ICS 141: Discrete Mathematics for Computer Science I Spring 2025

Problem Set 6

Kyle Berney *Due: Tuesday, April 8, 2025 at 1:30pm*

1 Basis Representation Conversions (15 pts)

Convert the following integers into the specified base representation. Show your work.

(a) **(3 pts)** Convert 234 into binary (base 2).

(b) **(3 pts)** Convert 5432 into quinary (base 5).

(c) **(3 pts)** Convert 6789 into septenary (base 7).

(d) **(3 pts)** Convert $(110100010111)_2$ into octal (base 8).

(e) **(3 pts)** Convert $(1010100001001110)_2$ into hexadecimal (base 16).

2 Integral Linear Combination (10 pts)

(a) **(10 pts)** Use the extended Euclidean algorithm to express the $\text{GCD}(1001, 100001)$ as a integral linear combination of 1001 and 100001. Show your work.

3 Systems of Linear Congruences (15 pts)

(a) **(5 pts)** Use the procedure based on trial division described in the lecture slides of Chapter 4.3 to find the prime factorization of 2093. Show your work.

(b) **(10 pts)** Use the Chinese Remainder Theorem to solve the congruence

$$5x \equiv 2 \pmod{2093}$$

Show your work.

4 Most Significant Digit (30 pts)

Let b be an integer such that $b > 1$. For all $n \in \mathbb{Z}^+$, prove that the most significant digit in the representation of n to the base b is strictly greater than the sum of all its k least significant digits. That is, for $n = (a_k a_{k-1} \dots a_0)_b$

$$a_k b^k > \sum_{i=0}^{k-1} a_i b^i \ .$$

Hint: The Basis Representation Theorem (Theorem 1 in the lecture notes of Chapter 4.2) states that each digit is non-negative and strictly less than b.

5 Sum of Perfect Squares (30 pts)

(a) **(5 pts)** Let $x \in \mathbb{Z}$. Prove that x is even if and only if $x^2 \equiv 0 \pmod 4$.

(b) **(5 pts)** Let $x \in \mathbb{Z}$. Prove that x is odd if and only if $x^2 \equiv 1 \pmod 4$.

(c) **(20 pts)** Let w, x, y, and z be positive integers and suppose that $w^2 = x^2 + y^2 + z^2$. Using part (a) and (b), prove that w is even only if x, y, and z are all even. *Hint: Consider all cases of x, y, and z being odd or even.*