ICS 141: Discrete Mathematics for Computer Science I Spring 2025

Problem Set 2

Kyle Berney *Due: Tuesday, February 11, 2025 at 1:30pm*

1 Proving Odd Parity (20 pts)

Prove that if n is an odd integer then $n^2 - n - 5$ is odd.

2 Difference of Two Squares (20 pts)

Let a and b be integers. Prove that if $a^2 - b^2$ is even, then a and b are both even or both odd.

(a) **(10 pts)** Provide a proof by contraposition.

(b) **(10 pts)** Provide a proof by contradiction.

3 Product of a Rational and Irrational Number (30 pts)

Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

4 Existence of a Power of Two (30 pts)

Let n be a positive integer. Prove that the closed interval $[n, 2n]$ contains a power of two. (A real number r is in the closed interval $[n, 2n]$ if $n \leq r \leq 2n$.) *Hint: Use a proof by cases.*