ICS 141: Discrete Mathematics for Computer Science I	Spring 2025
Problem Set 1	
Kyle Berney	*Due: Tuesday, January 28, 2025 at 1:30pm*

1 Truth Value of Propositions (20 pts)

Determine whether the proposition is true or false. Justify your answers.

(a) **(4 pts)** $2 + 2 = 3$ if and only if $2 \cdot 2 = 5$.

(b) **(4 pts)** If it is raining, then it is raining.

(c) **(4 pts)** If $1 < 0$, then $2 = 3$.

(d) **(4 pts)** If $2^3 = 8$, then $\sqrt{5} = 2$.

(e) **(4 pts)** If $1 + 1 = 2$ or $5 + 4 = 8$, then $2 + 2 = 4$ and $3^3 = 26$.

2 Truth Tables (30 pts)

Determine whether the proposition is a tautology, contradiction, or contingency by constructing a truth table. The truth table must be in the exact format as shown below or **zero points** will be awarded.

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(a) **(10 pts)** $(P \oplus Q) \Leftrightarrow (\neg P \vee R)$

(b) **(10 pts)** $((P \vee Q) \Rightarrow R) \vee \neg R$

(c) **(10 pts)** $((P \wedge Q) \Rightarrow R) \wedge (\neg R \wedge (P \wedge Q))$

3 Logical Equivalences (30 pts)

(a) **(15 pts)** Show that $((P \vee Q) \wedge \neg P) \Rightarrow Q \equiv \text{T}$ using only logical equivalences.

(b) **(15 pts)** Show that $((P \Rightarrow Q) \wedge (Q \Rightarrow \neg P)) \Rightarrow (P \wedge \neg P) \equiv P$ using only logical equivalences.

4 Translations using Quantifiers (20 pts)

Let $P(x, y)$ denote the statement "x's sibling is y", where the domain is all people. Express the following statements using $P(x, y)$, quantifiers, logical connectives, and mathematical operators. Any negations used should not preceed a quantifier (i.e., no negation is to the immediate left of a quantifier).

(a) **(5 pts)** Every person has exactly one sibling.

(b) **(5 pts)** Some people do not have siblings.

(c) **(5 pts)** Some people have at least two siblings.

(d) **(5 pts)** Everyone has exactly two siblings.