Chapter 23

Electrostatic Potential

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Topics for Chapter 23

- Electrostatic potential energy

- Electrostatic potential function, $V(r)$, generated by source charges

- Connection between electrostatic potential, $V(r)$, and the electrostatic field, $E(r)$, generated by source charges

- Intermission

- Find $V(r)$ and $E(r)$ for continuous charge distributions

- Equipotential surfaces
Compare gravitational potential energy with electrostatic potential energy

- When two masses are separated (such as a baseball and the Earth) by a distance \( r \), we say that there is a gravitational potential energy associated with this configuration because if we let go the masses, they will move (toward each other); potential energy is converted into kinetic energy.

- Similarly, if two charges are separated by a distance \( r \), there is a electrostatic potential energy associated with this configuration, if we let go the charges, they will move (away from each other for like charges; toward each other for opposite charges); potential energy is again converted into kinetic energy.
Expression for electrostatic potential energy of two point charges

Gravitational potential energy:

\[ U_g = -\frac{Gm_1m_2}{r} \]

Lower potential energy if the masses are closer. (potential energy is more negative)
Masses tends to move toward lower potential energy if they are free to move.

Electrostatic potential energy:

\[ U_e = \frac{kq_1q_2}{r} \]

(q's can be positive or negative)
For two unlike charges, lower potential energy if charges are closer (they like to move closer)
For two like charges, lower potential energy if charges are further apart (they like to move away)
Relating work done by electrostatic force to change in electrostatic potential energy

\[ W = \int_{i}^{f} \vec{F} \cdot d\vec{l} = \int_{i}^{f} kq_1q_2 \frac{1}{r^2} dr \]

\[ = -\left( \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i} \right) = -(U_f - U_i) \]

**Work – Kinetic Energy Theorem:**

\[ K_f - K_i = W = -(U_f - U_i) \]

\[ \Rightarrow K_f + U_f = K_i + U_i \]

*(Conservation of energy)*

( Electrostatic force is a conservative force

\[ \vec{F} = -\frac{dU}{dr} \hat{r} \]

Example: Given: \( q_1 = +2\text{C}, q_2 = +1\text{C}, r_i = 3m \)
\( m_1 = 10\text{kg}, m_2 = 5\text{kg} \).

Let go of \( q_2 \) and it moves to infinity.

Find the final kinetic energy of \( q_2 \).

Find the final speed of \( q_2 \).

**Challenge question:** Let go both \( q_1 \) and \( q_2 \); they both move to infinity. Find the final speeds of \( q_1 \) and \( q_2 \).
Electric field concept:

$q_1 \rightarrow$ generates \( \vec{E}(\vec{r}) \) everywhere \( \rightarrow q_2 \) at location \( \vec{r} \)
senses \( \vec{E}(\vec{r}) \) and experiences a force \( \vec{F} = q_2 \vec{E}(\vec{r}) \)

Electrostatic potential \((V(\vec{r}))\) concept:

$q_1 \rightarrow$ generates \( V(\vec{r}) \) everywhere \( \rightarrow q_2 \) at location \( \vec{r} \)
senses \( V(\vec{r}) \) and the potential energy of this interaction is \( U = q_2 V(\vec{r}) \)

If \( q_1 \) is a point charge, then \( V(\vec{r}) = \frac{kq_1}{r} = \frac{kq_1}{\sqrt{x^2 + y^2 + z^2}} \);

the origin is chosen to be the location of \( q_1 \).
Connection between electrostatic potential and electrostatic field

\[ \vec{F} = -\frac{dU}{dr} \hat{r} \]
\[ \vec{E} = \frac{\vec{F}}{q_2} = -\frac{d(U/q_2)}{dr} \hat{r} = -\frac{dV}{dr} \hat{r} \]

**Check:**
\[ V(\vec{r}) = \frac{kq_1}{r} \Rightarrow \vec{E} = -\frac{dV}{dr} \hat{r} = \frac{kq_1}{r^2} \hat{r} \]

If we express \( V(\vec{r}) = V(x,y,z) = \frac{kq_1}{\sqrt{x^2 + y^2 + z^2}} \)

then \( \vec{E} = \left( -\frac{\partial V}{\partial x} \right) \hat{i} + \left( -\frac{\partial V}{\partial y} \right) \hat{j} + \left( -\frac{\partial V}{\partial z} \right) \hat{k} \)

**Note:** \( V(\vec{r}) \) is a scalar (not a vector) so, it is easier to calculate than \( \vec{E}(\vec{r}) \). Once \( V(\vec{r}) \) is known, \( E(\vec{r}) \) can be calculated from \( V(\vec{r}) \).
More than one source charge

Potential energy of source charges $q_1, q_2, and q_3$

$$U_{source} = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$ (i.e. all possible pair combinations)

If a test charge $q_o$ is added, then the additional potential energy is

$$U_{source-test charge interaction} = q_o \left( \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} \right) \equiv q_o V_{source}$$

That is, the electrostatic potential generated by several source charges is the sum of the individual potentials.
More than one source charge - example

Given: \( q_1 = +12nC, q_2 = -12nC \)

(1) **Find** the electrostatic potential at points a, b, and c

(2) Is the potential at point c zero? Is the electric field at point c zero?

(3) A test charge \( q_0 = +1nC \) travels from point b to point c, how much work is done by the electrostatic force?
Intermission
Example—a line of charge

- Refer to Example 23.12 using Figure 23.22.

1. Calculate $V(x, y = 0, z = 0)$ for all $x$.
2. From the potential function $V(x, y = 0, z = 0)$, find $E_x(x, y = 0, z = 0)$.
3. How would you find $\vec{E}$ at any point?
Example—Calculate $V(x,y,z)$ for a line of charge

(1) Calculate $V(x,y)$ for all $x$ and $y$

$$V(x,y,z) = \frac{kdQ}{r} = \frac{dQ}{dy'}$$

$$V(x,y,z) = k \int_{-a}^{a} \frac{\lambda dy'}{\sqrt{x^2 + (y-y')^2 + z^2}}$$

Once this is calculated, then we can find $\vec{E}(x,y,z)$ by taking derivatives,

$$\vec{E}(x,y,z) = \left(-\frac{\partial V}{\partial x}\right)\hat{i} + \left(-\frac{\partial V}{\partial y}\right)\hat{j} + \left(-\frac{\partial V}{\partial z}\right)\hat{k}$$

You are NOT required to do the integrals, but need to understand the concept.
Calculation of electrical potential from E-field

- If the E-field is known (say using Gauss’s Law), then one find the potential from E(r).

\[ E(r) = -\frac{dV}{dr} \hat{r} \quad \Leftrightarrow \quad V(r) = -\int E(r') \cdot d\hat{r}' \]
Example—oppositely charged parallel plates

What is the electrostatic potential function for a constant E-field?
Potential energy vs. potential

A positive test charge (if let go) would move from a region of higher potential to a region of lower potential (fundamentally it moves from high potential energy to low potential energy; $U = qV$)

A negative test charge (if let go) would move from a region of lower potential to a region of higher potential (again, fundamentally it moves from high potential energy to low potential energy; $U = -|q|V$)
Equipotential surfaces and field lines

- Surfaces of equal potential are **perpendicular** to the field lines.
- Spacing between equi-potentials => strength of E-field
- closer spacing => higher E-field strength
Field lines and a conducting surface

- Refer to Figure 23.25 to illustrate the concept of field lines near a conducting surface.