Chapter 10: Step 5 Supplement

At this stage of the semester it is very important that if you feel you are struggling, you should do a self-diagnosis on what step you are at. **If you can't do the exercises in steps 1 and 4 quickly, you will have a miserable time on step 5.** If you can't do these steps quickly, you have to go back and repeat the learning steps outlined for these steps and then do the exercises again. I suggest the tutorials for steps 1 and 4.

If you did ok on steps 1 and 4, then you should be successful doing the step 5 exercises using the strategies on page 444. Some students will not need them because they will be good enough with the rules such that they just "see" the answers. But for most of you, at least at first, as in Chapter 9, you will need a strategy of staring. With 19 rules now, you will drive yourself absolutely crazy if you just randomly stare at any of the lines in the step 5 exercises and then randomly stare at the rules. This “stare-and-hope method” does not work!

The textbook preparation for step 5 exercises takes you through one problem step by step. I will do one more short one here that uses all three strategies.

1. \((A \lor B) \supset (\neg Y \supset \neg X)\)
2. \(A \lor B\)
3. \(Y \supset P \therefore X \supset P\)
4. \(\neg Y \supset \neg X\)
5. \(X \supset Y\)
6. \(X \supset P\)

We start with line (4). Using the strategies, we see that strategy 1 seems to apply. Line (4) is part of line (1). By now you should see that the \(\neg Y \supset \neg X\) in line (1) is in a perfect place for an MP and that line (2) is the match we need for MP.

1. \((A \lor B) \supset (\neg Y \supset \neg X)\)
2. \(A \lor B \therefore \neg Y \supset \neg X\)

\(p \supset q\)
\(p \therefore q\)

So, we have:

1. \((A \lor B) \supset (\neg Y \supset \neg X)\)
2. \(A \lor B\)
3. \(Y \supset P \therefore X \supset P\)
4. \(\neg Y \supset \neg X\) \(\text{(1)(2) MP}\)
5. \( X \supset Y \)
6. \( X \supset P \)

Next, in looking at line (5) we see that strategy 1 does not work. \( X \supset Y \) is not part of a premise. But in moving to strategy 3, we see that line (5) is close to (same letters as) line (4). Because only the rules of replacement apply to strategy 3, the only question is whether we have a rule of replacement that justifies moving from \(~Y \supset \neg X\) to \( X \supset Y \). We do. Transposition allows this move.

\(~Y \supset \neg X\) /\(\therefore X \supset Y\)

\((p \supset q) \equiv (\neg q \supset \neg p)\)

Remember that the rules of replacement go in both directions. If we have a line that fits \(~q \supset \neg p\), which we do in line (4), we can produce \( p \supset q \), which we have in line (5).

So, now we have:

1. \((A \lor B) \supset (~Y \supset \neg X)\)
2. \(A \lor B\)
3. \(Y \supset P \) /\(\therefore X \supset P\)
4. \(~Y \supset \neg X\) \((1)(2)\) MP
5. \(X \supset Y\) \((4)\) Trans.
6. \(X \supset P\)

Next we see that in looking at line (6) neither strategy 1 nor 3 seem to work. Line (6) is not part of a premise and there does not appear to be another line that is at all similar. So we try strategy 2. By now you should know that this means we focus on the connective (\(\supset\)) of line (6). Furthermore, it would mean we would be considering the rules of Absorption or Hypothetical Syllogism. But obviously Abs. does not apply -- line (6) does not fit \( p \supset (p \bullet q) \). So, we are left only with HS and would be looking to fill in:

\(X \supset ???\)

???(???) \(\supset P \) /\(\therefore X \supset P\)

\(p \supset q\)
\(q \supset r \) /\(\therefore p \supset q\)

Lines (5) and (3) match what we need. So, we can complete the proof as follows:

1. \((A \lor B) \supset (~Y \supset \neg X)\)
2. \(A \lor B\)
3. \(Y \supset P \) /\(\therefore X \supset P\)
4. \( \neg Y \supset \neg X \) (1)(2) MP
5. \( X \supset Y \) (4) Trans.
6. \( X \supset P \) (5)(3) HS

Now do the step 5 exercises. Be sure to resist the temptation of just looking at the answers before you attempt all the problems. When you are stuck on a line, at that point you have the opportunity to learn the most. Focus on what you are doing. What strategy are you trying? What rules apply to that strategy? If a strategy does not work, remember to go to the next one.