Chapter 22

Gauss’s Law

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– Hugh D. Young and Roger A. Freedman

Lectures by James Pazun  Modified by P. Lam 7_8_2008

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Topics for Chapter 22

• Concept of electric flux
• Gauss’s Law - relates electric flux to the amount of source charge
• (Intermission)
• Use Gauss’s Law to deduce the electric field of various symmetric charge distributions
• Conductor under electrostatic condition
Introduction

- Look at those electric field lines!

- In the last lecture, we impose a rule that the number of electric field lines should be proportional to the magnitude of the charge, we will dealt into the justification of that rule in this lecture (Gauss’s Law)
Electric Flux

- How much electric field “flows” out from a +1C of charge?

- One way to quantify the amount is to draw an imaginary “box” (doesn’t have to be a rectangular box, can be any shape) around the charge and “count” the amount of electric field vectors that pierce through the surface (an analogy is the amount of water flowing through a screen surrounding a sprinkler)

Electric Flux through a closed surface

\[ \Phi_E = (\vec{E} \cdot \hat{n}) \times \text{area} \] (qualitative idea)

Exact definition:

\[ \Phi_E = \int (\vec{E} \cdot \hat{n})dA \]
Why dot product?

- Using water flow as an analogy: the amount of water flowing through an area depends on whether that area is tiled or not.

(a) Water flux = $(\vec{v} \cdot \hat{n})A = |\vec{v}| |A|$

(b) Water flux = $(\vec{v} \cdot \hat{n})A = |\vec{v}| |A| \cos \phi$
Why dot product (cont)?

(a) Surface is face-on to electric field:
- \( \vec{E} \) and \( \vec{A} \) are parallel (the angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi = 0 \)).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \).

(b) Surface is tilted from a face-on orientation by an angle \( \phi \):
- The angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi \).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi \).

(c) Surface is edge-on to electric field:
- \( \vec{E} \) and \( \vec{A} \) are perpendicular (the angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi = 90^\circ \)).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90^\circ = 0 \).

\[ \vec{A} = \hat{n}A \]

Maximum flux when \( \vec{E} \parallel \vec{A} \)
\[ \vec{E} \cdot \vec{A} = |\vec{E} \parallel \vec{A} | \]

General case
\[ \vec{E} \cdot \vec{A} = |\vec{E} \parallel \vec{A} | \cos \phi \]

Minimum flux when \( \vec{E} \perp \vec{A} \)
\[ \vec{E} \cdot \vec{A} = |\vec{E} \parallel \vec{A} | \cos 90^\circ = 0 \]
Interesting observation

Consider the electric flux through an imaginary spherical surface center about the point source charge (q).

\[ \Phi_E = \int_{\text{closed surface}} \vec{E} \cdot \hat{n} \, dA \]

\( \vec{E} \) on the surface of the sphere = \( \frac{kq}{r^2} \hat{r} \)

\( \hat{n} = \) "outer normal unit vector" = \( \hat{r} \)

\[ \Rightarrow (\vec{E} \cdot \hat{n}) = \frac{kq}{r^2} \hat{r} \cdot \hat{r} = \frac{kq}{r^2} \]

*same number for every point on the surface*

\[ \Rightarrow \text{can take} (\vec{E} \cdot \hat{n}) \text{ outside the integral} \]

\[ \Rightarrow \Phi_E = (\vec{E} \cdot \hat{n}) \int dA = \frac{kq}{r^2} \int dA = \frac{kq}{r^2} (4\pi r^2) = 4\pi kq \]

\[ \Rightarrow (1) \text{ independent of the radius of the sphere.} \]

\[ \Rightarrow (2) \text{ linearly dependent on the source charge} \ (q) \]

This result is consistent with the analogy of water flux from a sprinkler - the total amount of water coming out a sprinkler is independent of the size of the imaginary surface as long as it encloses the sprinkler completely.

In fact, this analogy suggests that the result is true for arbitrary closed surfaces.
Gauss's Law

Gauss's Law:

\[ \Phi_E = \int (\vec{E} \cdot \hat{n}) dA = 4\pi k q_{\text{enclosed}} \]

defined on an arbitrary closed surface.

Define a new constant \( \frac{1}{\varepsilon_0} \equiv 4\pi k \)

\[ \Rightarrow \Phi_E = \int (\vec{E} \cdot \hat{n}) dA = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

defined on an arbitrary closed surface.

\( k \equiv 9.0 \times 10^9 \frac{Nm^2}{C^2} \);

\( \varepsilon_0 \equiv 8.854 \times 10^{-12} \frac{C^2}{Nm^2} \) "permittivity of free space"
What if the enclosed charge is negative?

\[ \Phi_E = \int (\vec{E} \cdot \hat{n})dA = 4\pi k q_{\text{enclosed}} \]

If the enclosed charge is negative then \((\vec{E} \cdot \hat{n})\) is negative and \(\Phi_E\) is negative.
What if the surface doesn’t enclosed any charge?

\[ \Phi_E = \oint (\vec{E} \cdot \hat{n})dA = 4\pi kq_{enclosed} \]

If the surface doesn't enclosed the charge
then \((\vec{E} \cdot \hat{n})\) is positive for some points on the surface
and negative for other points
\[ \Rightarrow \text{net } \Phi_E \text{ is zero.} \]
Examples

Find the electric flux through surface A, B, C, and D

Suppose I forgot to tell you that there is another charge outside surface C, would your answers above be different?
Examples

Given the electric field is uniform. Which orientation of the cube gives a larger flux?

(a)

(b)

Is there any charge enclosed inside the cube?
Intermission
Applications of Gauss’s Law

Gauss's Law:

\[ \Phi_E = \oint (\vec{E} \cdot \hat{n}) dA = \frac{q_{\text{enclosed}}}{\varepsilon_o} \]

(1) If the source charges are given then we can only determine the flux (the integral of E). However, if the source have special symmetries, then Gauss's Law can be used to determine the E-field in a much simpler way than the Coulomb's Law.

(2) If the know the E-field everywhere (do many measurements with a test charge), then you can determine where the source charges are located by calculating the flux at many locations.
Gauss’s Law application - I

- Find the E-field inside and outside of a thin spherical shell with radius $R$ and total charge $Q$. 

\[ E(R) = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \]

Inside the sphere, the electric field is zero: \( E = 0 \).

Outside the sphere, the magnitude of the electric field decreases with the square of the radial distance from the center of the sphere:

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \]
Gauss’s Law application - II

- Find the E-field outside and inside of a uniformly charged solid sphere with radius R and total charge Q.

\[
E(R) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2}
\]

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{Qr}{R^3}
\]
Gauss’s Law Application - III

- Find the E-field of outside an infinitely long, uniform line charge linear charge density $\lambda$ (charge/length)
Gauss’s Law Application - IV

- Find the E-field of an infinite, uniform charge sheet with surface charge density $\sigma$ (charge/area)
A field between parallel plates of opposing charge

- The capacitor is the actual device.

(a) Realistic drawing

Between the two plates the electric field is nearly uniform, pointing from the positive plate toward the negative one.

(b) Idealized model

In the idealized case we ignore “fringing” at the plate edges and treat the field between the plates as uniform.

Cylindrical Gaussian surfaces (seen from the side)
Charges on conductors under electrostatic condition

- A conductor is where the electrons are free to move.
- If one add some excess electrons to or remove some electrons from a conductor, all the electrons will move and re-distribute themselves under electrostatic condition is achieved, that is, the net force on every electron is zero and therefore no more movement of electrons
- Since the force on an electron is

\[ \vec{F} = (-e)\vec{E} = 0 \Rightarrow \vec{E} = 0 \] inside a conductor.

Applying Gauss’s Law=> no net charge enclosed inside the conductor; all the excess charge reside on the surface of a conductor
E-field inside a hollow conductor

- The E-field inside the cavity is also zero. The proof relies on the cavity being completely surrounded by regions of zero E-field.

(b) The same conductor with an internal cavity

Because \( \vec{E} = 0 \) at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

What happens to the field inside the cavity if we place an charge outside the conductor?
A Faraday cage shield the external E-fields

- A Faraday cage is an enclosed cage made with conductor.

Faraday cage protects the person inside from the high E-fields from the Van de Graaff.