INFORMATION & COMPUTER SCIENCES
UNIVERSITY of HAWAI'I at MĀNOA

Ch 1.1: Propositional Logic

ICS 141: Discrete Mathematics for Computer Science I

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Propositions

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2. One day is 20 hours.
3. $1 + 1 = 2$
4. $2 + 2 = 3$

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- *Why are the above examples propositions?*

1. True sentence
2. False sentence
3. True statement
4. False statement

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1. What time is it?
2. Pay attention.
3. $x + 1 = 2$
4. $x + y = z$

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2. Pay attention.
3. $x + 1 = 2$
4. $x + y = z$

- *Why are the above examples NOT propositions?*

1. A question is not a statement
2. A request is not a statement
3. Not true or false
4. Not true or false

Propositions

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- Ex. *NOT* propositions
  1. What time is it?
  2. Pay attention.
  3. $x + 1 = 2$
  4. $x + y = z$
- *Question:* What happens if we assign values to the variables in 3. and 4.?

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  1. What time is it?
  2. Pay attention.
  3. $x + 1 = 2$
  4. $x + y = z$
- *Question:* What happens if we assign values to the variables in 3. and 4.?
- *Answer:* They become propositions!
  - Will be either true or false statements (depending on the assigned values)

Propositions

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- Definition: A proposition is assigned one of two truth values:
  - P = True, denoted T, if the proposition P is true
  - P = False, denoted F, if the proposition P is false
- Definition: A compound proposition is a proposition formed by combining two or more propositions using logical operators.

Logical Operators – Negation

- Definition: Let P be a proposition. The negation of P is a logial operator denoted as $\neg P$ (also commonly denoted as $\overline{P}$ or $!P$).
- Read as "Not P"
- The truth value of $\neg P$ is the opposite of the truth value of P

P	$\neg P$
T	F
F	T

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- Definition: A truth table is a table used to show all possible truth values of a logical expression based on its component propositions.

Logical Operators – Conjunction

- Definition: Let P and Q be propositions. The conjunction of P and Q is a logical operator denoted as $P \wedge Q$.
- Read as "P and Q"
- The truth value of $P \wedge Q$ is true if both P and Q are true; and false otherwise.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Operators – Disjunction

- Definition: Let P and Q be propositions. The disjunction (also known as the inclusive or) of P and Q is a logical operator denoted as $P \vee Q$.
- Read as "P or Q"
- The truth value of $P \vee Q$ is true if at least one of P or Q are true; and false otherwise.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Operators – Exclusive Or

- Definition: Let P and Q be propositions. The exclusive or of P and Q is a logical operator denoted as $P \oplus Q$.
- Read as "P xor Q"
- The truth value of $P \oplus Q$ is true if exactly one of P or Q are true; and false otherwise.

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Logical Operators – Tips to Remember

- $P \wedge Q$ vs. $P \vee Q$
  - The operator $\wedge$ looks similar to an A (without the middle line), representing "And" (i.e., conjunction).

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- $P \wedge Q$ vs. $P \vee Q$
  - The operator $\wedge$ looks similar to an A (without the middle line), representing “And” (i.e., conjunction).
- Inclusive Or vs. Exclusive Or
  - Inclusive Or *includes* the case where both P and Q are true.
    - *Includes* more True values
  - Exclusive Or *excludes* the case where both P and Q are true.
    - *Excludes* more True values

Conditional Statements

- Definition: Let P and Q be propositions. The conditional statement is denoted as $P \Rightarrow Q$ (also commonly denoted as $P \to Q$).
- Read as "P implies Q" or "if P, then Q"
- The truth value of $P \Rightarrow Q$ is false if P is true and Q is false; and true otherwise.

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional Statements – Other Terminology

- A varitey of terminology is used to express $P \Rightarrow Q$.

  - “if P, then Q”
  - “if P, Q”
  - “P is sufficient for Q”
  - “Q if P
  - Q when P
  - “a necessary condition for P is Q”
  - “Q unless $\neg P$”
  - “P implies Q”
  - “P only if Q”
  - “a sufficient condition for Q is P”
  - “Q whenever P”
  - “Q is necessary for P”
  - “Q follows from P”
  - “Q provided that P”

Notable Conditional Statements – Converse

- Definition: Let P and Q be propositions. The converse of $P \Rightarrow Q$ is $Q \Rightarrow P$.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Notable Conditional Statements – Inverse

- Definition: Let P and Q be propositions. The inverse of $P \Rightarrow Q$ is $\neg P \Rightarrow \neg Q$.

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg P \Rightarrow \neg Q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Notable Conditional Statements – Contrapositive

- Definition: Let P and Q be propositions. The contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$.

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

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P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

- Definition: Two propositons R and S are logically equivalent, denoted $R \equiv S$, if the truth values for R and S are always the same.

Notable Conditional Statements – Contrapositive

- Definition: Let P and Q be propositions. The contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$.

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Logically Equivalent

Biconditionals

- Definition: Let P and Q be propositions. The biconditional is denoted as $P \Leftrightarrow Q$ (or P iff Q).
- Read as "P if and only if Q"
- The truth value of $P \Leftrightarrow Q$ is true when P and Q have the same truth values; and false otherwise.

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of Logical Operators

1. Negation: “not” ($\neg$)
2. Conjunction: “and” ($\wedge$)
3. Disjunction: “inclusive or” ($\vee$)
4. Conditional: “implies” ($\Rightarrow$)
5. Biconditional: “if and only if” ($\Leftrightarrow$)

Precedence of Logical Operators

1. Negation: "not" ($\neg$)
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3. Disjunction: "inclusive or" ($\vee$)
4. Conditional: "implies" ($\Rightarrow$)
5. Biconditional: "if and only if" ($\Leftrightarrow$)

- Note: Exclusive or ($\oplus$) has the same precedence as inclusive or ($\vee$)

Logic and Bit Operators

- Computers represent information using bits.
- Definition: A bit is a symbol with two possible values, 0 and 1.
  - The word bit comes from the usage of binary digit in the binary representation of numbers (i.e., base-2 representation).
  - Bits are used in logic to represent true and false values
    - 1 bit is used to represent true
    - 0 bit is used to represent false

Logic and Bit Operators

- Bit operators correspond to logical operators
  - AND operator ($\wedge$)
  - OR operator ($\vee$)
  - XOR operator ($\oplus$)

x	y	$x \wedge y$	$x \vee y$	$x \oplus y$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Logic and Bit Operators

- Definition: A bit string is a sequence of zero or more bits.
- Definition: The length of a bit string is the number of bits in the string.
- Bit operators can be extended to bit strings
  - Bitwise AND ($\wedge$)

$$\begin{array}{r} 1\,0\,1\,1\quad 0\,1\,1\,0 \\ \wedge\quad 0\,0\,0\,1\quad 1\,1\,0\,1 \\ \hline 0\,0\,0\,1\quad 0\,1\,0\,0 \end{array}$$

Logic and Bit Operators

- Definition: A bit string is a sequence of zero or more bits.
- Definition: The length of a bit string is the number of bits in the string.
- Bit operators can be extended to bit strings
  - Bitwise OR ($\vee$)

$$
\begin{array}{r}
1\,0\,1\,1\;\;0\,1\,1\,0 \\
\vee\quad 0\,0\,0\,1\;\;1\,1\,0\,1 \\
\hline
1\,0\,1\,1\;\;1\,1\,1\,1
\end{array}
$$

Logic and Bit Operators

- Definition: A bit string is a sequence of zero or more bits.
- Definition: The length of a bit string is the number of bits in the string.
- Bit operators can be extended to bit strings
  - Bitwise XOR ($\oplus$)

$$
\begin{array}{r}
1\,0\,1\,1\;\;0\,1\,1\,0 \\
\oplus\;\; 0\,0\,0\,1\;\;1\,1\,0\,1 \\
\hline
1\,0\,1\,0\;\;1\,0\,1\,1
\end{array}
$$

Exercises

- Construct a truth table for $(P \vee \neg Q) \Rightarrow Q$
- Recall that:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Exercises

- Construct a truth table for $(P \vee \neg Q) \Rightarrow Q$

P	Q	$\neg Q$	$P \vee \neg Q$	$(P \vee \neg Q) \Rightarrow Q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

Exercises

- Construct a truth table for $(P \Leftrightarrow Q) \oplus (\neg P \Leftrightarrow Q)$
- Recall that:

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercises

- Construct a truth table for $(P \Leftrightarrow Q) \oplus (\neg P \Leftrightarrow Q)$

P	Q	$P \Leftrightarrow Q$	$\neg P$	$\neg P \Leftrightarrow Q$	$(P \Leftrightarrow Q) \oplus (\neg P \Leftrightarrow Q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

Exercises

- Construct a truth table for $(P \Rightarrow Q) \wedge (\neg P \Rightarrow R)$
- Recall that:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Exercises

- Construct a truth table for $(P \Rightarrow Q) \wedge (\neg P \Rightarrow R)$

P	Q	R	$P \Rightarrow Q$	$\neg P$	$\neg P \Rightarrow R$	$(P \Rightarrow Q) \wedge (\neg P \Rightarrow R)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F