Chapter 9: Answers and Comments

Step 1 Exercises

1. Simplification.
2. Absorption.
3. See textbook.
5. Modus Ponens.
7. X -- A very common student mistake; can't use Simplification unless the major connective of the premise is a conjunction.
8. Disjunctive Syllogism.
9. X -- Fallacy of Denying the Antecedent.
10. X
11. Constructive Dilemma.
12. See textbook.
13. Hypothetical Syllogism.
15. Conjunction.
16. See textbook.
17. Addition.
18. Modus Ponens.
19. X -- Fallacy of Affirming the Consequent.
20. Disjunctive Syllogism.
21. X -- not HS, the (D v G) does not match (D v C). This is deliberate to make sure you don't just focus on generalities, and make sure the entire form fits.
22. Constructive Dilemma.
23. See textbook.
25. Modus Ponens.
27. See textbook.
29. Modus Ponens.
30. Disjunctive Syllogism.
Step 2 Exercises

#1
1 Z • A
2. (Z v B) ⊃ C / : Z • C
4. Z v B (3) Add.
5. C (2)(4) MP

For line 4 it is easy to get locked into line 2 and strategy 1. But they do not work.

#2
1. K ⊃ (B v I)
2. K
3. ~B
4. I ⊃ (~T • N)
5. N ⊃ T / : ~N
6. B v I (1)(2) MP
7. I (6)(3) DS
8. ~T • N (4)(7) MP
10. ~N (5)(9) MT

#3 See textbook.

#4
1. H ⊃ I
2. I ⊃ J
3. K ⊃ L
4. H v K / : J v L
5. H ⊃ J (1)(2) HS
6. (H ⊃ J) • (K ⊃ L) (5)(3) Conj.
7. J v L (6)(4) CD

#5
1. ~R ⊃ S
2. ~T ⊃ (U ⊃ V)
3. T v (~R v U)
4. ~T / : S v V
5. U ⊃ V (2)(4) MP
6. (~R ⊃ S) • (U ⊃ V) (1)(5) Conj.
7. ~R v U (3)(4) DS
8. S v V (6)(7) CD
#6
1. \( O \Rightarrow P \)
2. \((O \cdot P) \Rightarrow Q\)
3. \( \neg(O \cdot Q) \) /.: \( \neg O \)
4. \( O \Rightarrow (O \cdot P) \)  (1) Abs
5. \( O \Rightarrow Q \)  (4)(2) HS
6. \( O \Rightarrow (O \cdot Q) \)  (5) Abs.
7. \( \neg O \)  (6)(3) MT

#7
1 \( X \Rightarrow Y \)
2. \((X \Rightarrow Z) \Rightarrow (A \lor Y)\)
3. \( (X \cdot Y) \Rightarrow Z \)
4. \( \neg A \) /.: \( Y \)
5. \( X \Rightarrow (X \cdot Y) \)  (1) Abs.
6. \( X \Rightarrow Z \)  (5)(3) HS
7. \( A \lor Y \)  (2)(6) MP
8. \( Y \)  (7)(4) DS

#8  See Textbook.

#9
1. \( G \Rightarrow \neg H \)
2. \( \neg G \Rightarrow (I \Rightarrow \neg H) \)
3. \( (\neg J \lor \neg I) \Rightarrow \neg \neg H \)
4. \( \neg J \) /.: \( \neg I \)
5. \( \neg J \lor \neg I \)  (4) Add.
6. \( \neg \neg H \)  (3)(5) MP
7. \( \neg G \)  (1)(6) MT
8. \( I \Rightarrow \neg H \)  (2)(7) MP
9. \( \neg I \)  (8)(6) MT

#10
1. \( A \Rightarrow B \)
2. \( \neg (A \cdot B) \)
3. \( A \lor (\neg L \cdot \neg K) \)
4. \( P \Rightarrow \neg L \) /.: \( \neg P \lor \neg Y \)
5. \( A \Rightarrow (A \cdot B) \)  (1) Abs.
6. \( \neg A \)  (5)(2) MT
7. \( \neg L \cdot \neg K \)  (3)(6) DS
8. \( \neg L \)  (7) Simp.
9. \( \neg P \)  (4)(8) MT
10. \( \neg P \lor \neg Y \)  (9) Add.
Step 3 Exercises:

#1
1. A \implies B
2. B \implies C
3. \neg C \therefore \neg A
4. A \implies C \hspace{1cm} (1)(2) \text{ HS}
5. \neg A \hspace{1cm} (4)(3) \text{ MT}

#2
1. A \implies B
2. A \lor C
3. \neg B \therefore C
4. \neg A \hspace{1cm} (1)(3) \text{ MT}
5. C \hspace{1cm} (2)(4) \text{ DS}

#3
1. D \implies B
2. F \lor \neg B
3. \neg F \therefore \neg D
4. \neg B \hspace{1cm} (2)(3) \text{ DS}
5. \neg D \hspace{1cm} (1)(4) \text{ MT}

#4 See textbook.

#5
1. \neg D
2. \neg D \implies \neg B
3. C \implies B \therefore \neg C
4. \neg B \hspace{1cm} (1)(2) \text{ MP}
5. \neg C \hspace{1cm} (3)(4) \text{ MT}

#6
1. J \implies (K \bullet L)
2. S \lor J
3. \neg S
4. K \implies (S \lor T) \therefore T
5. J \hspace{1cm} (2)(3) \text{ DS}
6. K \bullet L \hspace{1cm} (1)(5) \text{ MP}
7. K \hspace{1cm} (6) \text{ Simp.}
8. S \lor T \hspace{1cm} (4)(7) \text{ MP}
9. T \quad (8)(3) \text{ DS}

#7 See textbook.

#8

1. A \supset B
2. C \supset D
3. A \lor C
4. \neg B
5. D \supset (S \supset B) \therefore \neg S
6. (A \supset B) \land (C \supset D) \quad (1)(2) \text{ Conj.} \quad \text{Most students will start this problem with} \neg A -- (1)(4) \text{ MT.}
7. B \lor D \quad (3)(6) \text{ CD}
8. D \quad (7)(4) \text{ DS} \quad \text{Good to show different trails.}
9. S \supset B \quad (5)(8) \text{ MP}
10. \neg S \quad (9)(4) \text{ MT}

#9

1. (D \equiv \neg H) \supset (P \lor \neg T)
2. \neg \neg X
3. D \equiv \neg H
4. P \supset \neg X
5. \neg T \supset Y
6. (\neg X \lor Y) \supset G \quad / \quad G \bullet \neg P
7. P \lor \neg T \quad (1)(3) \text{ MP}
8. \neg P \quad (4)(2) \text{ MT}
9. (P \supset \neg X) \bullet (\neg T \supset Y) \quad (4)(5) \text{ Conj.}
10. \neg X \lor Y \quad (9)(7) \text{ CD}
11. G \quad (6)(10) \text{ MP}
12. G \bullet \neg P \quad (11)(8) \text{ Conj.}

Line 9 is the key step. Many students will get Y as follows:

9. \neg T \quad (7)(8) \text{ DS}
10. Y \quad (5)(9) \text{ MP}

But we cannot get to \neg X \lor Y from just Y with the 9 rules only. As we will see in C10, to do this common sense move, we need more common sense rules.

#10

1. X \supset (Y \supset H)
2. X
3. H \supset P
4. Y
5. \(~(H \cdot P) / \vdash Q\)
6. Y \(\supset H\)  
   (1)(2) MP
7. H  
   (6)(4) MP
8. H \(\supset (H \cdot P)\)  
   (3) Abs.
9. \(~H\)  
   (8)(5) MT
10. H \lor Q  
    (7) Add
11. Q  
    (10)(9) DS

This last problem can also be solved by finding \((H \cdot P)\) and \(~(H \cdot P)\), and Y and \(~Y\) as the needed contradictions.

Translations for Chapter 9 exercises:

#1
1. C \(\supset (I \supset \sim B)\)
2. C \(\supset I\)
3. C \(\vdash \sim B\)

Many students will mistranslate the first premise, because they have forgotten how to render the phrase "provided that." See number 21 in the dictionary, C7. The first premise could also be I \(\supset (C \supset \sim B)\) or \((C \cdot I) \supset \sim B\).

#2
1. A \(\supset I\)
2. I \(\supset M\)
3. M \(\supset C\)  \(\vdash A \supset C\)

The key phrase here is "for the following reasons." This phrase indicates that the first statement is the conclusion and the sentences after the phrase are the premises.

#3
1. I
2. F
3. I \(\supset (F \supset P)\)  \(\vdash P \lor R\)

The key phrase here is "This is so, because. . ." It tells us the first sentence is the conclusion and the two statements after "because" are the premises.

#5 A challenging formal proof. If you want more practice on step 3 proofs, try this one.

1. V \(\supset C\)
2. \((V \cdot C) \supset L\)
3. \([(V \cdot C) \cdot L] \supset T\)
4. \{[(V \cdot C) \cdot L] \cdot T\} \supset A  \(\vdash V \supset A\)
The parentheses, brackets and brace need to be this way in order for the proof to be done. Notice that \((V \bullet C)\) in the second premise is repeated and stays consistent in premises 3 and 4. For a hint on the proof (the most difficult in all of Chapter 9), notice that the major connective of all the statements is \((\equiv)\).

#6

1. \((T \lor O) \equiv \sim E\)
2. \(E \lor R\)
3. \((R \bullet \sim D) \equiv W\)
4. \(T\)
5. \(\sim D \quad /:\:\sim W \sim R\)

An easy proof after the first step, but the very first step may require a little insight.

#7

1. \(E \equiv A\)
2. \(P \equiv C\)
3. \(\sim A\)
4. \(\sim E \equiv \sim C \quad /:\:\sim P\)

A good test on whether you remember how to translate necessary conditions and if-statements where the "if" is in the middle of the sentence. (See \#s 17 and 22 in the dictionary.). The phrase "it follows that" indicates the conclusion.

#8

1. \(E \equiv M\)
2. \(L \equiv V\)
3. \((E \lor L) \bullet (E \bullet L)\)
4. \(\sim M \quad /:\:\sim V\)

Search EL for ‘exclusive’ in C8 if you forgotten how to translate an exclusive disjunction (‘or’).

#9

1. \(\sim (V \equiv T)\)
2. \(\sim (V \equiv T) \equiv (V \bullet \sim T)\)
3. \((S \equiv T) \bullet (T \equiv (V \lor S))\)
4. \((S \equiv T) \equiv (S \cdot \sim T) \quad /:\:\sim (V \bullet \sim T) \cdot \sim (S \bullet \sim T)\)

Notice that \(\sim V \equiv T\) would be wrong for the first premise and the antecedent of the second premise. That would say, "If an argument is not valid, then the premises are true." That would be as dumb as someone saying, "If you fail the final, then you will pass the course." Remember, think about the difference between \(\sim (F \equiv C)\) and \(\sim F \equiv C\).

#10

1. \((F \bullet C) \equiv Q\)
2. \((P \bullet S) \equiv (F \bullet C)\)
3. \(\sim (P \bullet S) \equiv (\sim P \bullet \sim S)\)
4. \((\sim P \bullet \sim S) \equiv \sim H\)
5. \(\sim Q\)
6. \(\sim P \equiv \sim E \quad /:\:\sim (H \bullet \sim E) \lor (L \lor R)\)
Believe it or not, #s 9 and 10 are easy proofs. Both translations though take patience.