Chapter 29

Electromagnetic Induction

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– Hugh D. Young and Roger A. Freedman

Lectures by James Pazun

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Topics for Chapter 29

• Faraday’s Law of Induction

• Lenz’s Law - direction of induced current

• Some Faraday’s Law experiments can be explained by \( \mathbf{v} \times \mathbf{B} \) force - motional emf

Intermission

• New concept - \( \frac{dB}{dt} \) induces electric field

• Another new concept - \( \frac{dE}{dt} \) induces magnetic field.

• Maxwell’s equations for classical E&M
Faraday's Law of Induction:
Changing magnetic flux through a conducting loop induces an emf which causes an induced current.

\[-\frac{d}{dt} \left[ \int (\vec{B} \cdot \hat{n}) dA \right] = \mathcal{E}_{\text{induced}} \rightarrow I_{\text{induced}} = \frac{\mathcal{E}_{\text{induced}}}{R}\]

Minus sign tells us the direction of induced current, to be discussed later using Lenz's Law.
Many ways to change the magnetic flux

Magnetic flux through a conducting loop: \( \Phi_m \equiv \int (\vec{B} \cdot \hat{n})dA \)

\( \frac{d\Phi_m}{dt} \) can be caused by

1. moving a magnet (non-uniform field) near a stationary, conducting loop thus causing a time-dependent \( \vec{B}(\vec{r},t) \) at the loop,
2. placing a stationary conducting loop in a time-dependent magnetic field,
3. moving a conducting loop in a static but non-uniform magnetic field, \( \vec{B}(\vec{r}) \),
4. rotating a conducting loop in a static and uniform magnetic field \( \vec{B} \), changing \( \vec{B} \cdot \hat{n} \),
5. varying the area of the conducting loop with time.

Any combinations of the above.

"Regardless what causes \( \frac{d\Phi_m}{dt} \); \( -\frac{d\Phi_m}{dt} = E_{induced} \)"
Faraday’s Law - Example 1

(1) Moving a magnet (with non-uniform B-field) near a stationary conducting loop causing a time-dependent $\mathbf{B}(\mathbf{r},t)$ at the loop.
Faraday’s Law - Example 2

(2) A stationary conducting loop in a time-dependent magnetic field

• Follow Example 29.1. A uniform B-field in space that varies with time. Find the magnetic flux as a function of time, the induced emf and induced current.
(3) Moving a conducting loop in a static but non-uniform magnetic field, \( \vec{B}(\vec{r}) \).

(b) Moving the magnet toward or away from the coil

Meter shows induced current.
Faraday’s Law - Example 4

(4) Rotating a conducting loop in a static and uniform magnetic field $\vec{B}$, changing $\vec{B} \cdot \hat{n}$

Find $\Phi_m(t)$ and $\varepsilon_{\text{induced}}(t)$ in terms of $B$, $\omega$, and $A$

This is the basic principle of (AC) electric generator.
Faraday’s Law - Example 5

(5) Varying the area of the conducting loop with time

Find $\Phi_m(t)$ and $\varepsilon_{\text{induced}}(t)$ in terms of $B$, $v$, and $L$
Applications

• A credit card has a magnet strip on its back, “swiping” the card can generate tiny currents that send information to cash registers.

• Electric generator

• Metal detector, etc.
Find the direction of an induced emf

- Consult Figure 29.6 and the text on the bottom of page 997 and top of page 998 and solve.

(a) Flux is positive ($\Phi_B > 0$) ...
... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

(b) Flux is positive ($\Phi_B > 0$) ...
... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

(c) Flux is negative ($\Phi_B < 0$) ...
... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

(d) Flux is negative ($\Phi_B < 0$) ...
... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).
Find direction of induced current - Lenz’s Law

- The direction of any magnetic induction effect is such as to oppose the cause of the effect.

- Follow Conceptual Example 29.8.

- Follow Example 29.9; Figure 29.14 illustrates the example.

(a) Motion of magnet causes increasing downward flux through loop. The induced magnetic field is upward to oppose the flux change. To produce this induced field, the induced current must be counterclockwise as seen from above the loop.

(b) Motion of magnet causes decreasing upward flux through loop. The induced magnetic field is downward to oppose the flux change. To produce this induced field, the induced current must be clockwise as seen from above the loop.
Can we explain Faraday’s induction using already known concepts?

Yes, if the Faraday induction is caused by moving the conducting loop, then the induction can be explained in terms of $q(v \times B)$ force.

No, if the Faraday induction is caused by moving the magnetic or having a time-dependent magnetic field - needs a new concept.

"$\frac{\partial B}{\partial t}$ induces an electric field $\vec{E}(\vec{r}, t)$"
Re-examine this Faraday’s induction using $\mathbf{v} \times \mathbf{B}$ force

“Motional emf”

\[
\varepsilon = \oint \frac{\mathbf{F}}{q} \cdot d\mathbf{l} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}
\]

Calculate induced emf using this formula; you should get the same answer as in the previous slide which uses the Faraday's Law.
There is more to the previous example - magnetic braking effect

- Follow Example 29.7.
- Figure 29.12 illustrates Example 29.7.

\[ \vec{v} \times \vec{B} \text{ force} \rightarrow \varepsilon_{induced} \rightarrow I_{induced} \rightarrow I \times \vec{B} \text{ force (F)} \]

which slows down the motion of the conducting bar.
Motional emf ($v \times B$ force) can explain situations where there is no “magnetic flux”

A solid conductor moving in a B-field.

Find induced emf in both cases
Intermission
Review yesterday’s materials

1. State Faraday’s Law in English and in mathematical formula
2. List some of the ways that the magnetic flux can have a time dependence
3. How to find the direction of induced emf and induced current?
4. Practice calculating magnetic flux ($\Phi_m$), $d\Phi_m/dt$, $\varepsilon_{\text{induced}}$, and $I_{\text{induced}}$
5. Know when and how to calculate motional emf.
New concept - dB/dt (partial derivative) induces an electric field

- The windings of a long solenoid carrying a time-varying current induces an electric field everywhere.

- Note: The induced electric field is there even if there is no wire loop; the wire loop is there to measure the induced emf.

**Faraday's Law**

\[
\varepsilon = -\frac{d\Phi}{dt} = -\int \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot \hat{n} \, dA
\]

*New concept:*

\[
- \frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E};
\]

\[
\oint \vec{E} \cdot d\vec{l} = \varepsilon = -\int \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot \hat{n} \, dA
\]
Another new concept - \( \frac{dE}{dt} \) (partial derivative) induces a B-field

- A varying electric field will give rise to a magnetic field.
- Consider the magnetic field generated by a circuit with a capacitor - the magnetic field inside the capacitor must be created by \( \frac{dE}{dt} \) because \( I=0 \) in that region - Modified Ampere’s Law
Maxwell’s Equation (Integral form) for classical E&M

\(1\) \[ \int (\vec{E} \cdot \hat{n}) dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \quad \text{(Gauss's Law)} \]

\(2\) \[ \oint \vec{E} \cdot d\vec{l} = - \int \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot \hat{n} dA \quad \text{(from Faraday's Law)} \]

\(3\) \[ \int (\vec{B} \cdot \hat{n}) dA = 0 \quad \text{(No magnetic monopole)} \]

\(4\) \[ \oint \vec{B} \cdot d\vec{l} = \mu_0 \left[ I_{\text{enclosed}} + \varepsilon_0 \int \left( \frac{\partial \vec{E}}{\partial t} \right) \cdot \hat{n} dA \right] \quad \text{Modified Ampere's Law} \]

**Lorentz Force Law:** \[ \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \]

Maxwell’s Equations plus the Lorentz force Law describe ALL electromagnetic phenomena in the classical (non-quantum mechanics) limit.