Chapter 8: Exercises III

1. \begin{array}{cccc}
   \text{p1} & \text{p2} & \text{c} \\
   \text{A} & \text{B} & \text{A} \\
   \text{T} & \text{T} & \text{T} \\
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2. \begin{array}{cccc}
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\end{array}

3. See textbook.

4. \begin{array}{cccccccc}
   \text{p1} & \text{p2} & \text{p3} & \text{c} \\
   \text{X} & \text{Y} & \text{Z} & \text{X} & \text{(~Y} & \text{Z}) & \text{Y} & \text{v} & \text{X} & \text{~Y} & \text{Z} & \text{(~Y} \\
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Exercise IV

#1

1. I \lor D  
2. D \pimp; \neg I  

Inclusive or; he could be both.

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#2

1. \( W \)
2. \( J \)
3. \( T \supset (W \cdot J) /\therefore T \)

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#3

1. \( G \supset P \)
2. \( (G \cdot P) \supset U \)
3. \( U /\therefore G \)

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#4

1. \( S \supset M \)
2. \( R \bullet (R \supset M) \vdash S \)

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<th>M</th>
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#5

1. \( (~I \supset ~Y) \bullet (~Y \supset ~I) \)
2. \( (~I \bullet ~Y) \supset R \vdash (~I \lor ~Y) \supset R \)

<table>
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<th>( ~Y \supset ~I)</th>
<th>(~I \bullet ~Y)</th>
<th>(~I \lor ~Y)</th>
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Valid

#6 See textbook
#7

1. \( R \lor \neg R \)
2. \( R \supset (F \cdot S) \)
3. \( \neg R \supset \neg(F \lor S) \therefore (F \cdot \neg S) \)

\[
P_1 & | P_2 & | P_3 & | C \\
| R & F & S & | R \lor \neg R & | R \supset (F \cdot S) & | \neg R \supset \neg (F \lor S) & | (F \cdot \neg S) & | \therefore \\
T & T & T & | T & | T & | F & | F & | T & F \\
T & T & F & | T & | F & | F & | T & | F & T \\
T & F & T & | T & | F & | F & | T & | F & T \\
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Valid

#8

1. \( C \supset F \)
2. \( E \supset P \)
3. \( C \supset \neg E \therefore F \supset \neg P \)

\[
P_1 & | P_2 & | P_3 & | P & | C & | E & | F & | \therefore \\
| C & F & E & P & | C \supset F & | E \supset P & | C \supset \neg E & | F \supset \neg P & | \\
T & T & T & T & | T & | T & | F & | F & \\
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Invalid
#9

1. \( \text{U} \supset \text{R} \)
2. \( \text{P} \supset \text{U} \quad \therefore \quad \neg(\text{P} \cdot \neg\text{R}) \)

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#10

1. \( \text{C} \supset (\text{S} \cdot \text{O}) \)
2. \( \text{D} \supset \neg\text{S} \)
3. \( \neg(\text{C} \cdot \text{D}) \quad \therefore \quad \neg\text{O} \lor \text{S} \)

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#11

1. \( C \supset \neg L \)
2. \( P \supset \neg C \)
3. \( \neg C \supset L \) \( \therefore (P \supset L) \cdot (L \supset \neg C) \)

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#12

1. \( B \supset T \)
2. \( T = E \)
3. \( (T \supset E) \supset \neg S \) \( \therefore (B \supset E) \cdot \neg S \)

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