INFORMATION & COMPUTER SCIENCES
UNIVERSITY of HAWAIʻI at MĀNOA

Ch 1.3: Propositional Equivalences

ICS 141: Discrete Mathematics for Computer Science I

KYLE BERNEY
DEPARTMENT OF ICS, UNIVERSITY OF HAWAII AT MANOA

Terminology

- Definition: A proposition that is always true is called a tautology.
- Definition: A proposition that is always false is called a contradiction.
- Definition: A proposition that is neither a tautology nor contradiction is called a contingency.

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
F	T	T	F

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Tautology ($P \vee \neg P$) Contradiction ($P \wedge \neg P$)

Logical Equivalences

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- Alternate Definition: Two propositons P and Q are logically equivalent, denoted $P \equiv Q$, if $P \Leftrightarrow Q$ is a tautology.
- Truth tables are typically used to determine whether two propositions are logically equivalent.

De Morgan Laws

- Let P and Q be propositions.
- De Morgan laws:
  - $\neg(P \land Q) \equiv \neg P \lor \neg Q$
  - $\neg(P \lor Q) \equiv \neg P \land \neg Q$

De Morgan Laws

- Let P and Q be propositions.
- De Morgan laws:
  - $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
  - $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- Intuitively:
  1. Distribute the negation ($\neg$)
  2. Flip the AND ($\wedge$) and OR ($\vee$)

De Morgan Laws

- Let P and Q be propositions.
- De Morgan laws:
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P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

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P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Logically Equivalent

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P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

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- Let P and Q be propositions.
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P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Logically Equivalent

Conditional Disjunction Equivalence

- Let P and Q be propositions.
- Conditional Disjunction Equivalence
  - $P \Rightarrow Q \equiv \neg P \vee Q$

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P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

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P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Logically Equivalent

Distributive Law of Disjunction Over Conjunction

- Let P, Q, and R are propositions.
- Distributive law of disjunction over conjunction
  - $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Distributive Law of Disjunction Over Conjunction

- Let P, Q, and R are propositions.
- Distributive law of disjunction over conjunction
  - $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
- *Note:* A truth table with n propositional variables require 2^n rows.
  - $n = 3$ propositional variables
  - $2^n = 2^3 = 8$ rows

Distributive Law of Disjunction Over Conjunction

- Let P, Q, and R are propositions.
- Distributive law of disjunction over conjunction
  - $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Distributive Law of Disjunction Over Conjunction

- Let P, Q, and R are propositions.
- Distributive law of disjunction over conjunction
  - $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

← Logically Equivalent →

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Identity laws:
  - $P \wedge \mathrm{T} \equiv P$
  - $P \vee \mathrm{F} \equiv P$
  - *Intuition:* The added true or false proposition does not change the truth value of P.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Domination laws:
  - $P \vee \text{T} \equiv \text{T}$
  - $P \wedge \text{F} \equiv \text{F}$
  - *Intuition:* The added true or false proposition dominates the truth value of P.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Idempotent laws:
  - $P \vee P \equiv P$
  - $P \wedge P \equiv P$
  - *Intuition:* Repeating the same logical statement is redundant
    - Let P = “I am wearing shoes”.
    - “I am wearing shoes or I am wearing shoes.”
    - “I am wearing shoes and I am wearing shoes.”
    - Both statements are equivalent to “I am wearing shoes”.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Double Negation law:
  - $\neg(\neg P) \equiv P$
  - *Intuition:* Negating a statement twice cancels the effect of negation.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Commutative laws:
  - $P \vee Q \equiv Q \vee P$
  - $P \wedge Q \equiv Q \wedge P$
  - *Intuition:* The order of propositions in an AND ($\wedge$) or OR ($\vee$) does not affect the outcome.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Associative laws:
  - $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$
  - $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
  - *Intuition:* The way propositions are grouped in logical AND ($\wedge$) or OR ($\vee$) does not affect the outcome.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Distributive laws:
  - $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
  - $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Absorption laws:
  - $P \vee (P \wedge Q) \equiv P$
  - $P \wedge (P \vee Q) \equiv P$
  - *Intuition:* Shows how redundancy is logical statements can be eliminated.
    - If P is true, the entire expression is true regardless of Q.
    - If P is false, the expression reduces to $P \wedge Q$, which is also false, since P is false.

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  - $P \wedge (P \vee Q) \equiv P$
  - *Intuition:* Shows how redundancy is logical statements can be eliminated.
    - If P is false, the expression is false regardless of Q.
    - If P is true, the entire expression is reduced to $P \vee Q$, which is also true, since P is true.

More Logical Equivalences

- Let P, Q and R be propositions.
- Let T and F be any proposition that is always true or false, respectively.

- Negation laws:
  - $P \vee \neg P \equiv \mathrm{T}$
    - Known as the <u>law of contradiction</u>
    - P cannot be both true and false simultaneously.
  - $P \wedge \neg P \equiv \mathrm{F}$
    - Known as the <u>law of excluded middle</u>
    - P must be either true or false (i.e., there is no middle ground).

More Logical Equivalences – Conditional Statements

- Let P, Q and R be propositions.

- $P \Rightarrow Q \equiv \neg P \vee Q$
- $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$
- $P \vee Q \equiv \neg P \Rightarrow Q$
- $P \wedge Q \equiv \neg(P \Rightarrow \neg Q)$
- $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$
- $(P \Rightarrow Q) \wedge (P \Rightarrow R) \equiv P \Rightarrow (Q \wedge R)$
- $(P \Rightarrow R) \wedge (Q \Rightarrow R) \equiv (P \vee Q) \Rightarrow R$
- $(P \Rightarrow Q) \vee (P \Rightarrow R) \equiv P \Rightarrow (Q \vee R)$
- $(P \Rightarrow R) \vee (Q \Rightarrow R) \equiv (P \wedge Q) \Rightarrow R$

- Table 7 in the textbook.

More Logical Equivalences – Biconditional Statements

- Let P and Q be propositions.

- $P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
- $P \Leftrightarrow Q \equiv \neg P \Leftrightarrow \neg Q$
- $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$
- $\neg(P \Leftrightarrow Q) \equiv P \Leftrightarrow \neg Q$

- Table 8 in the textbook.

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \Rightarrow Q)$ and $P \wedge \neg Q$ are logically equivalent.

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- Let P and Q be propositions.
- Show that $\neg(P \Rightarrow Q)$ and $P \wedge \neg Q$ are logically equivalent.

$$\neg(P \Rightarrow Q) \equiv \neg(\neg P \vee Q)$$

- Using the logical equivalence:

$$P \Rightarrow Q \equiv \neg P \vee Q$$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \Rightarrow Q)$ and $P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned}\neg(P \Rightarrow Q) &\equiv \neg(\neg P \vee Q) \\ &\equiv \neg(\neg P) \wedge \neg Q\end{aligned}$$

- Using De Morgan's law:

$$\neg(P' \vee Q) \equiv \neg P' \wedge \neg Q$$

- Where $P' = \neg P$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \Rightarrow Q)$ and $P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned} \neg(P \Rightarrow Q) &\equiv \neg(\neg P \vee Q) \\ &\equiv \neg(\neg P) \wedge \neg Q \\ &\equiv P \wedge \neg Q\,. \end{aligned}$$

- Using the double negation law:

$$\neg(\neg P) \equiv P$$

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- Let P and Q be propositions.
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

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- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

$$\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg(\neg P \wedge Q)$$

- Using De Morgan's law:

$$\neg(P \vee Q') \equiv \neg P \wedge \neg Q'$$

- Where $Q' = \neg P \wedge Q$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned}\neg(P \vee (\neg P \wedge Q)) &\equiv \neg P \wedge \neg(\neg P \wedge Q) \\ &\equiv \neg P \wedge (\neg(\neg P) \vee \neg Q)\end{aligned}$$

- Using De Morgan's law:

$$\neg(P' \wedge Q) \equiv \neg P' \vee \neg Q$$

- Where $P' = \neg P$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \lor (\neg P \land Q))$ and $\neg P \land \neg Q$ are logically equivalent.

$$\begin{aligned} \neg(P \lor (\neg P \land Q)) &\equiv \neg P \land \neg(\neg P \land Q) \\ &\equiv \neg P \land (\neg(\neg P) \lor \neg Q) \\ &\equiv \neg P \land (P \lor \neg Q) \end{aligned}$$

- Using the double negation law:

$$\neg(\neg P) \equiv P$$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned}
\neg(P \vee (\neg P \wedge Q)) &\equiv \neg P \wedge \neg(\neg P \wedge Q) \\
&\equiv \neg P \wedge (\neg(\neg P) \vee \neg Q) \\
&\equiv \neg P \wedge (P \vee \neg Q) \\
&\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q)
\end{aligned}$$

- Using the distributive law:

$$P' \wedge (P \vee Q') \equiv (P' \wedge P) \vee (P' \wedge Q')$$

- Where $P' = \neg P$ and $Q' = \neg Q$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned}
\neg(P \vee (\neg P \wedge Q)) &\equiv \neg P \wedge \neg(\neg P \wedge Q) \\
&\equiv \neg P \wedge (\neg(\neg P) \vee \neg Q) \\
&\equiv \neg P \wedge (P \vee \neg Q) \\
&\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q) \\
&\equiv \mathrm{F} \vee (\neg P \wedge \neg Q)
\end{aligned}$$

- Using negation law:

$$P \wedge \neg P \equiv \mathrm{F}$$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically equivalent.

$$\begin{aligned} \neg(P \vee (\neg P \wedge Q)) &\equiv \neg P \wedge \neg(\neg P \wedge Q) \\ &\equiv \neg P \wedge (\neg(\neg P) \vee \neg Q) \\ &\equiv \neg P \wedge (P \vee \neg Q) \\ &\equiv (\neg P \wedge P) \vee (\neg P \wedge \neg Q) \\ &\equiv \mathrm{F} \vee (\neg P \wedge \neg Q) \\ &\equiv \neg P \wedge \neg Q \,. \end{aligned}$$

- Using identity law:

$$P' \vee \mathrm{F} \equiv P'$$

- Where $P' = \neg P \wedge \neg Q$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

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- Show that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

$$(P \wedge Q) \Rightarrow (P \vee Q) \equiv \neg(P \wedge Q) \vee (P \vee Q)$$

- Using the conditional disjunction equivalence:

$$P' \Rightarrow Q' \equiv \neg P' \vee Q'$$

- Where $P' = P \wedge Q$ and $Q' = P \vee Q$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

$$\begin{aligned}(P \wedge Q) \Rightarrow (P \vee Q) &\equiv \neg(P \wedge Q) \vee (P \vee Q) \\ &\equiv (\neg P \vee \neg Q) \vee (P \vee Q)\end{aligned}$$

- Using De Morgan's law:

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $(P \land Q) \Rightarrow (P \lor Q)$ is a tautology.

$$
\begin{aligned}
(P \land Q) \Rightarrow (P \lor Q) &\equiv \neg(P \land Q) \lor (P \lor Q) \\
&\equiv (\neg P \lor \neg Q) \lor (P \lor Q) \\
&\equiv (\neg P \lor P) \lor (\neg Q \lor Q)
\end{aligned}
$$

- Reorder and regroup using the associative and commutative laws for disjunction

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

$$\begin{aligned}
(P \wedge Q) \Rightarrow (P \vee Q) &\equiv \neg(P \wedge Q) \vee (P \vee Q) \\
&\equiv (\neg P \vee \neg Q) \vee (P \vee Q) \\
&\equiv (\neg P \vee P) \vee (\neg Q \vee Q) \\
&\equiv \mathrm{T} \vee \mathrm{T}
\end{aligned}$$

- Using truth table from slide 2:

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
F	T	T	F

Constructing New Logical Equivalences

- Let P and Q be propositions.
- Show that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

$$
\begin{aligned}
(P \wedge Q) \Rightarrow (P \vee Q) &\equiv \neg(P \wedge Q) \vee (P \vee Q) \\
&\equiv (\neg P \vee \neg Q) \vee (P \vee Q) \\
&\equiv (\neg P \vee P) \vee (\neg Q \vee Q) \\
&\equiv \mathrm{T} \vee \mathrm{T} \\
&\equiv \mathrm{T}\,.
\end{aligned}
$$

Satisfiability

- Definition: A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true.
  - We call an assignment of truth values to its variables that makes the compound proposition true a solution of the satisfiability problem.

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- Definition: A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true.
  - We call an assignment of truth values to its variables that makes the compound proposition true a solution of the satisfiability problem.
- Definition: A compound proposition is unsatisfiable if it is false for all assignments of truth values to its variables.
- Read textbook section 1.3.6 for examples of the applications of satisfiability

Exercises

- Show that $(P \wedge Q) \Rightarrow P$ is a tautology using logical equivalences.
- Recall that: $P \Rightarrow Q \equiv \neg P \vee Q$

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(P \wedge Q) \Rightarrow P &\equiv \neg(P \wedge Q) \vee P \\
&\equiv \neg P \vee \neg Q \vee P \\
&\equiv (\neg P \vee P) \vee \neg Q \\
&\equiv \mathrm{T} \vee \neg Q \\
&\equiv \mathrm{T}\,.
\end{aligned}
$$

Exercises

- Show that $(P \wedge Q) \Rightarrow (P \Rightarrow Q)$ is a tautology using logical equivalences.
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\begin{aligned}
(P \wedge Q) \Rightarrow (P \Rightarrow Q) &\equiv \neg(P \wedge Q) \vee (P \Rightarrow Q) \\
&\equiv \neg(P \wedge Q) \vee (\neg P \vee Q) \\
&\equiv (\neg P \vee \neg Q) \vee (\neg P \vee Q) \\
&\equiv (\neg P \vee \neg P) \vee (\neg Q \vee Q) \\
&\equiv \neg P \vee \mathrm{T} \\
&\equiv \mathrm{T}\,.
\end{aligned}
$$

Exercises

- Show that $\neg(P \Rightarrow Q) \Rightarrow \neg Q$ is a tautology using logical equivalences.
- Recall that: $P \Rightarrow Q \equiv \neg P \vee Q$

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\begin{aligned}
\neg(P \Rightarrow Q) \Rightarrow \neg Q &\equiv \neg(\neg P \vee Q) \Rightarrow \neg Q \\
&\equiv \neg(\neg(\neg P \vee Q) \vee \neg Q \\
&\equiv (\neg P \vee Q) \vee \neg Q \\
&\equiv \neg P \vee \mathrm{T} \\
&\equiv \mathrm{T}\,.
\end{aligned}
$$

Exercises

- Show that $\neg(P \Leftrightarrow Q) \equiv (P \Leftrightarrow \neg Q)$
- Recall that: $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

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- Show that $\neg(P \Leftrightarrow Q) \equiv (P \Leftrightarrow \neg Q)$
- Recall that: $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$$\begin{aligned}\neg(P \Leftrightarrow Q) &\equiv \neg((P \wedge Q) \vee (\neg P \wedge \neg Q)) \\ &\equiv \neg(P \wedge Q) \wedge \neg(\neg P \wedge \neg Q) \\ &\equiv (\neg P \vee \neg Q) \wedge (P \vee Q) \ .\end{aligned}$$

Exercises

- Show that $\neg(P \Leftrightarrow Q) \equiv (P \Leftrightarrow \neg Q)$
- Recall that: $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$$\begin{aligned}
\neg(P \Leftrightarrow Q) &\equiv \neg((P \wedge Q) \vee (\neg P \wedge \neg Q)) \\
&\equiv \neg(P \wedge Q) \wedge \neg(\neg P \wedge \neg Q) \\
&\equiv (\neg P \vee \neg Q) \wedge (P \vee Q) \ .
\end{aligned}$$

$$\begin{aligned}
P \Leftrightarrow \neg Q &\equiv (P \wedge \neg Q) \vee (\neg P \wedge Q) \\
&\equiv ((P \wedge \neg Q) \vee \neg P) \wedge ((P \wedge \neg Q) \vee Q) \\
&\equiv ((\neg P \vee P) \wedge (\neg P \vee \neg Q)) \wedge ((Q \vee P) \wedge (\neg Q \vee Q)) \\
&\equiv \mathrm{T} \wedge (\neg P \vee \neg Q) \wedge (P \vee Q) \wedge \mathrm{T} \\
&\equiv (\neg P \vee \neg Q) \wedge (P \vee Q) \ .
\end{aligned}$$

Exercises

- Show that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology.

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- Hint: Use a truth table

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- Hint: Use a truth table

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$P \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T