INFORMATION & COMPUTER SCIENCES
UNIVERSITY of HAWAIʻI at MĀNOA

Ch 1.4: Predicates and Quantifiers

ICS 141: Discrete Mathematics for Computer Science I

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Predicate Logic

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- We can denote predicates using a propositional function
  - Ex.
    - Let $P(x)$ denote the statement "x is greater than 3"
    - $Q(x, y)$: "$x = y + 3$"

Predicate Logic

- Once a value has been assigned to the variable(s) in a predicate, the statement becomes a proposition and has a truth value.
  - Ex.
    - Let $P(x)$ denote the statement "x is greater than 3"
    - $P(4)$ sets the value of $x = 4$.
      "4 is greater than 3" is **true**.
    - $P(2)$ sets the value of $x = 2$.
      "2 is greater than 3" is **false**.

Predicate Logic

- Once a value has been assigned to the variable(s) in a predicate, the statement becomes a proposition and has a truth value.
  - Ex.
    - $Q(x, y)$: "$x = y + 3$"
    - $Q(1, 2)$ sets the value of $x = 1$ and $y = 2$.
      $1 = 2 + 3$ is **false**.
    - $Q(3, 0)$ sets the value of $x = 3$ and $y = 0$.
      $3 = 0 + 3$ is **true**.

Preconditions and Postconditions

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- Definition: Preconditions are the conditions that must be true before execution.
- Definition: Postconditions are the conditions that must be true after execution, provided that the preconditions were satisfied.

Preconditions and Postconditions

- Ex. Consider the following program, designed to interchange the values of two variables `x` and `y`:

```
x = y;
y = z;
z = x;
```

- What are the preconditions and postconditions to verify correctness of this program?

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- *Hint:* The precondition should state something about the validity of the variables `x`, `y`, and `z`.

Preconditions and Postconditions

- Ex. Consider the following program, designed to interchange the values of two variables `x` and `y`:

```
x = y;
y = z;
z = x;
```

- What are the preconditions and postconditions to verify correctness of this program?

- Precondition: “The variables `x`,`y`, `z` have been correctly initialized such that, `x` and `y` are intialized to values `a` and `b`, respectively. ”

Preconditions and Postconditions

- Ex. Consider the following program, designed to interchange the values of two variables `x` and `y`:

```
x = y;
y = z;
z = x;
```

- What are the preconditions and postconditions to verify correctness of this program?
- *Hint #1:* What values do we expect a correct execution to assign to the variables?
- *Hint #2:* Which variables do we care about?

Preconditions and Postconditions

- Ex. Consider the following program, designed to interchange the values of two variables `x` and `y`:

```
x = y;
y = z;
z = x;
```

- What are the preconditions and postconditions to verify correctness of this program?

- Postcondition: "The variables `x` and `y` are set to values `b` and `a`, respectively."

Quantifiers

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  - Allows us to make statements about more than one object

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  - Allows us to make statements about more than one object
- Ex.
  - "All students completed the homework."
  - "Some students have perfect attendance."
  - "No one has an umbrella."

Universal Quantifier

- Definition: The universal quantifier of a propositional function $P(x)$, denoted $\forall x(P(x))$, is the statement:

  "$P(x)$ for all values of x in the domain."

  - The domain specifies the possible values of the variable.
- Read as "for all x, $P(x)$" or "for every x, $P(x)$".

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  - Known as a counterexample to $\forall x(P(x))$
- *Note:* Only a single counterexample is needed to show that $\forall x(P(x))$ is false!

Universal Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x < 2$"
  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?

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  - What is the truth value of $\forall x(P(x))$?
- Solution:
  - $P(3)$ produces the statement $3 < 2$, which is **false**.
  - Since we produced a counterexample,

$\forall x(P(x))$ is **false**.

Universal Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x^2 \geq x$"
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1. Simplify the inequality:

$$\begin{aligned} x^2 &\geq x \\ \Rightarrow x^2 - x &\geq 0 \\ \Rightarrow x(x-1) &\geq 0 \end{aligned}$$

Universal Quantifier

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  - Let $P(x)$ be the statement "$x^2 \geq x$"
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  - What is the truth value of $\forall x(P(x))$?

2. Solve for x at critical points:

$$x(x-1) = 0$$
$$\Rightarrow x = 0 \text{ and } x = 1$$

Universal Quantifier

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  - Let $P(x)$ be the statement "$x^2 \geq x$"
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3. Test values in intervals:
   (a) $(-\infty, 0)$

- Let $x = -1$

$$\begin{aligned}(-1)(-1-1) &= (-1)(-2) \\ &= 2 \\ &\geq 0\ .\ \textbf{True.}\end{aligned}$$

Universal Quantifier

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  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?

3. Test values in intervals:

   (b) $(1, \infty)$

- Let $x = 2$

$$
\begin{aligned}
(2)(2-1) &= (2)(1) \\
&= 2 \\
&\geq 0 \text{ . } \textbf{True.}
\end{aligned}
$$

Universal Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x^2 \geq x$"
  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?

3. Test values in intervals:

   (c) $(0, 1)$

- Let $x = 0.5$

$$(0.5)(0.5 - 1) = (0.5)(-0.5)$$
$$= -0.25$$
$$\not\geq 0$$. **False.**

Universal Quantifier

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  - Let $P(x)$ be the statement "$x^2 \geq x$"
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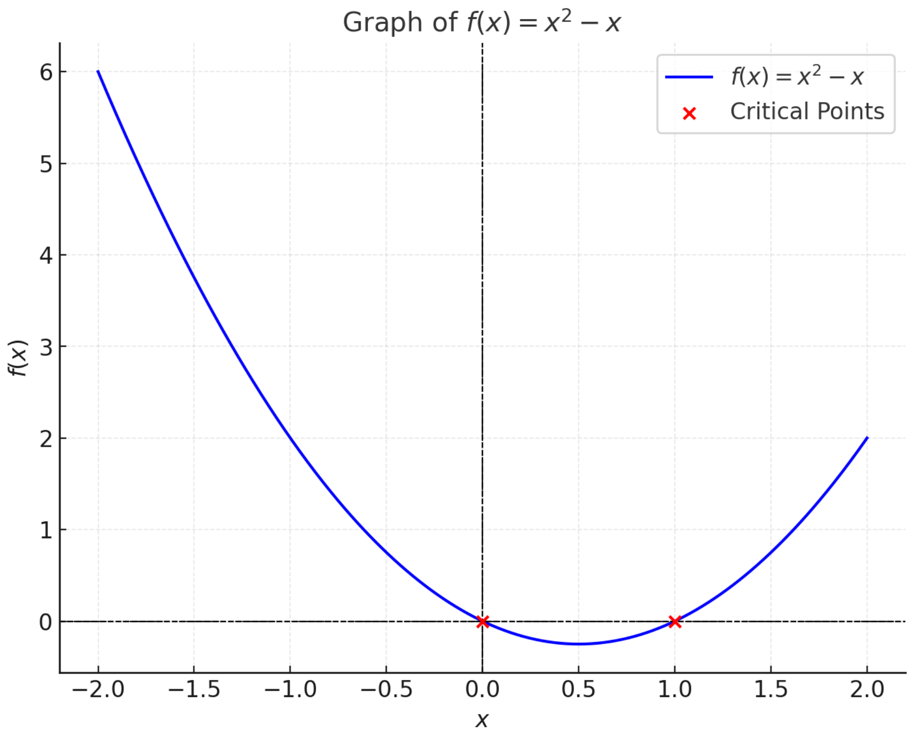

Universal Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x^2 \geq x$"
  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?

- Solution:
  - $P(0.5)$ produces the statement "$0.25 \geq 0.5$" which is **false**.
  - Since we produced a counterexample,

$\forall x(P(x))$ is **false**.

Universal Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x^2 \geq x$"
  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?
- What if the domain consisted of only integers?

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  - Let $P(x)$ be the statement "$x^2 \geq x$"
  - Domain consists of all real numbers
  - What is the truth value of $\forall x(P(x))$?
- What if the domain consisted of only integers?
  - $\forall x(P(x))$ is **true**, since there are no integer values in the interval $(0, 1)$

Existential Quantifier

- Definition: The existential quantifier of a propositional function $P(x)$, denoted $\exists x(P(x))$, is the statement:

  "There exists an element x in the domain such that $P(x)$."

- Also read as "for some x, $P(x)$" or "there is at least one x such that $P(x)$".

Existential Quantifier

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  "There exists an element x in the domain such that $P(x)$."
- Also read as "for some x, $P(x)$" or "there is at least one x such that $P(x)$".
- A statement $\exists x(P(x))$ is **true** if and only if there is at least one element x in the domain for which $P(x)$ is true.

Existential Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x > 3$"
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- Ex.
  - Let $P(x)$ be the statement "$x > 3$"
  - Domain consists of all real numbers
  - What is the truth value of $\exists x(P(x))$?

- Solution:
  - $P(4)$ produces the statement $4 > 2$, which is **true**.
  - Since we produced a valid example,

$\exists x(P(x))$ is **true**.

Empty Domain

- We generally assume that the domain is non-empty
- If the domain is empty:
  - Universal quantifier ($\forall$)
    - $\forall x(P(x))$ is **true**, since there no elements x in the domain for which $P(x)$ is false.
  - Existential quantifier ($\exists$)
    - $\exists x(P(x))$ is **false**, since there no elements x in the domain for which $P(x)$ is true.

Uniqueness Quantifier

- Definition: The uniqueness quantifier of a propositional function $P(x)$, denoted $\exists! x(P(x))$, is the statement:

"There exists a unique element x in the domain such that $P(x)$."

- Also read as "there is exactly one x such that $P(x)$

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- Also read as "there is exactly one x such that $P(x)$

- A statement $\exists! x(P(x))$ is **true** if and only if there is exactly one element x in the domain for which $P(x)$ is true.
- To show that $\exists! x(P(x))$ is **false**:
  - There does not exist any element x such that $P(x)$ is true
  - There exists more than one element x such that $P(x)$ is true

Uniqueness Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x - 1 = 0$"
  - Domain consists of all real numbers
  - What is the truth value of $\exists! x(P(x))$?

Uniqueness Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x - 1 = 0$"
  - Domain consists of all real numbers
  - What is the truth value of $\exists! x(P(x))$?
- Solution:
  - $x = 1$ is the only real number such that $x - 1 = 0$

$\exists! x(P(x))$ is **true**.

Uniqueness Quantifier

- Ex.
  - Let $P(x)$ be the statement "$x^2 = 4$"
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- Ex.
  - Let $P(x)$ be the statement "$x^2 = 4$"
  - Domain consists of all real numbers
  - What is the truth value of $\exists! x(P(x))$?

- Solution:
  - $P(2)$ produces the statement $4 = 4$, which is **true**.
  - $P(-2)$ produces the statement $4 = 4$, which is **true**.
  - Since we produced two valid values of x for which $P(x)$ is true,

$$\exists! x(P(x)) \text{ is } \textbf{false}.$$

Quantifiers and Finite Domains

- For finite domains (i.e., all elements can be listed), quantified statements can be expressed using propositional logic.
- Let the elements of the domain be:

$$x_1, x_2, x_3, \ldots, x_n$$

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- Universal quantification, $\forall x(P(x))$, is equivalent to

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- Universal quantification, $\forall x(P(x))$, is equivalent to

$$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots P(x_n)$$

- Existential quantification, $\exists x(P(x))$, is equivalent to

$$P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots P(x_n)$$

Quantifiers with Restricted Domains

- We can limit the domain by providing additional conditions that the elements of a domain must satisfy.
- Ex. Domain is all real numbers
  - Restrict domain to "all real numbers less than 0."
    $\forall x < 0(x^2 \geq 0)$
  - Restrict domain to "all real numbers not equal to 0."
    $\forall y \neq 0(y^3 \neq 0)$
  - Restrict domain to "all real numbers greater than 0."
    $\exists z > 0(z^2 = 0)$

Precedence of Quantifiers

- Quantifiers ($\forall$, $\exists$, $\exists!$) have higher precedence than all logical operators in propositional logic.
- Ex.
  - Consider the statement:

$$\forall x P(x) \vee Q(x)$$

  - Is equivalent to:

$$(\forall x P(x)) \vee Q(x)$$

  - Is NOT equivalent to:

$$\forall x (P(x) \vee Q(x))$$

Binding Variables

- When a quantifier is used on a variable x, we say that this occurence of the variable is bound.
- An occurence of a variable that is not bound by a quantifier or set equal to a particular value is said to be free.

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Binding Variables

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- An occurence of a variable that is not bound by a quantifier or set equal to a particular value is said to be free.
- All variables that occur in a propositional function must be bound or set equal to a particular value to turn it into a proposition.
- The part of a logical expression to which a quantifier is applied is called the scope of the quantifier.

Logical Equivalences Involving Quantifiers

- Definition: Statements involving predicates and quantifiers are logically equivalent if and only if they have the same truth value no matter:
  1. Which predicates are substitued into these statements and which domain
  2. Which domain is used for the variables in these propositional functions
- Denote logical equivalence of two statements involving predicates and quantifiers, S and T, as $S \equiv T$.

Negating Quantified Expressions

- Consider the proposition "Every student in this class has taken a course in calculus."
- Domain is all students in this class.
- $P(x)$: "Student x has taken a course in calculus".
- We can express the proposition as:

$$\forall x(P(x))$$

Negating Quantified Expressions

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- The negation of the proposition, denoted $\neg\forall x(P(x))$, is the statement
  - "It is not the case that every student in this class has taken a course in calculus."
  - "There is a student in this class who has not taken a course in calculus."

$$\exists x(\neg P(x))$$

Negating Quantified Expressions

- Consider the proposition "There is a student in this class who has taken a course in calculus."
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- The negation of the proposition, denoted $\neg\exists x(P(x))$, is the statement
  - "It is not the case that there is a student in this class who has taken a course in calculus."
  - "Every student in this class has not taken calculus."

$$\forall x(\neg P(x))$$

De Morgan's Laws for Quantifiers

1. $\neg \forall x(P(x)) \equiv \exists x(\neg P(x))$
2. $\neg \exists x(P(x)) \equiv \forall x(\neg P(x))$

De Morgan's Laws for Quantifiers

1. $\neg\forall x(P(x)) \equiv \exists x(\neg P(x))$
2. $\neg\exists x(P(x)) \equiv \forall x(\neg P(x))$

- Proposition 1. is **true** if
  - There exists an x in the domain for which $P(x)$ is false.
- Proposition 1. is **false** if
  - For every x in the domain, $P(x)$ is true.

De Morgan's Laws for Quantifiers

1. $\neg\forall x(P(x)) \equiv \exists x(\neg P(x))$
2. $\neg\exists x(P(x)) \equiv \forall x(\neg P(x))$

- Proposition 2. is **true** if
  - For every x in the domain, $P(x)$ is false.
- Proposition 2. is **false** if
  - There exists an x in the domain for which $P(x)$ is true.

De Morgan's Laws for Quantifiers

- Ex.
  - What is the negation of the statement

$$\forall x (x^2 > x)$$

De Morgan's Laws for Quantifiers

- Ex.
  - What is the negation of the statement

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- Solution:

$$\neg\forall x(x^2 < x) \equiv \exists x(\neg(x^2 > x))$$
$$\equiv \exists x(x^2 \leq x)\ .$$

De Morgan's Laws for Quantifiers

- Ex.
  - What is the negation of the statement

$$\exists x(x^2 = 2)$$

De Morgan's Laws for Quantifiers

- Ex.
  - What is the negation of the statement

$$\exists x(x^2 = 2)$$

- Solution:

$$\neg \exists x(x^2 = 2) \equiv \forall x(\neg(x^2 = 2))$$
$$\equiv \forall x(x^2 \neq 2) \ .$$

De Morgan's Laws for Quantifiers

- Ex.
  - Show that

$$\neg\forall x(P(x) \Rightarrow Q(x)) \equiv \exists x(P(x) \wedge \neg Q(x))$$

De Morgan's Laws for Quantifiers

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- Solution:

$$\neg\forall x(P(x) \Rightarrow Q(x)) \equiv \exists x(\neg(P(x) \Rightarrow Q(x)))$$

- Using De Morgan's Law for Quantifiers:

$$\neg\forall x(P'(x)) \equiv \exists x(\neg P'(x))$$

- Where $P'(x) = P(x) \Rightarrow Q(x)$

De Morgan's Laws for Quantifiers

- Ex.
  - Show that

$$\neg\forall x(P(x) \Rightarrow Q(x)) \equiv \exists x(P(x) \land \neg Q(x))$$

- Solution:

$$\begin{aligned}\neg\forall x(P(x) \Rightarrow Q(x)) &\equiv \exists x(\neg(P(x) \Rightarrow Q(x))) \\ &\equiv \exists x(P(x) \land \neg Q(x)) \ .\end{aligned}$$

- Using logical equivalences for conditional statements:

$$\neg(P(x) \Rightarrow Q(x)) \equiv P(x) \land \neg Q(x)$$

Exercises

- Translate each of these statements into logical expressions using predicates, quantifiers, and logical conncetives.

(a) No one is perfect
(b) Not everyone is perfect
(c) All your friends are perfect
(d) At least one of your friends is perfect
(e) Everyone is your friend and is perfect
(f) Not everybody is your friend or someone is not perfect

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- *Hint:* Define predicates for "__ is perfect" and "__ is your friend"

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

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(a) No one is perfect

$$\forall x \neg P(x)$$

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

(b) Not everyone is perfect

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

(c) All your friends are perfect

$$\forall x(F(x) \Rightarrow P(x))$$

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

(d) At least one of your friends is perfect

$$\exists x(F(x) \land P(x))$$

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

(e) Everyone is your friend and is perfect

$$\forall x(F(x) \wedge P(x))$$

Exercises

- Let $P(x)$ be "x is perfect"
- Let $F(x)$ be "x is your friend"

(f) Not everybody is your friend or someone is not perfect

$$(\neg\forall x F(x)) \vee (\exists x \neg P(x))$$
$$\equiv (\exists x \neg F(x)) \vee (\exists x \neg P(x))$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(a) $\forall x(x > 1)$
(b) $\forall x(x \leq 2)$
(c) $\exists x(x \geq 4)$
(d) $\exists x(x < 0)$
(e) $\forall x((x < -1) \vee (x > 2))$
(f) $\exists x((x < 4) \vee (x > 7))$

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(a)

$$\neg \forall x (x > 1) \equiv \exists x (x \leq 1)$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(b)

$$\neg \forall x (x \leq 2) \equiv \exists x (x > 2)$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(c)

$$\neg \exists x (x \geq 4) \equiv \forall x (x < 4)$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(d)

$$\neg\exists x(x < 0) \equiv \forall x(x \geq 0)$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(e)

$$\begin{aligned} &\neg\forall x((x < -1) \vee (x > 2)) \\ \equiv&\exists x\neg((x < -1) \vee (x > 2)) \\ \equiv&\exists x((x \geq -1) \wedge (x \leq 2)) \end{aligned}$$

Exercises

- Express the negation of each of these statements in terms of quantifiers without using the negation symbol

(f)

$$
\begin{aligned}
&\neg\exists x((x < 4) \vee (x > 7)) \\
\equiv&\forall x\neg((x < 4) \vee (x > 7)) \\
\equiv&\forall x((x \geq 4) \wedge (x \leq 7))
\end{aligned}
$$

Exercises

- Express each of these system specifications using predicates, quantifiers, and logical connectives.

(a) At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space.

(b) Whenever there is an active alert, all queued messages are transmitted.

(c) The diagnostic monitor tracks the status of all systems except the main console.

(d) Each participant on the conference call whom the host of the call did not put on a special list was billed.

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(a) At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space.
  - Let $F(x, y)$ be “Disk x has more than y kilobytes”
  - Let $S(x)$ be “Mail message x can be saved”

$$(\exists x F(x, 10)) \Rightarrow (\exists x S(x))$$

Exercises

- Express each of these system specifications using predicates, quantifiers, and logical connectives.

  (b) Whenever there is an active alert, all queued messages are transmitted.
  - Let $A(x)$ be "Alert x is active"
  - Let $Q(x)$ be "Message x is queued"
  - Let $T(x)$ be "Message x is transmitted"

$$(\exists x A(x)) \Rightarrow (\forall x (Q(x) \Rightarrow T(x))$$

Exercises

- Express each of these system specifications using predicates, quantifiers, and logical connectives.

  (c) The diagnostic monitor tracks the status of all systems except the main console.
  - Let $T(x)$ be "The diagnostic monitor tracks the status of system x"

$$\forall x((x \neq \text{main console}) \Rightarrow T(x))$$

Exercises

- Express each of these system specifications using predicates, quantifiers, and logical connectives.

(d) Each participant on the conference call whom the host of the call did not put on a special list was billed.

- Let $L(x)$ be "The host of the conference call put participant x on a special list"
- Let $B(x)$ be "Participant x was billed"

$$\forall x(\neg L(x) \Rightarrow B(x))$$