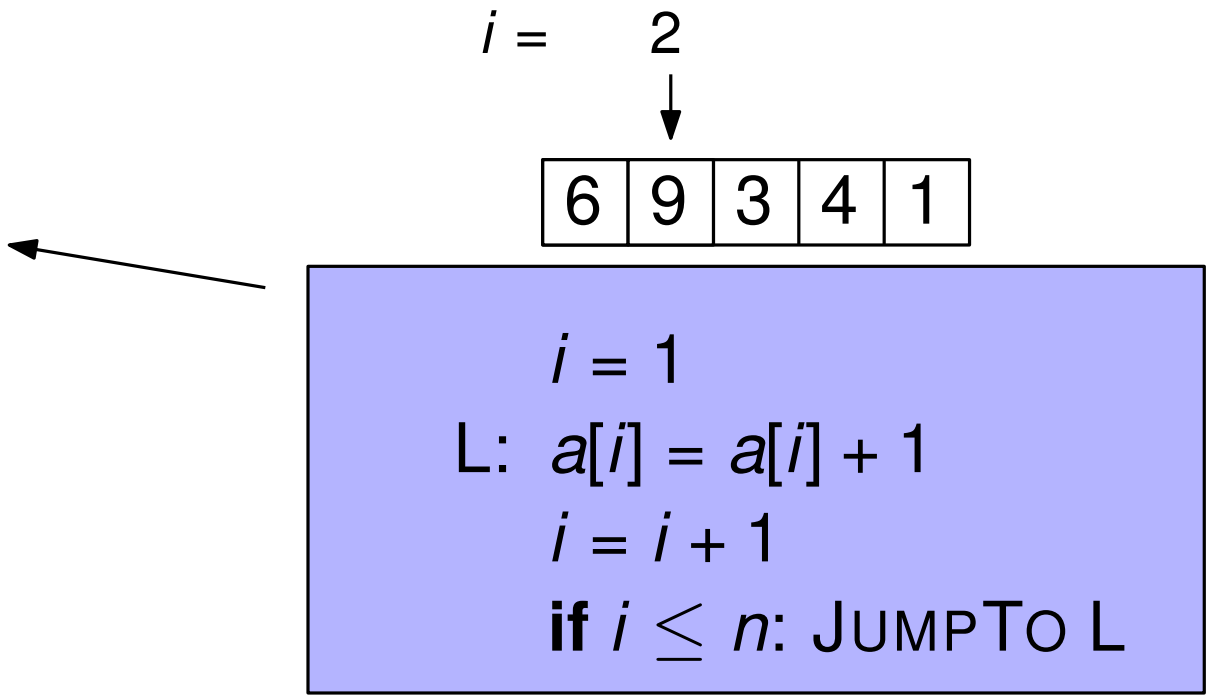

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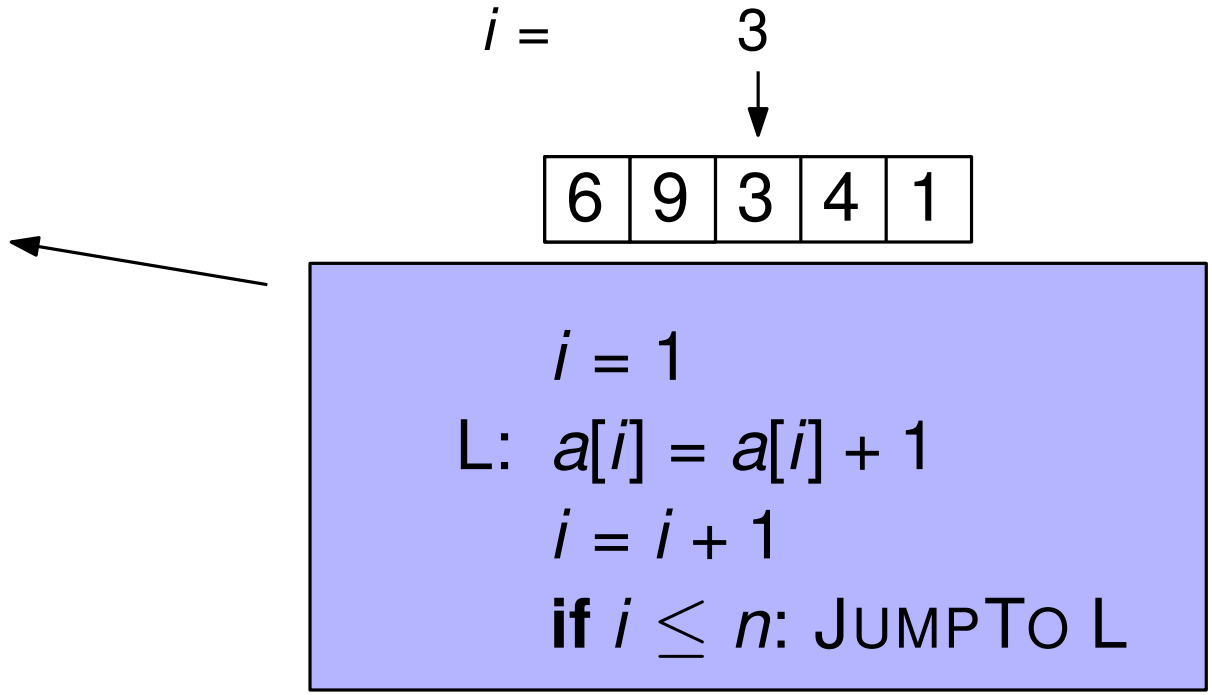

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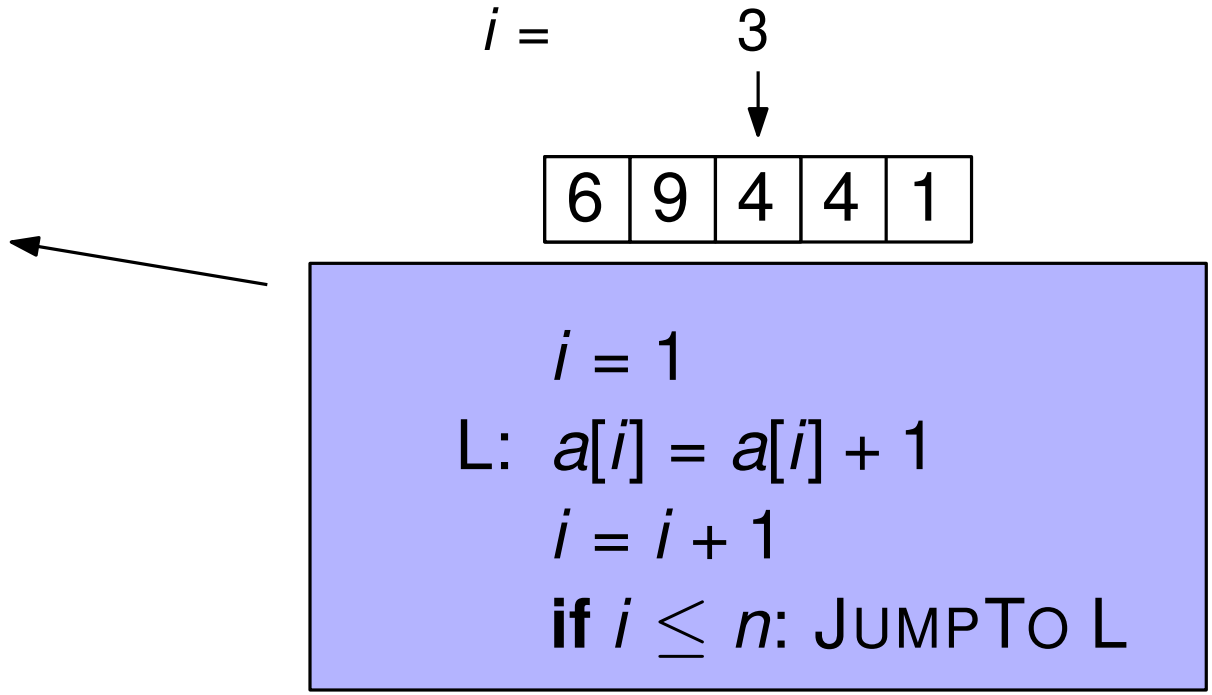

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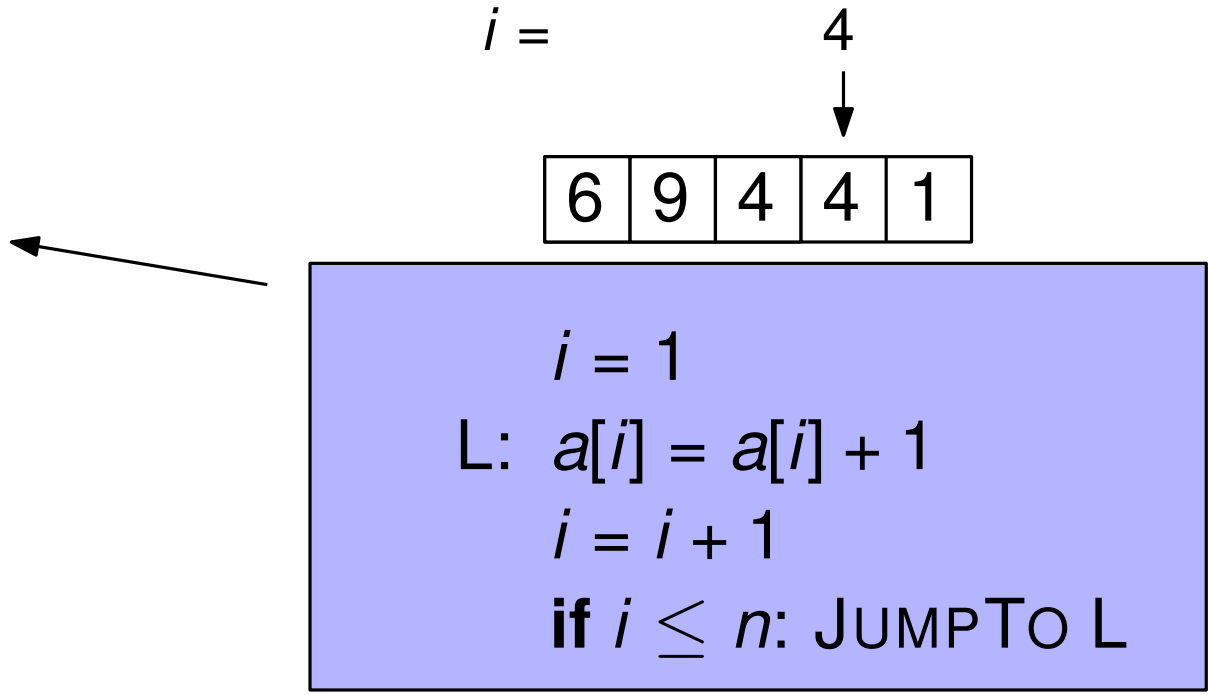

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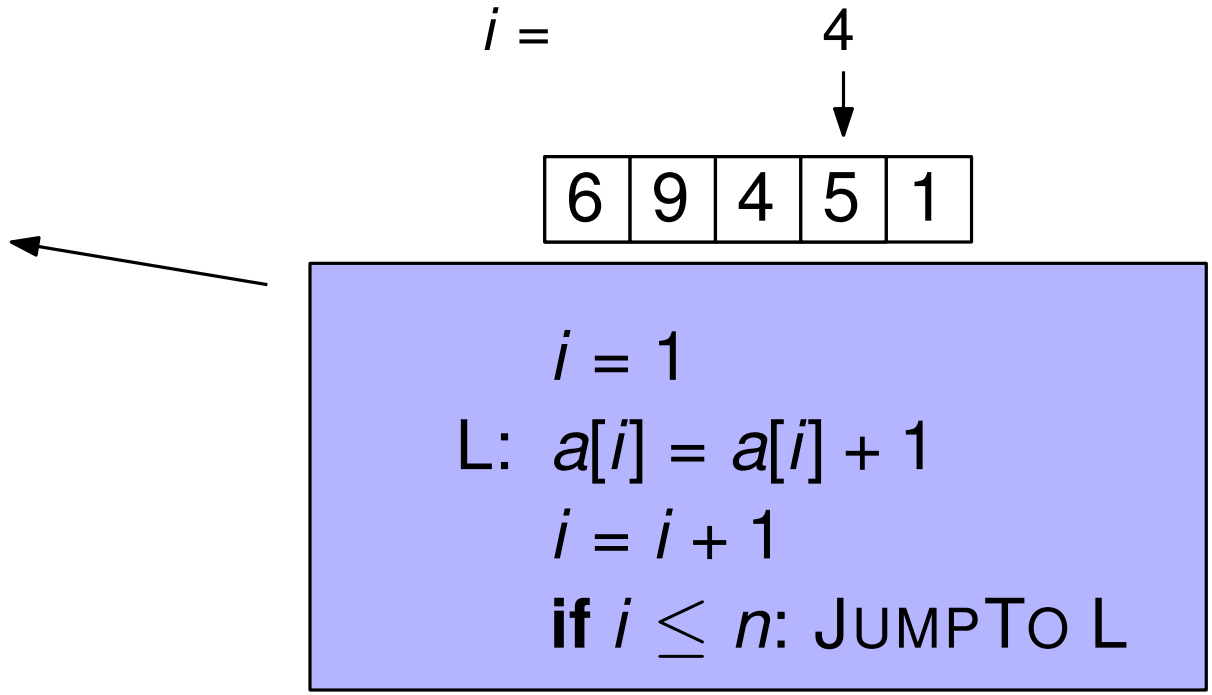

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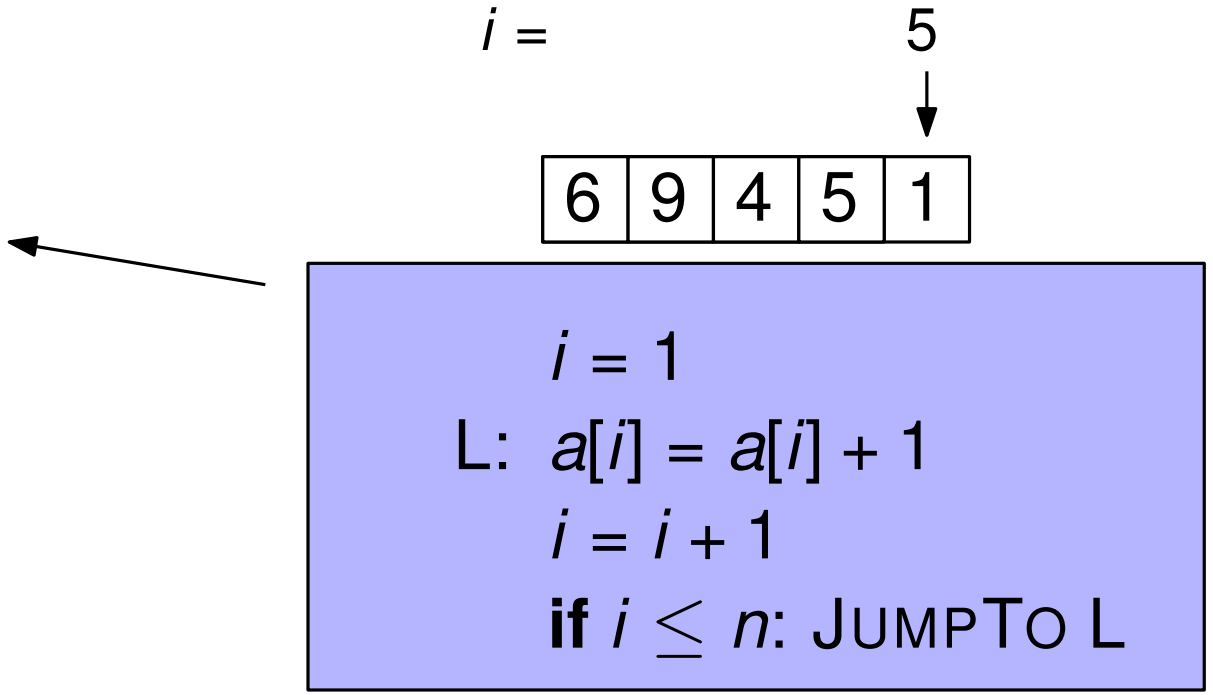

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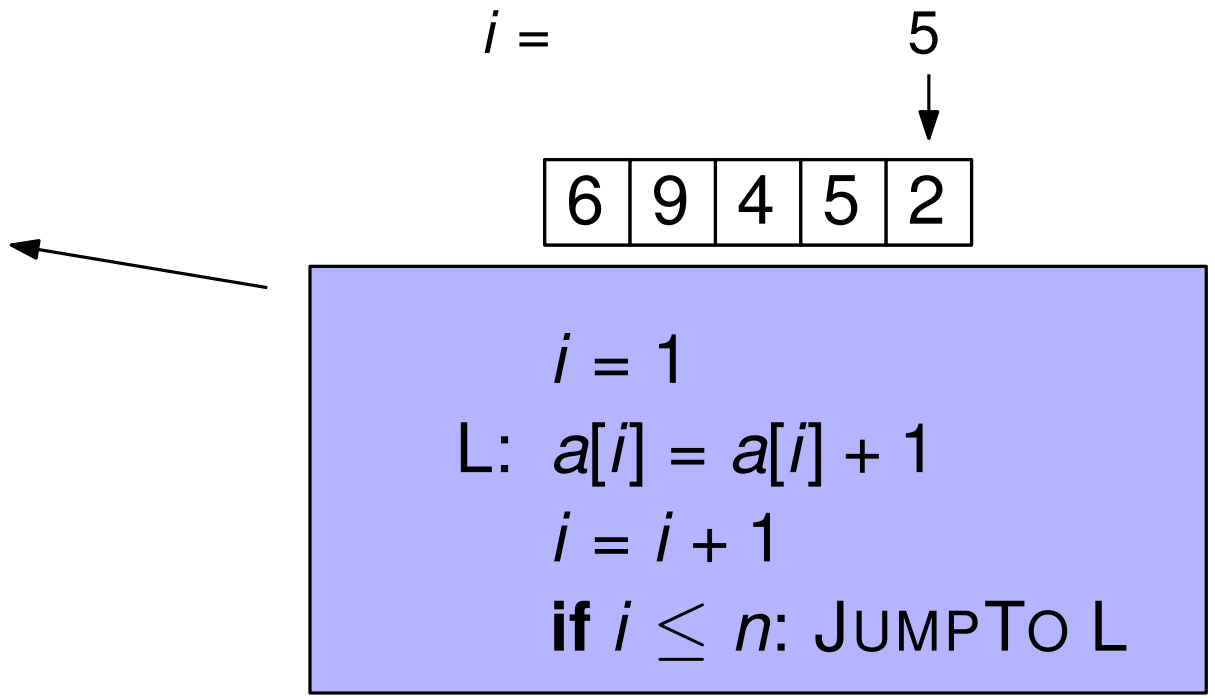

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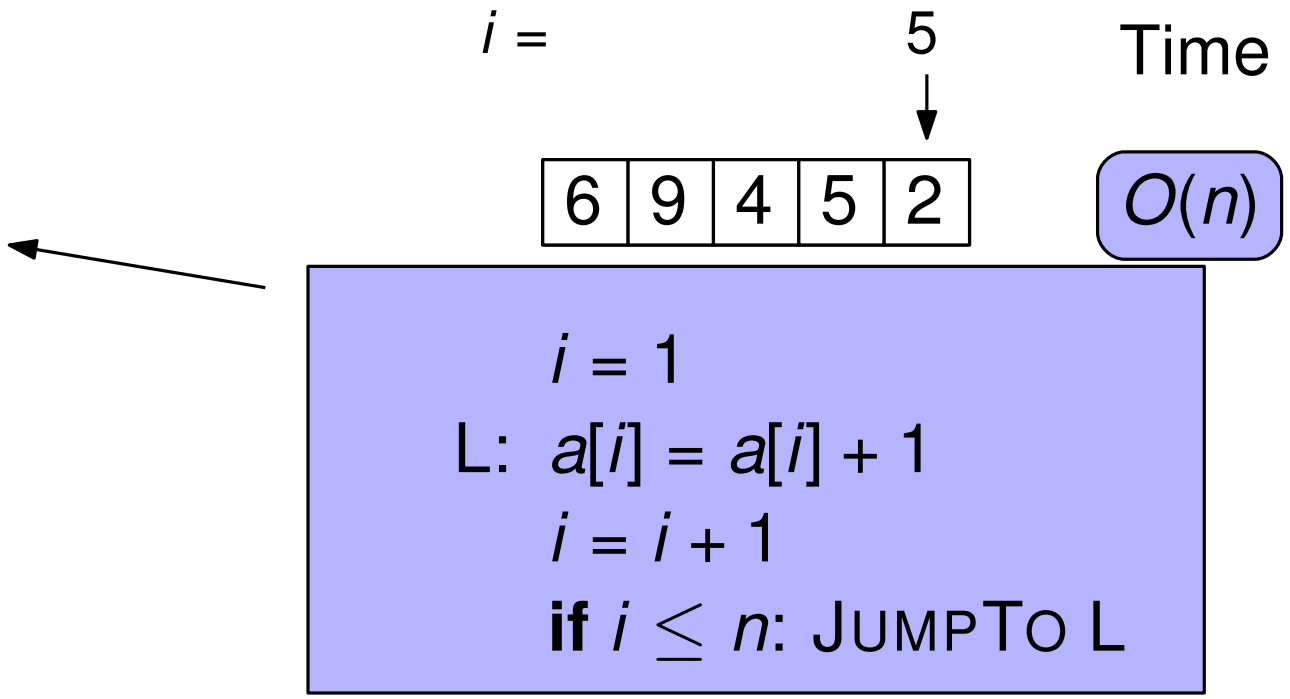

Simple example

Time

for $i = 1$ to n **do**
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$O(n)$

for $i = 1$ to n **in parallel do**
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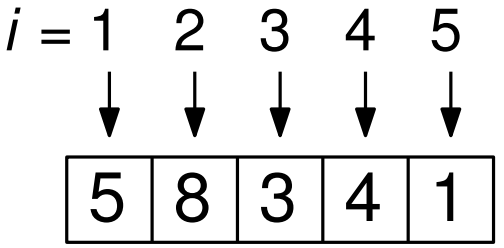

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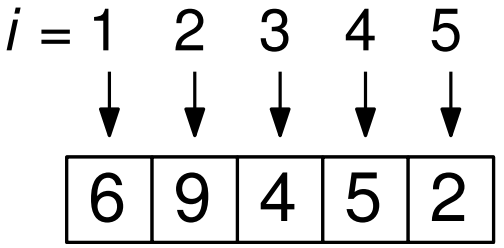

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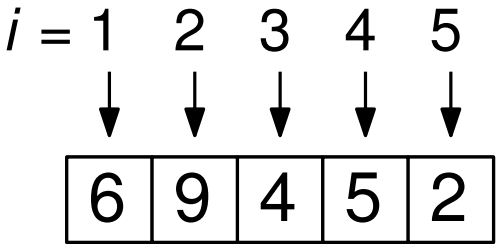

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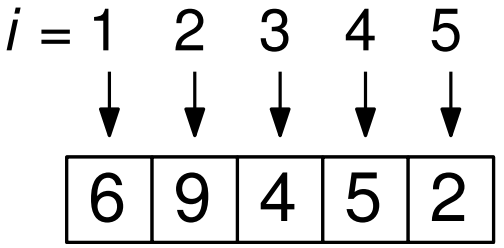

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Parallel Time = time of the slowest thread

More complex example: Prefix Sums

for $i = 2$ to n **do**

    $a[i] = a[i] + a[i-1]$

return $a[n]$

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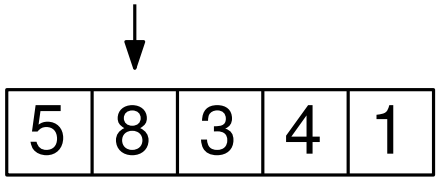

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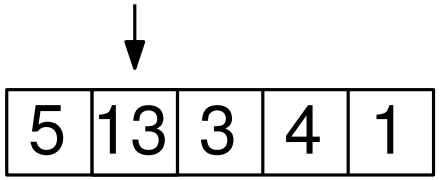

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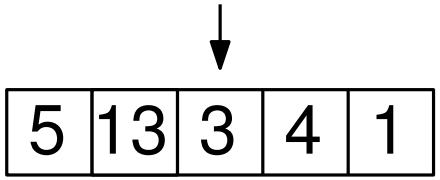

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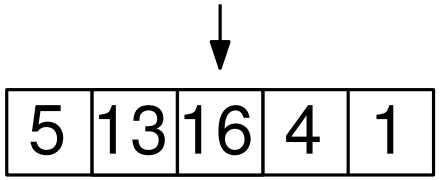

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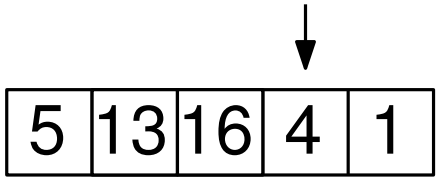

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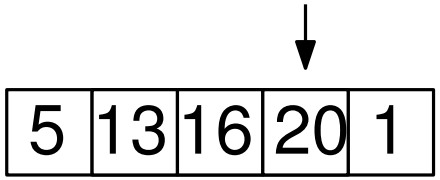

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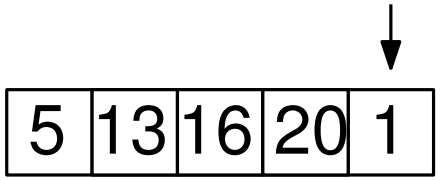

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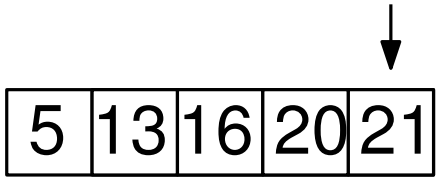

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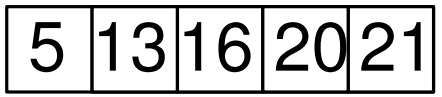

$O(n)$

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5	13	16	20	21

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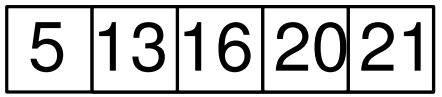

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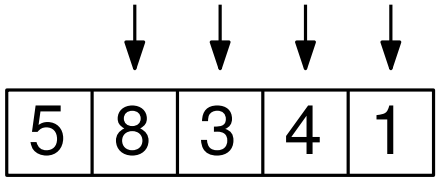

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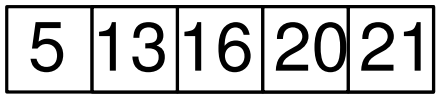

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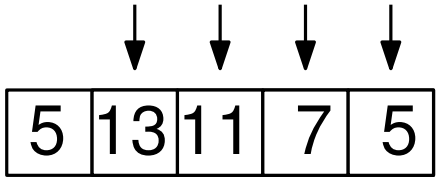

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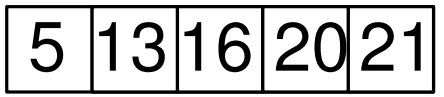

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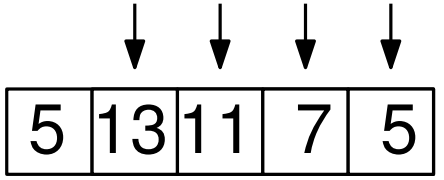

$O(1)$

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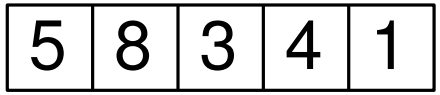
Parallel Prefix Sums



Parallel Prefix Sums

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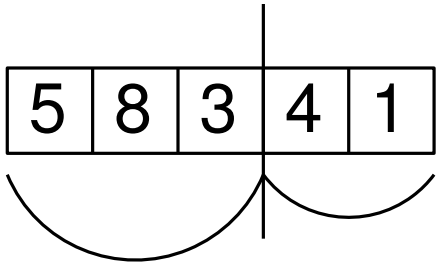

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5	8	3	4	1

5	8	3

4	1

Parallel Prefix Sums

5	8	3	4	1

5	13	16

4	5

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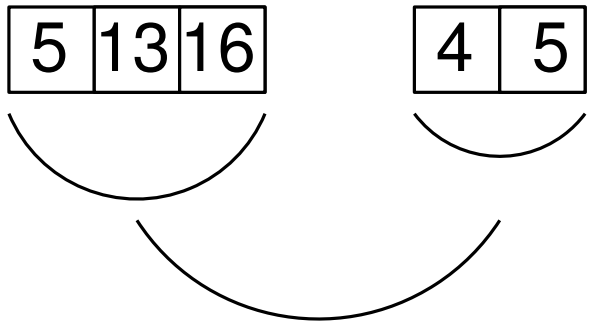

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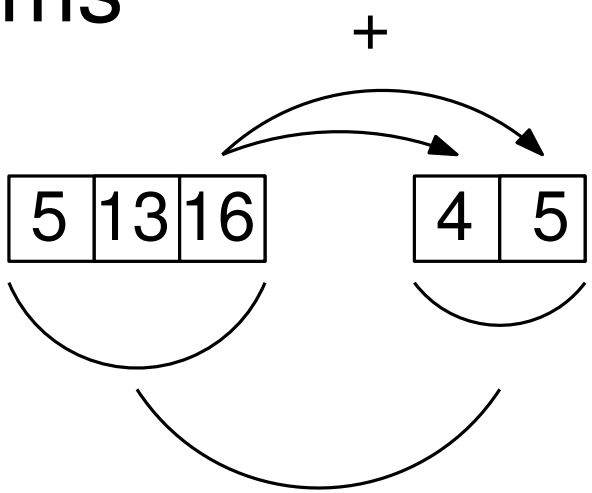

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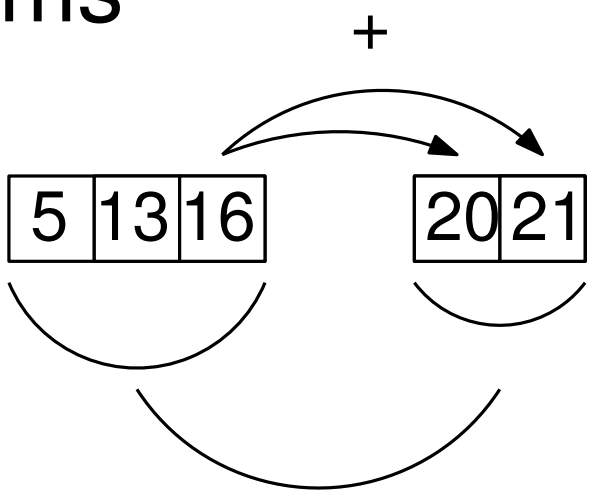

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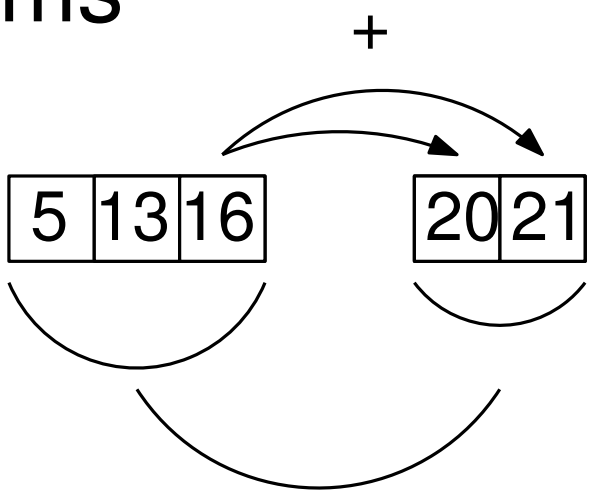


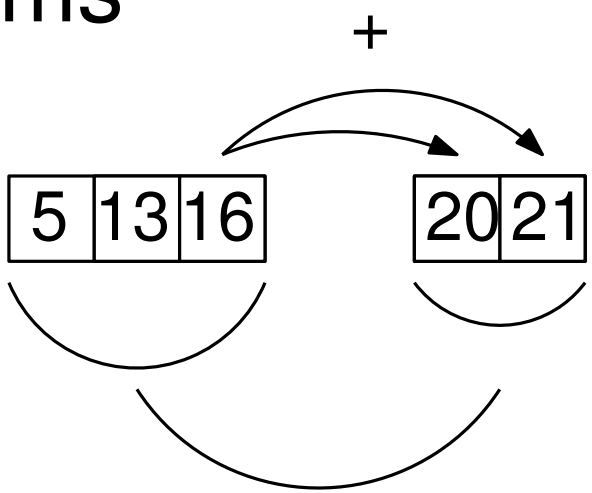
function PREFIX-SUMS(A, i, j)
  if $i \geq j$ **then return** ▷ Base case
  $mid = \left\lfloor \frac{i+j}{2} \right\rfloor$

  PREFIX-SUMS(A, i, mid)
  PREFIX-SUMS($A, mid + 1, j$)

  for $k = mid + 1$ to j **do**
    $A[k] = A[k] + A[mid]$

Parallel Prefix Sums

5	8	3	4	1



function PREFIX-SUMS(A, i, j)
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  $mid = \left\lfloor \frac{i+j}{2} \right\rfloor$

  PREFIX-SUMS(A, i, mid)
  PREFIX-SUMS($A, mid + 1, j$)

  for $k = mid + 1$ to j **do**
    $A[k] = A[k] + A[mid]$

$$\begin{aligned} T(n) &= 2T(n/2) + O(n) \\ &= O(n \log n) \end{aligned}$$

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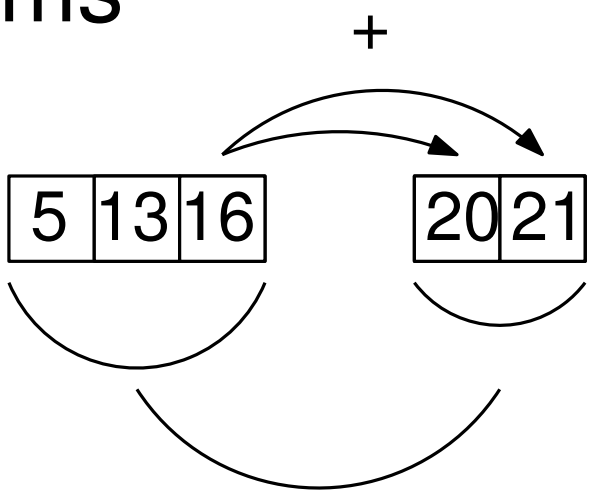

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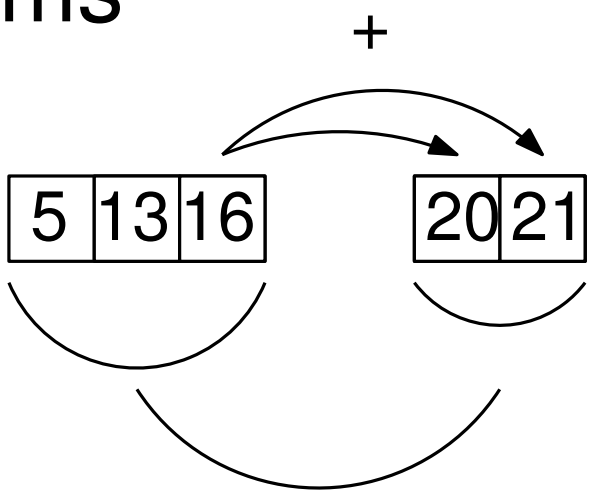

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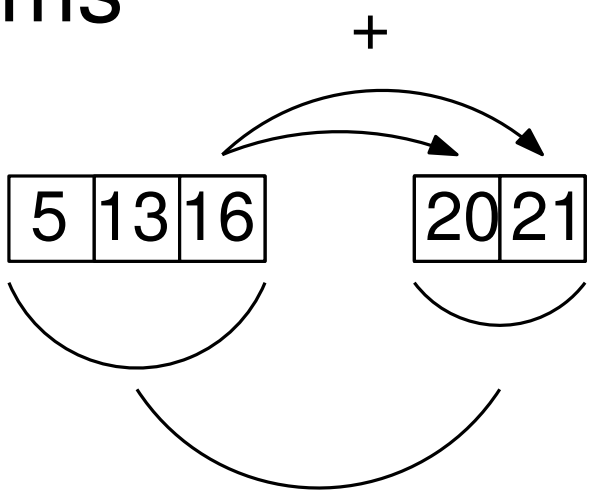

function PREFIX-SUMS(A, i, j)
  if $i \geq j$ **then return** ▷ Base case
  $mid = \left\lfloor \frac{i+j}{2} \right\rfloor$
  spawn
    PREFIX-SUMS(A, i, mid)
    PREFIX-SUMS($A, mid + 1, j$)
  sync
  for $k = mid + 1$ to j **in parallel** **do**
    $A[k] = A[k] + A[mid]$

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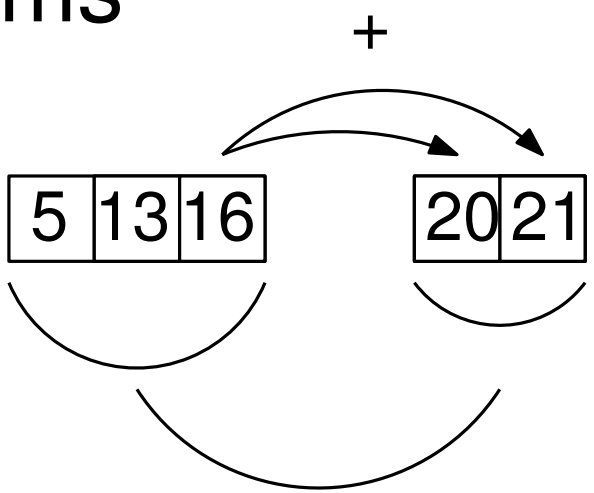

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    $A[k] = A[k] + A[mid]$

(t_1, t_2) = STARTTWOTHREADS()
t_1 **do**: PREFIX-SUMS(A, i, mid)
t_2 **do**: PREFIX-SUMS($A, mid + 1, j$)
WAITUNTILFINISHED(t_1, t_2)

$$\begin{aligned} T(n) &= 2T(n/2) + O(1) \\ &= O(n) \end{aligned}$$

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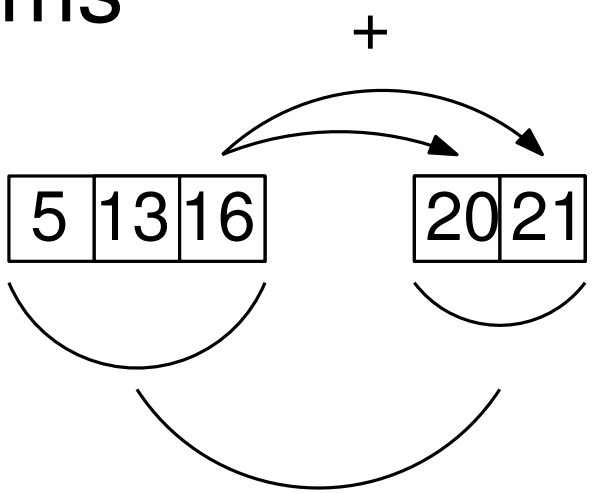

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$$T(n) = \max\left\{T\left(\left\lceil \tfrac{n}{2} \right\rceil\right), T\left(\left\lfloor \tfrac{n}{2} \right\rfloor\right)\right\} + O(1)$$
$$\leq T(n/2) + O(1)$$

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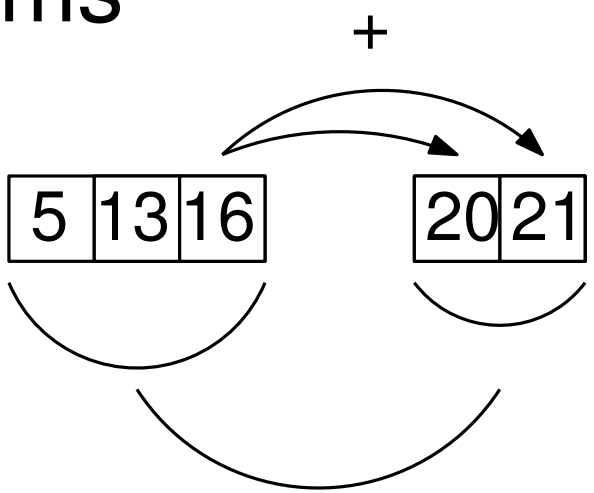

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$$\leq T(n/2) + O(1)$$
$$= O(\log n)$$

Recursion vs. parallel **for** loop

Parallel Sorting

Parallel Sorting

function MERGESORT(A, i, j)
  if $i \geq j$ **then return** ▷ Base case
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Parallel Sorting

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$$T(n) = 2T(n/2) + T_{\text{MERGE}}$$
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$$\begin{aligned} T(n) &= 2T(n/2) + T_{\text{MERGE}} \\ &= 2T(n/2) + O(n) \\ &= O(n \log n) \end{aligned}$$

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With parallel merging

Parallel Sorting

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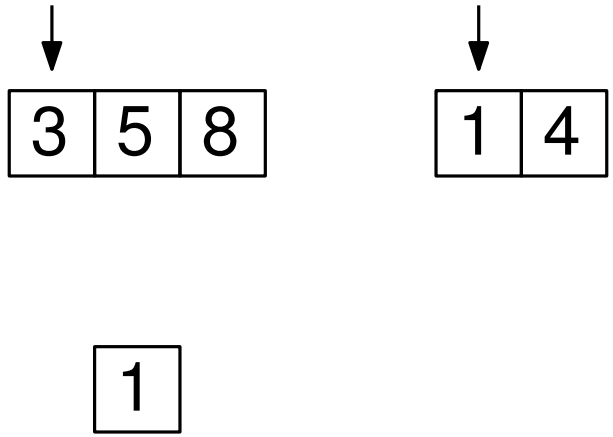
$$T(n) = T(n/2) + T_{\text{MERGE}}$$
$$= T(n/2) + O(\log n)$$
$$= O(\log^2 n)$$

With parallel merging

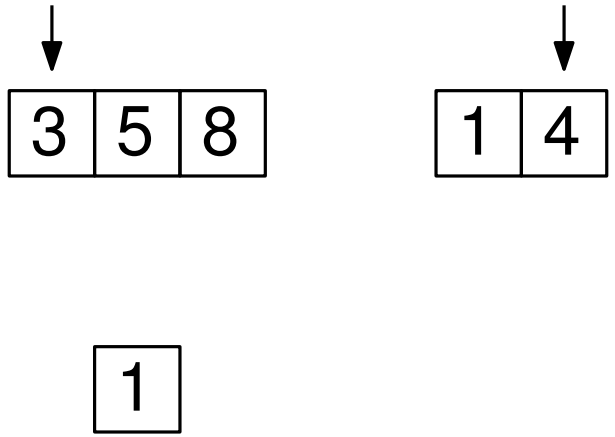
Parallel Merging

Parallel Merging

Parallel Merging



Parallel Merging



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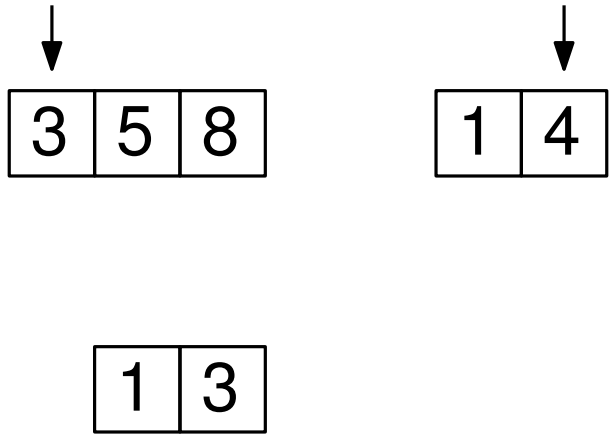

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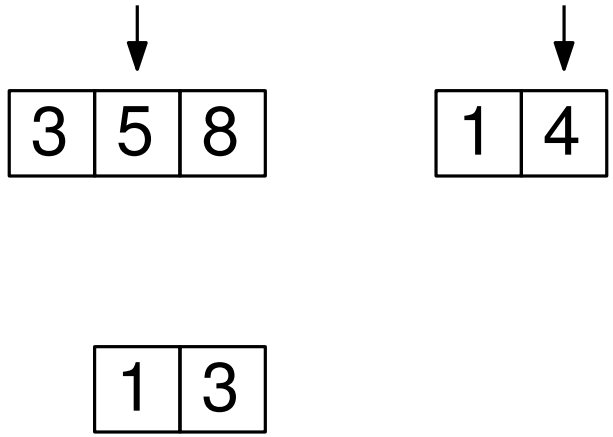

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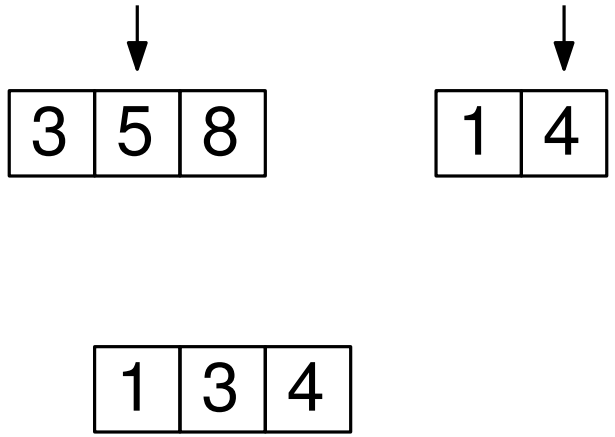

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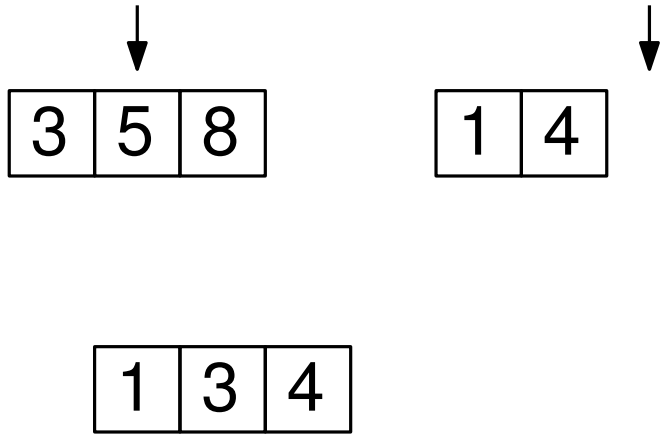

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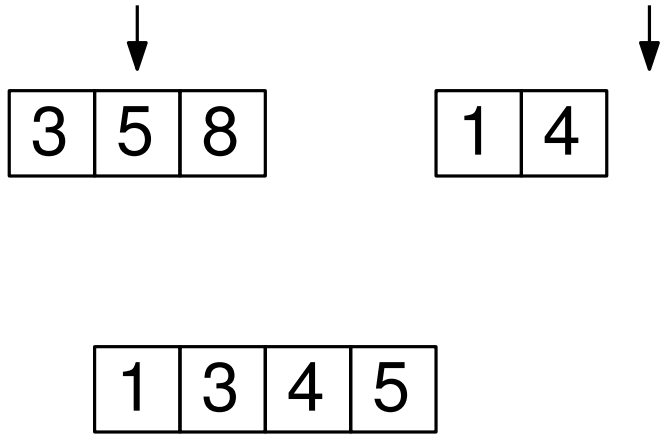

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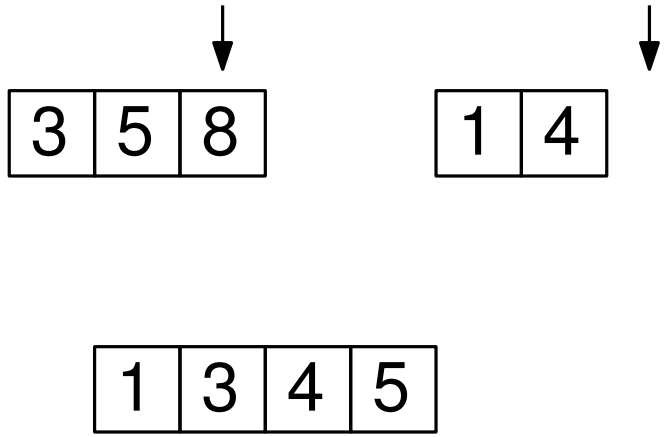

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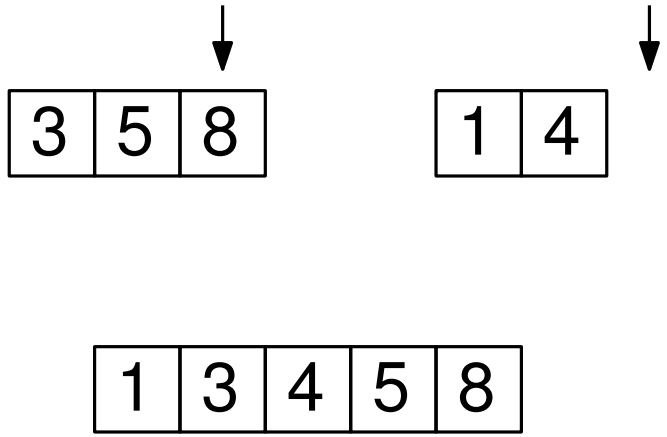

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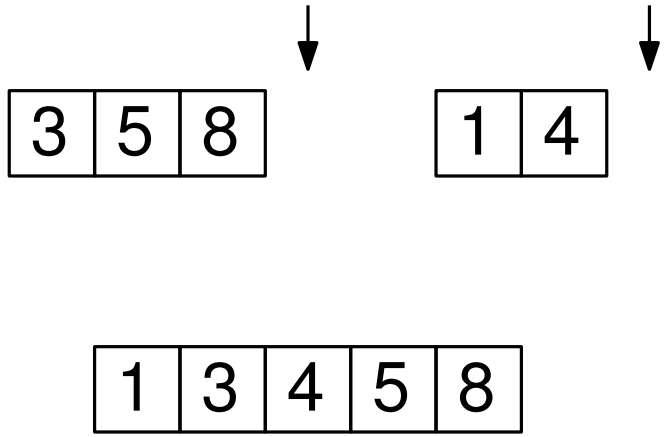

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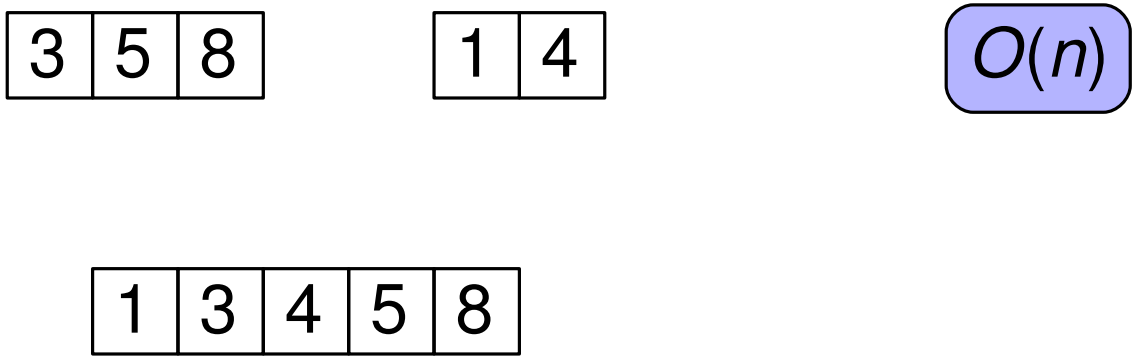

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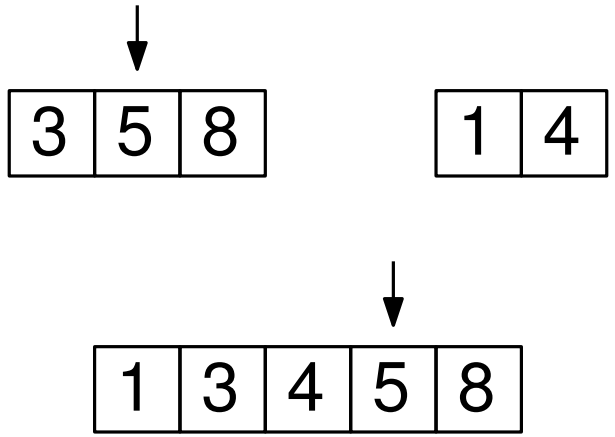
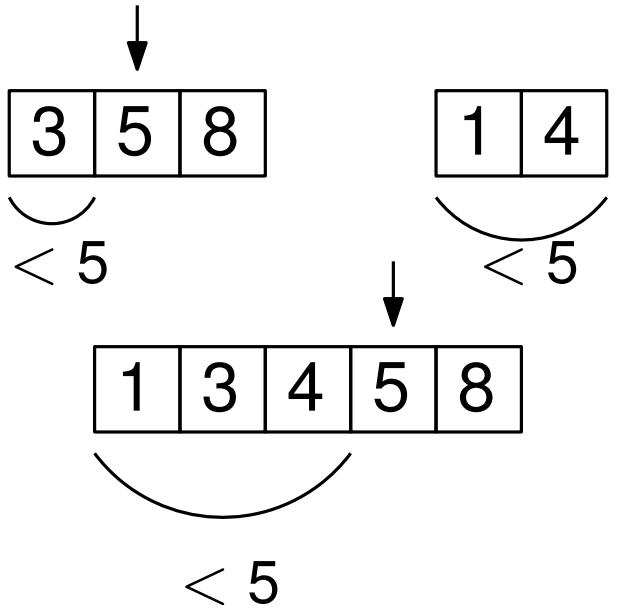


$O(n)$

Parallel Merging



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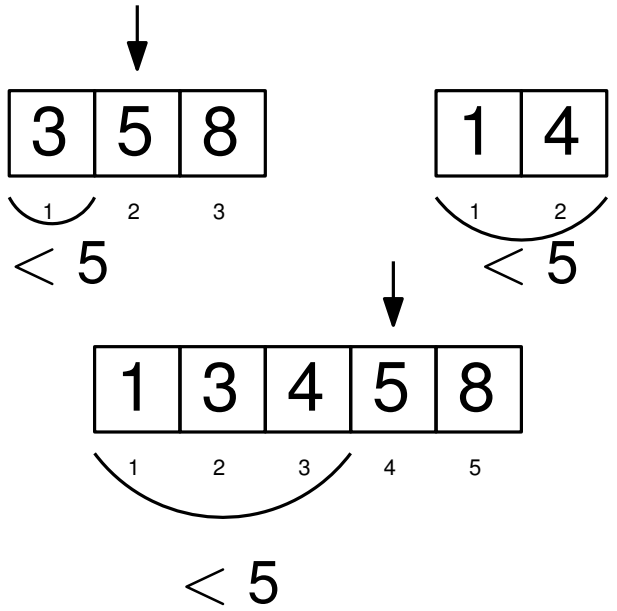

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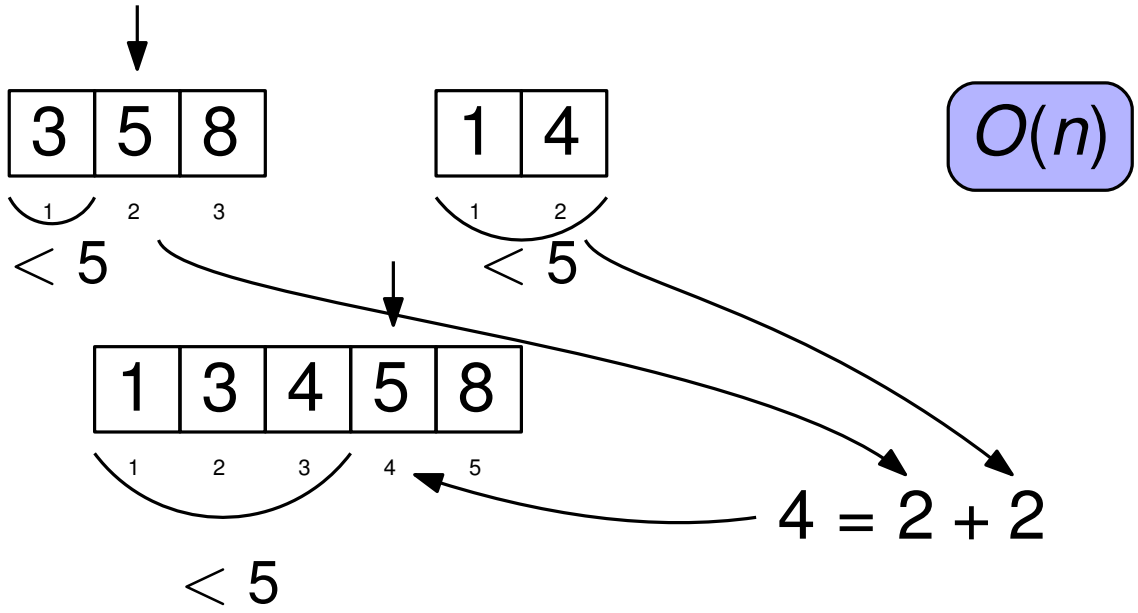

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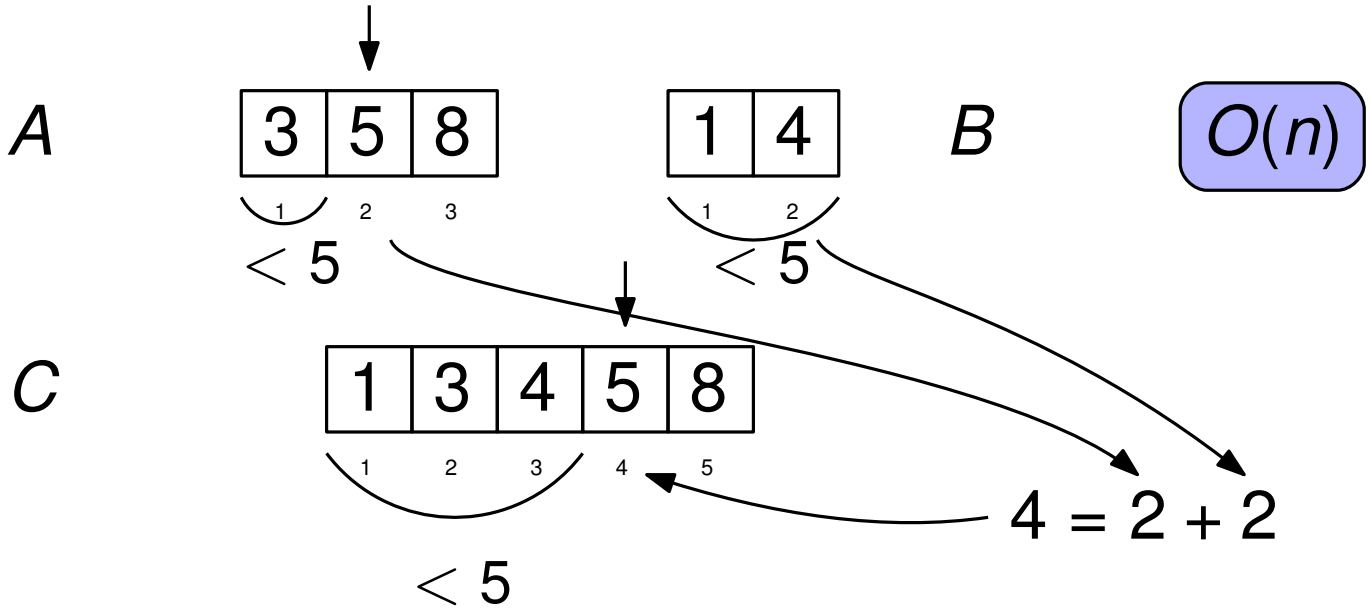

function MERGE(A, B, C)
    for $i = 1$ to $|A|$ **in parallel do**
        $k = i$ + PREDECESSOR($A[i]$, B)
        $C[k] = A[i]$
    for $j = 1$ to $|B|$ **in parallel do**
        $k = j$ + PREDECESSOR($B[j]$, A)
        $C[k] = B[j]$

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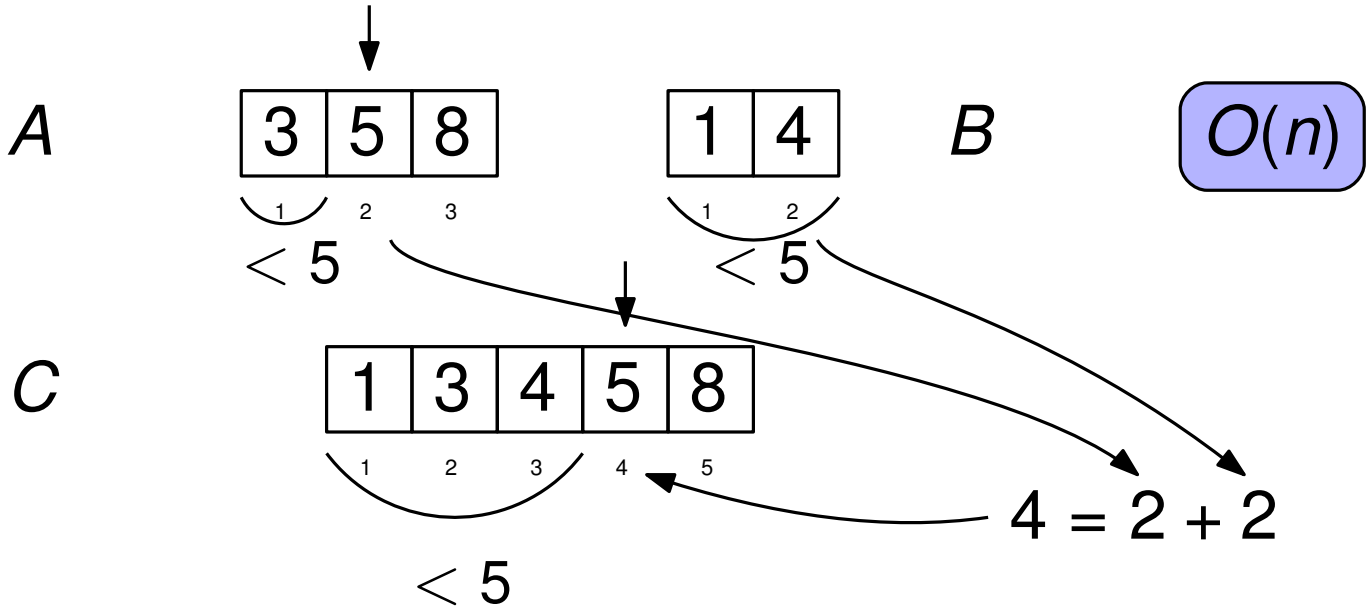

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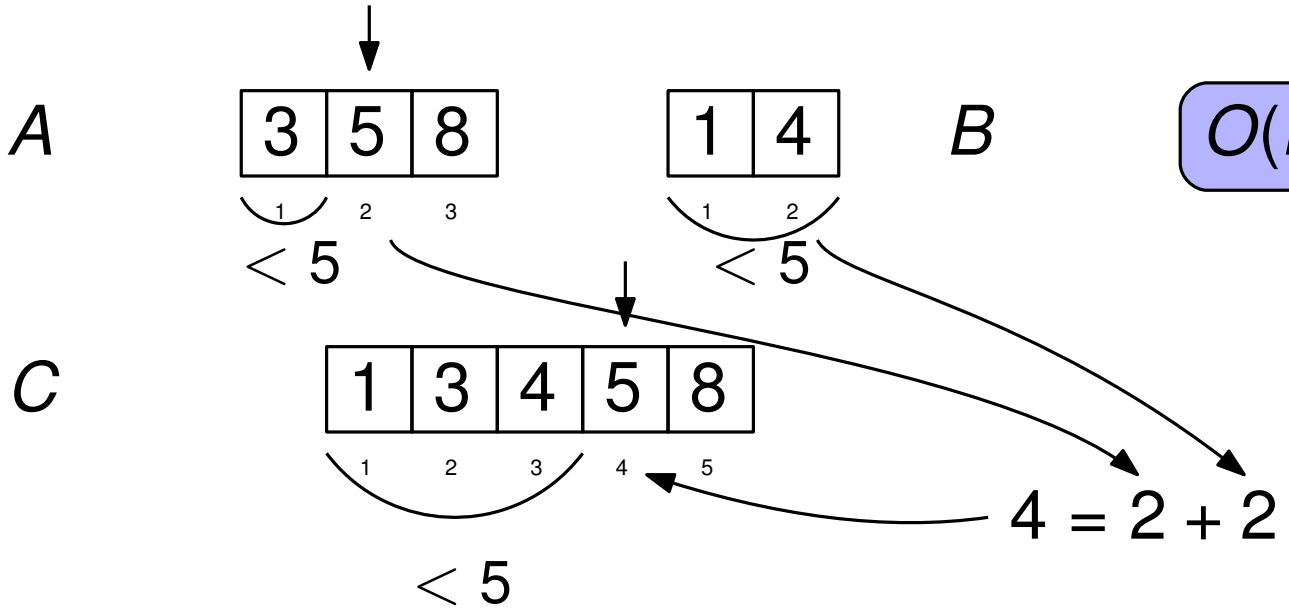
function PREDECESSOR(x, A)
  for $i = 1$ to $|A|$ **do**
    if $A[i] > x$ **then**
      return $i - 1$
  return $|A|$

Parallel Merging



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Still $O(n)$

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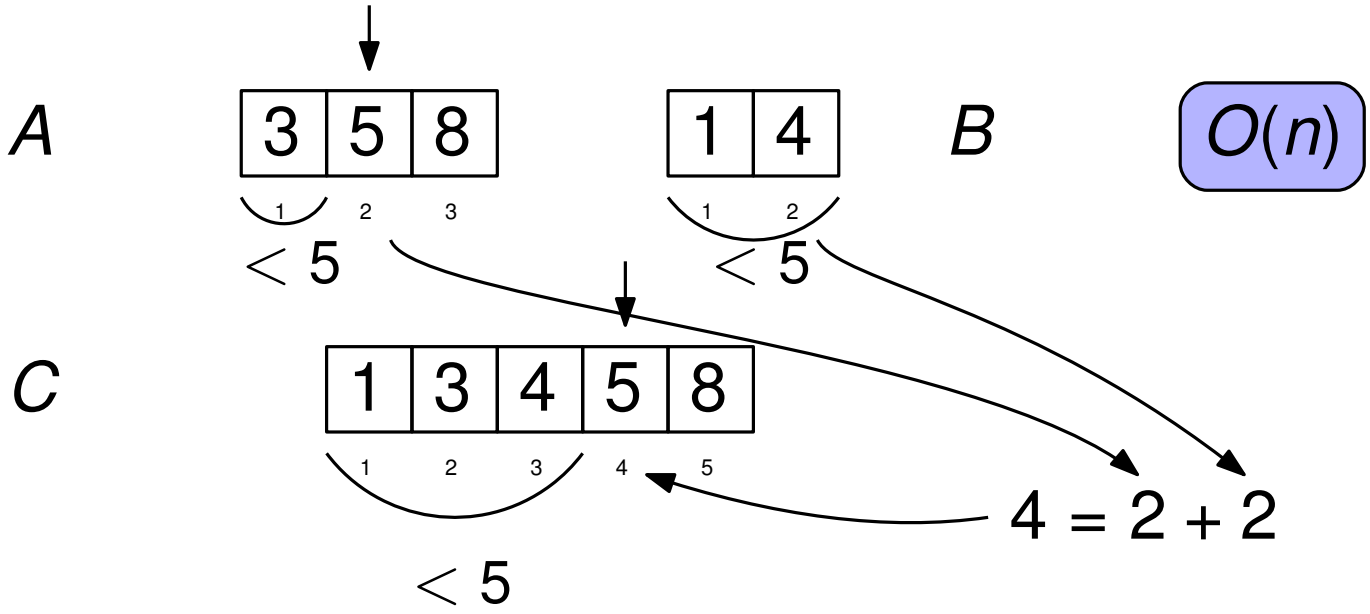

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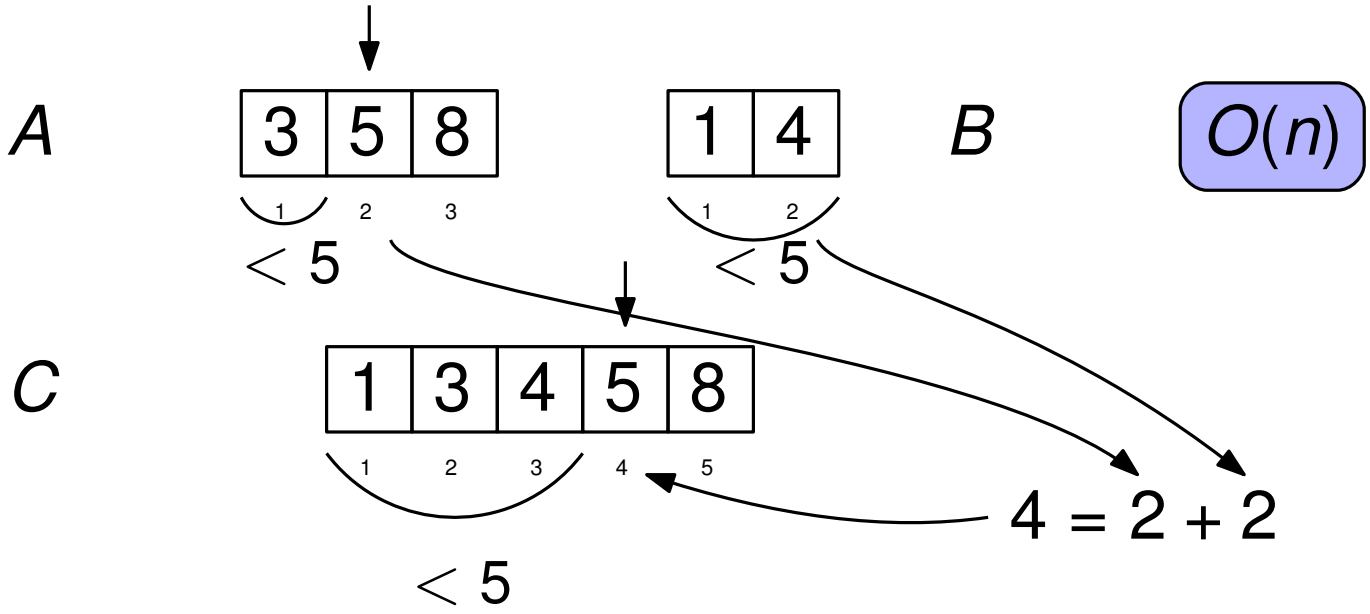
function PREDECESSOR(x, A)
  return BINARYSEARCH(x, A)

Parallel Merging



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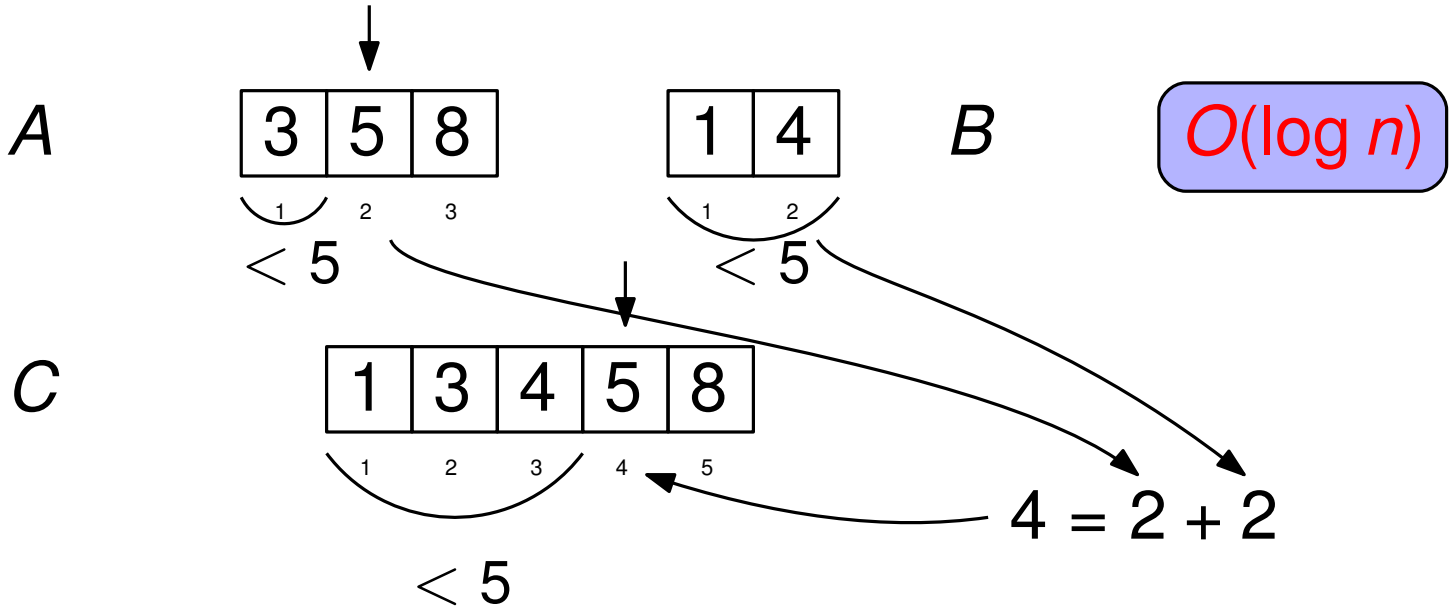
  return BINARYSEARCH(x, A)

$O(\log n)$

Parallel Merging



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Work vs Parallel Time

- Work: Total # of operations = Sequential runtime: $W = T_1$
- (Parallel) Time: # of operations of slowest thread: T_∞

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$W = O(n)$
$T_\infty = O(1)$

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        PREFIX-SUMS(A, i, mid)
        PREFIX-SUMS($A, mid + 1, j$)
    sync
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  for $k = mid + 1$ to j **in parallel do**
    $A[k] = A[k] + A[mid]$

$W = 2W(n/2) + O(n)$

Work vs Parallel Time

- Work: Total # of operations = Sequential runtime: $W = T_1$
- (Parallel) Time: # of operations of slowest thread: T_∞

function PREFIX-SUMS(A, i, j)
  if $i \geq j$ **then return** ▷ Base case
  $mid = \left\lfloor \frac{i+j}{2} \right\rfloor$
  spawn
    PREFIX-SUMS(A, i, mid)
    PREFIX-SUMS($A, mid + 1, j$)
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Work vs Parallel Time

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function MERGE(A, B, C)
  for $i = 1$ to $|A|$ **in parallel do**
    $k = i +$ PREDECESSOR$(A[i], B)$
    $C[k] = A[i]$
  for $j = 1$ to $|B|$ **in parallel do**
    $k = j +$ PREDECESSOR$(B[j], A)$
    $C[k] = B[j]$

function PREDECESSOR(x, A)
  return BINARYSEARCH(x, A)

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$$T_p = O(\frac{W}{P} + T_\infty)$$