Chapter 30

Inductance

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– Hugh D. Young and Roger A. Freedman

Lectures by James Pazun  Modified by P. Lam 8_6_2008
Topics for Chapter 30

• Mutual inductance
• Self-inductance
• Magnetic-field energy
• R-L circuits
• Intermission
• L-C circuits
• L-R-C circuits
Mutual inductance

- Faraday’s Law of Induction states that changing the current in one coil can induce a current in a nearby coil.

- This opens up a whole new way of transferring signal or energy from one coil (one device) to another device without contact!

- To quantify this coupling, we define a “mutual inductance” $M$:

  $$\varepsilon_2 = -M_{21} \frac{di_1}{dt} \quad \text{Likewise:} \quad \varepsilon_1 = -M_{12} \frac{di_2}{dt}$$

  It can shown that $M_{21} = M_{12}$; the mutual inductance depends on the geometry of coil 1 and coil 2.
Example - Mutual inductance of two concentric solenoids

- Refer to Example 30.1. To calculate the mutual inductance \( M \), we imagine putting a current \( i_1(t) \) through coil 1.

Use the long solenoid formula: \( B_1(t) = \mu_o \frac{N_1}{\ell} i_1(t) \)

The magnetic flux through coil 2: \( \Phi_2 = N_2 B_1(t) A \)

The induced emf in coil 2: \( \varepsilon_2 = -\frac{d\Phi_2}{dt} = -\mu_o \frac{N_1 N_2}{\ell} A \frac{di_1}{dt} \)

\[ \Rightarrow M = \mu_o \frac{N_1 N_2}{\ell} A \]
**Numerical Example - Mutual inductance**

**Numerical example**: \( \ell = 0.5\, \text{m}, A = 10^{-3}\, \text{m}^2, \)
\( N_1 = 1,000, N_2 = 10 \)

\[
M = \mu_o \frac{N_1 N_2}{\ell} A = \left(4\pi \times 10^{-7}\right) \left(\frac{1,000 \times 10}{0.5\, \text{m}}\right) (10^{-3})
\approx 25 \times 10^{-6} \text{ (unit?)}
\]

\[
M = 25 \times 10^{-6} \text{ Henry} = 25 \mu\text{H}
\]

**Suppose** \( i_1(t) = (2 \times 10^6 \frac{A}{s})t, \) find \( \varepsilon_2. \)

\[
\varepsilon_2 = -M \frac{di_1}{dt} = -(25 \mu\text{H})(2 \times 10^6 \frac{A}{s}) = -50 \text{ volts}
\]

**Suppose instead of putting the current through coil 1, we put the current through coil 2, i.e.**

\( i_2(t) = (2 \times 10^6 \frac{A}{s})t. \) **What is the induced emf on coil 1?**
Self-inductance (L)

Even when there is only one coil, there is an induced emf the coil if the current in the coil is changing with time.

Self-inductance: If the current \( i \) in the coil is changing, the changing flux through the coil induces an emf in the coil.

\[
\varepsilon = -L \frac{di}{dt}
\]

For a long solenoid, the self-inductance is \( L = \frac{\mu_0 N^2 A}{\ell} \).

For a straight conductor, \( L = 0 \).
How to increase Self-inductance and Mutual inductance

Recall vacuum (or air) gap capacitor has very small capacitance:

\[ C = \varepsilon_0 \frac{A}{d} \]  

because \( \varepsilon_0 \) is a very small number.

We can increase the capacitance by filling the gap with a dielectric material (electrically polarizable material),

\[ C = K\varepsilon_0 \frac{A}{d} \quad ; \quad K = \text{dielectric constant}. \]

Similarly, coils in vacuum (or air) have very small self and mutual inductances because \( \mu_0 \) is a very small number.

We can increase the inductance by filling the space inside the coil with a magnetically polarizable material such as iron and the inductance can be increased by several thousand times.
Voltage across an inductor (a coil)

(a) Resistor with current $i$ flowing from $a$ to $b$: potential drops from $a$ to $b$.

$\begin{align*}
  \begin{array}{c}
    a \\
    \downarrow
  \end{array} & \quad \begin{array}{c}
    b \\
    \uparrow
  \end{array} \\
  + & \quad - \\
  R & \quad V_{ab} = iR > 0
\end{align*}$

(b) Inductor with constant current $i$ flowing from $a$ to $b$: no potential difference.

$\begin{align*}
  \begin{array}{c}
    a \\
    \downarrow
  \end{array} & \quad \begin{array}{c}
    b \\
    \uparrow
  \end{array} \\
  \varepsilon = 0
\end{align*} \quad V_{ab} = L \frac{di}{dt} = 0$

(c) Inductor with increasing current $i$ flowing from $a$ to $b$: potential drops from $a$ to $b$.

$\begin{align*}
  \begin{array}{c}
    a \\
    \downarrow
  \end{array} & \quad \begin{array}{c}
    b \\
    \uparrow
  \end{array} \\
  \varepsilon & \quad V_{ab} = L \frac{di}{dt} > 0
\end{align*}$

(d) Inductor with decreasing current $i$ flowing from $a$ to $b$: potential increases from $a$ to $b$.

$\begin{align*}
  \begin{array}{c}
    a \\
    \downarrow
  \end{array} & \quad \begin{array}{c}
    b \\
    \uparrow
  \end{array} \\
  \varepsilon & \quad V_{ab} = L \frac{di}{dt} < 0
\end{align*}$

\[\varepsilon = -L \frac{di}{dt}\]

\[V_{ab} = -\varepsilon = L \frac{di}{dt}\]
Magnetic field energy

A charge capacitor stores energy: \( U = \frac{1}{2} CV^2 \)

so does an inductor with a current: \( U = \frac{1}{2} LI^2 \)

Derivation:

\[
\text{Power} = \frac{dU}{dt} = iV = iL \frac{di}{dt}
\]

\[
\Rightarrow U = \int_0^l \frac{dU}{dt} dt = \int_0^l iL \frac{di}{dt} dt = \frac{1}{2} Li^2 \bigg|_0^l = \frac{1}{2} Li^2
\]
The R-L circuit

The switch is closed at \( t = 0 \).
Before \( t = 0 \), the current is zero.
Since the inductor (L) opposes change current, the current CANNOT change abruptly thus the current increases gradually (In RC circuit, the current can change abruptly but the voltage across the capacitor has to change gradually because charge has to build up).

After a long time, the current stops changing

\[
\left( \frac{di}{dt} = 0 \right), \text{ the voltage across the inductor } = L \frac{di}{dt} = 0
\]

\[\Rightarrow\] the voltage across R is \( \varepsilon \)

\[\Rightarrow I_{\text{final}} = \frac{\varepsilon}{R} \]
The R-L circuit - Differential equation

Kirchoff's loop rule:

\[ \varepsilon - iR - L \frac{di}{dt} = 0 \]

\[ \Rightarrow L \frac{di}{dt} + Ri = \varepsilon \]

(Inhomogeneous, first order linear DE)

We have solved this type of DE when we consider charging a capacitor:

\[ R \frac{dQ}{dt} + \frac{1}{C} Q = \varepsilon \]

The solution consists of two parts:

The steady state (or inhomogeneous solution) + the transient (or homogeneous solution)

\[ \Rightarrow i(t) = \frac{\varepsilon}{R} + Ae^{-t/\tau}; \quad \tau = \frac{L}{R} \]

\[ A \text{ is determined by the initial current.} \]

If \( i(t = 0) = 0 \), then \( A = -\frac{\varepsilon}{R} \Rightarrow i(t) = \frac{\varepsilon}{R} \left[1 - e^{-t/\tau}\right] \)
**The R-L circuit - Mechanical analog**

*Kirchoff's loop rule:*

\[
\varepsilon - iR - L \frac{di}{dt} = 0
\]

\[
\Rightarrow L \frac{di}{dt} + Ri = \varepsilon
\]

\[i \leftrightarrow \text{velocity}\]

\[\frac{di}{dt} \leftrightarrow \text{acceleration}\]

\[L \leftrightarrow \text{mass}\]

\[\varepsilon \leftrightarrow \text{constant force (say, due to gravity)}\]

\[Ri \leftrightarrow \text{air - resistance force}\]

Drop a mass in the air, the velocity increases gradually until it reaches terminal velocity.
The R-L circuit - “discharging the current”

**Kirchoff’s Rule:**

\[-(iR) + (-L \frac{di}{dt}) = 0\]

\[\Rightarrow L \frac{di}{dt} + iR = 0\]

\[\Rightarrow i(t) = Ae^{-t/\tau}; \quad \tau = \frac{L}{R}\]

When \(S_2\) is open, the "initial" current is \(\frac{\varepsilon}{R}\)

\[\Rightarrow i(t) = \frac{\varepsilon}{R}e^{-t/\tau}\]
The R-L circuit - “discharging the current” -II

- Follow Example 30.7.

Switch $S_2$ is closed at $t = 0$. The equation $t = \frac{L}{R}$ is represented in the diagram.
Intermission
The L-C circuit

• Consider Figure 30.14.

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Compare LC circuit with mass-spring system

- Consider Figure 30.15.
- Use Table 30.1.
- Follow Example 30.9.
- Follow Example 30.10.

Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an L-C Circuit

<table>
<thead>
<tr>
<th>Mass-Spring System</th>
<th>Inductor-Capacitor Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic energy = $\frac{1}{2}mv_x^2$</td>
<td>Magnetic energy = $\frac{1}{2}Li^2$</td>
</tr>
<tr>
<td>Potential energy = $\frac{1}{2}kx^2$</td>
<td>Electric energy = $\frac{q^2}{2C}$</td>
</tr>
<tr>
<td>$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$</td>
<td>$\frac{1}{2}Li^2 + q^2/2C = \frac{Q^2}{2C}$</td>
</tr>
<tr>
<td>$v_x = \pm \sqrt{k/m}\sqrt{A^2 - x^2}$</td>
<td>$i = \pm \sqrt{1/LC}\sqrt{Q^2 - q^2}$</td>
</tr>
<tr>
<td>$v_x = dx/dt$</td>
<td>$i = dq/dt$</td>
</tr>
<tr>
<td>$\omega = \sqrt{\frac{k}{m}}$</td>
<td>$\omega = \sqrt{\frac{1}{LC}}$</td>
</tr>
<tr>
<td>$x = A\cos(\omega t + \phi)$</td>
<td>$q = Q\cos(\omega t + \phi)$</td>
</tr>
</tbody>
</table>
The L-R-C circuit - damped harmonic oscillator

- An application of the damped oscillator.
- Consider Figure 30.16.
- Consider Figure 30.17.
- Follow Example 30.11.