Topics for Chapter 24

• I. Capacitors and capacitance
• II. Energy stored in a charged capacitor
• III. Capacitors with dielectrics

Intermission

• IV. Dielectric breakdown
• V. Capacitors in series an parallel circuits
I. Capacitor and capacitance

- If we pull a positive charge and negative charge apart, it requires energy; this energy is stored as electrostatic potential energy in this charge arrangement. If the charges are allowed to move, they will move toward each other and gain kinetic energy (one can use this kinetic energy to push something to do work or to light up a light bulb).

- A **capacitor** is a device which can hold these separated charges.
How do we build a capacitor?

- The basic components of a capacitor are two conductors separated by a distance.
- The “parallel-plate capacitor is the easiest to analyze.
- Connect the plates to the two terminals of a battery (see figure below). Electrons will move from the top plate to the battery and then enter the bottom plate; leaving a net positive charge on the top plate and net negative charge on the bottom plate. The electrons will stop moving when the voltage difference between the two plates \(V_{ab}\) equal to the voltage of the battery \((\varepsilon)\).
- Disconnect the battery and the capacitor holds the charge.

\[\text{Potential difference } = V_{ab}\]

![Diagram of capacitor](image)

When the separation of the plates is small compared to their size, the fringing of the field is slight.
Capacitance

How much charge \( Q \) a capacitor can hold for a given voltage difference \( V_{ab} \) between the plates depends on the size and shape of the capacitor and it is called the capacitance \( C \).

Definition: \[
C = \frac{Q}{V_{ab}}
\]

For parallel-plate capacitor with area \( A \) and separation \( d \),
\[
C = \varepsilon_0 \frac{A}{d} \quad \text{(if } d \ll \sqrt{A})
\]

**Derivation:** \( d \ll \sqrt{A} \Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \); \( V_{ab} = Ed = \frac{Qd}{\varepsilon_0 A} \Rightarrow C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d} \)

(a) Arrangement of the capacitor plates

(b) Side view of the electric field \( E \)

When the separation of the plates is small compared to their size, the fringing of the field is slight.
Unit of capacitance

Definition: \( C = \frac{Q}{V_{ab}} \Rightarrow \text{unit} = \frac{\text{Coulomb}}{\text{volt}} = \text{Farad} \)

Example: The separation of a parallel plate capacitor is 1 mm, what is the area of the plate in order for \( C = 1 \text{ Farad} \)?

Answer: \( C = \varepsilon_0 \frac{A}{d}; \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m} \)

\[ \Rightarrow A = \frac{Cd}{\varepsilon_0} = \frac{(1 \text{ Farad})(10^{-3} \text{ m})}{8.85 \times 10^{-12} \text{ Farad/m}} \approx 1 \times 10^8 \text{ m}^2 \]

Common capacitances are pF = 10^{-12} Farad or \( \mu \text{F} = 10^{-6} \text{ Farad} \).

Note: Even if \( C = 1 \mu \text{F} \), the area is still 100m^2!!

(1) Large area can be accommodated by rolling the two plates into a cylinder.

(2) Capacitance can also be increased by inserting an insulating material (called dielectrics) between the plates (will be discussed later).

Electrolytic dielectrics capacitor can have a capacitance of 1 Farad and yet of very small size.
A cylindrical capacitor is formed by two concentric cylinders (e.g., a coaxial cable); the inner conductor is typically a solid cylinder, the outer conductor is a thin cylindrical shell.

Suppose we connect this capacitor to a battery; when it is fully charged, the charge is $Q$. If $r_a$, $r_b << L$, then we can use the infinite line cylinder approximation.

Using Gauss's Law $\Rightarrow E = \frac{Q}{2\pi rL\varepsilon_o}$ for $r_a < r < r_b$

$$V_{ab} = \int_a^b Edr = \frac{Q}{2\pi L\varepsilon_o} \ln \frac{r_b}{r_a} \Rightarrow C \equiv \frac{Q}{V_{ab}} = \varepsilon_o \frac{2\pi L}{\ln(r_b/r_a)}$$

**Note:** Let $d \equiv r_b - r_a$,

$$\ln\left(\frac{r_b}{r_a}\right) = \ln(1 + \frac{d}{r_a}) \approx \frac{d}{r_a} \quad \text{if } d << r_a$$

$\Rightarrow C = \varepsilon_o \frac{2\pi L}{\ln(r_b/r_a)} \approx \varepsilon_o \frac{2\pi r_a L}{d} = \varepsilon_o \frac{A}{d}$ (same formula as for parallel-plate)
Commercial capacitors

- Commercial capacitors for home electronics have sizes range from a grain of rice to that of a large cigar.

- Capacitors like those mentioned above and pictured at right are microfarad capacitors.

- Note: Even though the exterior shape is a cylinder, it doesn’t mean that it is cylindrical capacitor; it could be a rolled-up parallel-plate capacitor.
Note on capacitance

Although the definition $C$ involves $Q$ and $V_{ab}$: $C = \frac{Q}{V_{ab}}$

The capacitance depends on the size and geometry of the capacitor,

Example:

$$C = \varepsilon_0 \frac{A}{d} \quad \text{for parallel-plate capacitor}$$

$$C = \varepsilon_0 \frac{2\pi L}{\ln(r_b / r_a)} \quad \text{for cylindrical capacitor}$$

If we plot the charge $(Q)$ vs the voltage across the capacitor $(V_{ab})$, then $C$ is the slope of the linear graph.
Example

Example 24.2. The dimensions of a parallel-plate capacitor are d=5mm and A=2 m². A charge (Q) of 35.4 μC is each plate

(a) What is the capacitance (C) ?

(b) What is the voltage ($V_{ab}$) across the capacitor?

(c) What is the magnitude of the electric field between the plates?

(d) Now pull the plates apart to d=10mm, what are the values for C, Q, $V_{ab}$, and E? Do you have to apply energy to pull the plates apart?
II. Energy stored in a charged capacitor

The energy stored in a charged capacitor is

\[ U = \frac{1}{2} QV_{ab} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV_{ab}^2 \]

Refer to the example in the previous slide, find the energy stored in the capacitor when \( d=5\text{mm} \) and the energy stored after you pull it apart to \( d=10\text{mm} \).

How much energy did you have to apply to pull the plates apart?

If the capacitor is connected to a battery (10 kV) while you are pulling the plates apart, what is the stored energy after you have pulled the plates to \( d=10\text{mm} \)?
Capacitor with dielectrics

- The potential difference between the parallel plates of a capacitor **decreases** when a dielectric material is inserted between the plates because the dielectric weakens the electric field (explanation in next slide).

Since \( C = \frac{Q}{V} \),
decreasing \( V \) while \( Q \) remains the same \( \Rightarrow \) increasing \( C \).
Dielectric partially shields the electric field

\[ E = \frac{E_0}{K}; \quad K = \text{dielectric constant of the material} \Rightarrow C = K \varepsilon_0 \frac{A}{d} \]
### Table 24.1—Dielectric constants

<table>
<thead>
<tr>
<th>Material</th>
<th>$K$</th>
<th>Material</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
<td>Polyvinyl chloride</td>
<td>3.18</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>1.00059</td>
<td>Plexiglas</td>
<td>3.40</td>
</tr>
<tr>
<td>Air (100 atm)</td>
<td>1.0548</td>
<td>Glass</td>
<td>5–10</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>Neoprene</td>
<td>6.70</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>2.25</td>
<td>Germanium</td>
<td>16</td>
</tr>
<tr>
<td>Benzene</td>
<td>2.28</td>
<td>Glycerin</td>
<td>42.5</td>
</tr>
<tr>
<td>Mica</td>
<td>3–6</td>
<td>Water</td>
<td>80.4</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.1</td>
<td>Strontium titanate</td>
<td>310</td>
</tr>
</tbody>
</table>
Intermission
IV. Dielectric breakdown

- A very strong electrical field can exceed the strength of the dielectric to contain it. Table 24.2 at the bottom of the page lists some limits.

- For a given capacitor separation (d), the dielectric strength sets a limit on the maximum voltage (max V=Emd) that can be sustained by the capacitor.

<table>
<thead>
<tr>
<th>Material</th>
<th>Constant, K</th>
<th>$E_m$ (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polycarbonate</td>
<td>2.8</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td>Polyester</td>
<td>3.3</td>
<td>$6 \times 10^7$</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>2.2</td>
<td>$7 \times 10^7$</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
<td>$2 \times 10^7$</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>4.7</td>
<td>$1 \times 10^7$</td>
</tr>
<tr>
<td>air</td>
<td>~1</td>
<td>$3 \times 10^6$</td>
</tr>
</tbody>
</table>
V. Capacitors in series and parallel circuits

**Series:**

\[ Q_1 = Q_2 \]

\[ V_1 + V_2 = V \]

**Parallel:**

\[ Q_1 + Q_2 = Q \]

\[ V_1 = V_2 = V \]

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**Capacitors in series:**
- The capacitors have the same charge \( Q \).
- Their potential differences add: \( V_{ac} + V_{cb} = V_{ab} \).

**Capacitors in parallel:**
- The capacitors have the same potential \( V \).
- The charge on each capacitor depends on its capacitance: \( Q_1 = C_1 V, Q_2 = C_2 V \).
Calculations regarding capacitance

Replace these series capacitors by an equivalent capacitor...

... replace these parallel capacitors by an equivalent capacitor...

... replace these series capacitors by an equivalent capacitor.
Energy stored in a charge capacitor re-visit.

The energy stored in a charged capacitor is

\[ U = \frac{1}{2} \frac{Q^2}{C} \]

Consider a parallel-plate capacitor, \( C = \varepsilon_o \frac{A}{d} \)

\[ \Rightarrow U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{\varepsilon_o A} = \frac{1}{2} \frac{Q^2}{\varepsilon_o^2 A^2} d \cdot A \cdot \varepsilon_o \]

\[ E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_o A} \Rightarrow U = \left( \frac{1}{2} \frac{\varepsilon_o E^2}{A^2} \right) \cdot \text{Volume} \]

\[ \left( \frac{1}{2} \frac{\varepsilon_o E^2}{A^2} \right) = \text{energy density (energy/volume)} \]

That is, one can think of energy being stored as separated positive and negative charges OR being stored in the E-field.

The second interpretation is useful when we discuss energy carried by electromagnetic waves.