

Problem Set 7

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Due: Tuesday, April 29, 2025 at 1:30pm

1 Counting Bitstrings (25 pts)

How many bitstrings of length 10 contain either at least five consecutive 0's or at least five consecutive 1's? Explain/justify your answer. *Hint: Take careful consideration to ensure that you do not overcount bitstrings.*

2 Farmer's Fruit (15 pts)

A farmer has 8 apples, 5 bananas, and 6 mangos for sale. As this farmer only grows organic fruit, each of the pieces of fruit are of different shapes and sizes (i.e., they are distinct from each other). How many ways are there to buy 5 pieces of fruit such that at least 1 fruit of each type is bought? Explain/justify your answer. (Do not simplify your answer, leave the expression in a form that allows you to concisely justify its correctness.)

3 Baker's Dozen (20 pts)

A bakery has croissants, apple turnovers, glazed donuts, malasadas, and cheesecake. Due to the bakery's professional standards, all pastries and cakes of the same type are identical.

- (5 pts) How many ways are there to purchase one baker's dozen (i.e., 13) of any of the bakery's products? Explain/justify your answer.
- (5 pts) How many ways are there to purchase two baker's dozen (i.e., 26) of the bakery's products with at least two of each product? Explain/justify your answer.
- (10 pts) How many ways are there to purchase two baker's dozen (i.e., 26) of the bakery's products with at least two croissants, at least three apple turnovers, at least one glazed donut, at least one malasada, and no more than two cheesecakes? Explain/justify your answer.

4 Splitting Piles (40 pts)

Let n be a positive integer. Bob has a pile of n colored balls (all different colors). First he splits them into two piles; then he picks one of the piles with at least two balls in it and splits it into two; he repeats this until each pile consists of a single ball.

- (5 pts) How many steps of splitting piles into two does it take until each pile consists of a single ball?
- (15 pts) Using strong induction, prove that your answer to part (a) is correct. *Hint 1: Consider splitting a pile into two piles of variable sizes. Hint 2: Only a single base case is needed.*
- (20 pts) What is the number of different ways in which Bob can carry out this procedure of splitting piles? Explain/justify your answer. (Do not simplify your answer, leave the expression in a form that allows you to concisely justify its correctness.) *Hint: The number of ways to perform the procedure of splitting piles is equivalent to the number of ways to perform the reverse procedure of combining piles into a single pile.*