ICS 141: Discrete Mathematics for Computer Science I Spring 2025

Problem Set 3

Kyle Berney *Due: Tuesday, February 25, 2025 at 1:30pm*

1 Subset Method for Proving Equality (20 pts)

Prove the following statement using the subset method for showing set equality.
For sets A, B, and C, if $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then $B = C$. *Hint: Use a proof by cases.*

2 Proving Bijections (30 pts)

Let $f : \mathbb{R} - \{-1\} \to \mathbb{R} - \{1\}$ be a function such that

$$f(x) = \frac{x}{x+1} .$$

Prove or disprove whether f is a bijection by showing that either: f is both an injection and a surjection; or f is not an injection or not a surjection.

3 Countable Sets (30 pts)

Prove that the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable by listing the elements of the set as a sequence. *Hint: Read Chapter 2.5 Example 4 in the textbook, which shows that $\mathbb{Q}^+$ is countable.*

4 Simplifying Logarithms (20 pts)

Simplify the following expressions. Show your work.

(a)

$$\frac{\log_5 \sqrt{n} \cdot \log_5 25 + 2 \log_5 n}{\log_5 9}$$

(b)

$$\frac{\log_2(8n^3) - \log_4 n^2}{\log_8 n}$$