Question 1 on page 306 (20 points)
Answer:

Given Information:
y = 100
y' = 120
t = 20
t' = 10
r = 0.1

(a) To calculate wealth, we compute:
w = y - t + \frac{y' - t'}{1 + r} = 80 + \frac{110}{1.1} = 180

(b) In the perfect complements case, the indifference curve are like I₁ and I₂ in Figure 8.1.

(c) The consumer’s optimal consumption bundle is at point A. Point A simultaneously solves:

Figure 8.1
\[ c = c', \quad \text{and} \]
\[ c + \frac{c'}{1+r} = c + 0.91c' = 180 \]

Upon solving, we find that \( c = c' = 94.2 \). Savings is therefore given by:

\[ s = y - t - c = 80 - 94.2 = -14.2 \]

The consumer is a borrower. In Figure 8.1, the endowment point is \( E_1 \) and the consumer chooses A.

(d) First-period income rises from 100 to 140. We now recomputed \( w = 220 \). Solving as in part c, we find that \( c = c' = 115.2 \), and \( s = 4.8 \). In Figure 8.1, the endowment point is \( E_2 \) and the consumer chooses B.

(e) In part c, the consumer is a borrower. In part d, first-period income increases and savings has consequently increased enough that the consumer is now a lender.

**Question 2 on page 306 (10 points)**

**Answer:**

In this problem, there is a simultaneous increase in both future income and the real interest rate. The increase in future income is a positive income effect for both borrowers and lenders. The increase in the real interest rate includes a pure substitution effect and a pure income effect. The substitution effect induces the consumer to consume less in the current period and more in the second period. The direction of the pure income effect part of the real interest rate change depends on whether the consumer is a borrower or a lender. Lenders are better off with a higher real interest rate and borrowers are worse off with a higher real interest rate.

The top figure below shows the case of a borrower. The consumer starts out with endowment \( E_1 \) and picks point A on indifference curve \( I \). The diagram shows the case in which the positive income effect of the increase in \( y \) is exactly canceled out by the negative income effect of the increase in \( r \). In this particular case, \( c \) falls and \( c' \) increases. Since current income is fixed, the consumer must increase saving. For the borrower, this amounts to a reduction in borrowing. The consumer therefore picks point B, which is also on indifference curve \( I \), but which is parallel to a budget line that passes through \( E_2 \).

The bottom figure below shows the case of a lender. The consumer starts out with endowment \( E_3 \). The consumer chooses point D that is a tangency of indifference curve \( I_1 \) with the budget line that passes through point \( E_3 \). The disturbance shifts the budget line out to the line that passes through \( E_4 \), the new endowment point. The substitution effect moves the consumer from point D to point G on \( I_1 \). The pure substitution effect induces a reduction in \( c \) and an increase in \( c' \). The net income effect is then represented by a parallel shift in the line through G to the new budget line. In this case, the two income effects move in the same direction. Therefore both \( c \) and \( c' \) increase from point G to point F. Second-period consumption unambiguously increases. First-period consumption (and therefore savings) may either rise or fall. The bottom figure below shows the case in which \( c \) increases. If \( c \) increases, \( s \) must fall.
Question 4 on page 306 (10 points)
Answer:
Temporary and Permanent Tax Increases.
(a) The increase in first-period taxes induces a parallel leftward shift in the budget line. The original budget line passes through the initial endowment, E₁. The new budget line passes through E₂. The consumer reduces both current and future consumption. In Figure 8.4 the consumer’s optimum point moves from point A to point B. First-period consumption falls by less than the increase in taxes and so savings falls.
(b) Next consider a permanent increase in taxes. A permanent tax increase adds a second tax increase to the first tax increase, the current-period tax increase. The increase in second-period taxes induces a parallel downward shift in the budget line. The new budget line passes through $E_2$ in Figure 8.4. The second part of the tax increase also reduces both first-period and second-period consumption. The consumer moves from point $B$ to point $D$. Because the second tax increase reduces first-period consumption holding first-period disposable income fixed, savings must rise. Since the permanent tax increase is the sum of the two individual tax increases, the permanent tax increase reduces both first-period and second-period consumption, but on net, savings may either rise, fall, or remain unchanged.
Question 5 on page 307 (10 points)

Answer:

A tax on interest income.

(a) Initially, AB in Figure 8.5 depicts the consumer’s budget constraint. The introduction of the tax results in a kink in the budget constraint, since the interest rate at which the consumer can lend, \( r(1-t) \), is now smaller than the interest rate at which the consumer borrows, \( r \). The kink occurs at the endowment, \( E \).

(b) The top panel of Figure 8.5 shows the case of a consumer who was a borrower before the imposition of the tax. This consumer is unaffected by the introduction of the tax. The bottom panel of Figure 8.5 shows the case of a consumer who was a lender before the imposition of the tax. Initially the consumer chooses point \( G \), and then chooses
point H after the imposition of the tax. There is a substitution effect that results in an increase in first-period consumption and a reduction in second-period consumption, and moves the consumer from point G to point J. Savings also falls from point G to point J. The income effect is the movement from point J to point H, and the income effect reduces both first-period and second-period consumption, and increases savings. On net, consumption must fall in period 2, but in period 1, consumption may rise or fall. Figure 8.5 shows the case in which first-period consumption increases, which is a case where the substitution effect dominates.

**Question 8 on page 307 (15 points)**

**Answer:**

Given information:

\[ y = 200 \]
\[ y' = 150 \]
\[ t = 40 \]
\[ t' = 50 \]
\[ r = 0.05 \]

(a) If the consumer could borrow and lend at the real interest rate, \( r = 0.05 \), then the consumer’s lifetime budget constraint would be given by:

\[
c + \frac{c'}{(1+r)} = y - t + \frac{y' - t'}{(1+r)}.
\]

Plugging in the numbers from this problem, we obtain:

\[ c + 0.95c' = 255.2. \]

In the figure below, the initial budget constraint is given by BE1D. The budget constraint has a kink at the initial endowment point \( E_1 = (160,100) \), because the consumer cannot borrow, and therefore cannot consume more than 160 in the first period. Because the consumer has perfect-complements preferences, the indifference curves are kinked at \( c = c' \).
With perfect-complements preferences, the consumer picks point A in the figure on the previous page. Plugging in \( c = c' \) into the budget constraint and solving, we find that \( c = c' = 130.7 \) and so \( s = y - t - c = 160 - 130.7 = 29.3 \). In this case, the fact that the consumer cannot borrow does not matter for the consumer’s choice, as the consumer decides to be a lender.

When \( t = 20 \) and \( t' = 71 \), the consumer’s lifetime wealth remains unchanged at 255.2. However, the budget constraint shifts to \( BE_2F \), figure on the previous page, with the new endowment point at \( E_2 = (180, 79) \). This change does not matter for the consumer’s choice, again because he or she chooses to be a lender. Consumption is still 130.7, but now savings is \( s = y - t - c = 180 - 130.7 = 49.3 \).

Now first-period income falls to 100. Wealth is now equal to \( w = 155.2 \). In the figure above, the budget constraint for the consumer is \( AE_1D \), so when the consumer chooses the point on his or her budget constraint that is on the highest indifference curve, any point on the line segment \( BE_1 \) will do. Suppose that the consumer chooses the endowment point \( E_1 \), where \( c = 60 \) and \( c' = 100 \). This implies that \( s = 0 \), and the consumer is credit-constrained in that he or she would like to borrow, but cannot. Now with the tax change, the budget constraint shifts to \( AE_2G \), with the endowment point \( E_2 = (80, 79) \). Thus the consumer can choose \( c = c' \) on the new budget constraint, and solving for consumption in each period using the budget constraint

\[
c' + 0.95c' = 155.2,
\]

we get \( c = c' = 79.5 \), and \( s = 0.5 \). Here, notice that first-period consumption increased by almost the same amount as the tax cut, although lifetime wealth remains unchanged at 155.2. Effectively, the budget constraint for the consumer is relaxed. Therefore, for tax cuts that leave lifetime wealth unchanged, lenders will not change their current consumption, but credit-constrained borrowers will increase current consumption.
Real Intertemporal Model
Question 1, 2, 3, 8, 9, 11, 12 on page 358-359. (4th edition: no. 1, 2, 3, 8, 9, 11, 12 on page 391-392)

**Question 1 on page 358. (10 points)**

**Answer:**

There are two effects of an increase in the depreciation rate. First, there is the direct effect, which implies that, given the marginal product of capital in period two, $MP'_K$, the net marginal product of capital, $MP'_K - d$, will decrease when the depreciation rate increases. For any given real interest rate, this effect lowers investment demand, and so the investment demand schedule shifts to the left. This direct effect is the result of the fact that a higher depreciation rate implies that the scrap value of the capital the firm invests in will be lower at the end of period two.

In addition to this direct effect, there is also an indirect effect of the depreciation rate on investment. Since $K' = (1 - d)K + I$, given the initial capital stock, $K$, the quantity of capital in period two will be smaller, for any $I$, if the depreciation rate is higher.

Therefore, when $d$ increases, the investment schedule shifts to the right (Since $K'$ implies the future marginal product of capital $MP'_K$ will increase). The direct and indirect effects work in opposite directions, and so, given the real rate of interest, investment may either rise or fall with an increase in the depreciation rate.

**Question 2 on page 358. (15 points)**

**Answer:**

The problem supplies the following production function, where future output only depends on the level of second-period capital, in this case the number of trees.

<table>
<thead>
<tr>
<th>Future Trees</th>
<th>Future Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>155.0</td>
</tr>
<tr>
<td>16</td>
<td>162.0</td>
</tr>
<tr>
<td>17</td>
<td>168.0</td>
</tr>
<tr>
<td>18</td>
<td>173.0</td>
</tr>
<tr>
<td>19</td>
<td>177.0</td>
</tr>
<tr>
<td>20</td>
<td>180.0</td>
</tr>
<tr>
<td>21</td>
<td>182.0</td>
</tr>
<tr>
<td>22</td>
<td>183.8</td>
</tr>
<tr>
<td>23</td>
<td>184.8</td>
</tr>
<tr>
<td>24</td>
<td>185.2</td>
</tr>
<tr>
<td>25</td>
<td>185.4</td>
</tr>
</tbody>
</table>

(a) The production function is depicted in Figure
The marginal product of capital schedule is computed from the previous table. In table form:

<table>
<thead>
<tr>
<th>Future Trees</th>
<th>Future Output</th>
<th>$MP'_{K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>155.0</td>
<td>—</td>
</tr>
<tr>
<td>16</td>
<td>162.0</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>168.0</td>
<td>6</td>
</tr>
<tr>
<td>18</td>
<td>173.0</td>
<td>5</td>
</tr>
<tr>
<td>19</td>
<td>177.0</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>180.0</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>182.0</td>
<td>2</td>
</tr>
<tr>
<td>22</td>
<td>183.8</td>
<td>1.8</td>
</tr>
<tr>
<td>23</td>
<td>184.8</td>
<td>1.0</td>
</tr>
<tr>
<td>24</td>
<td>185.2</td>
<td>0.4</td>
</tr>
<tr>
<td>25</td>
<td>185.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

These data are plotted in Figure 9.4.
(c) Tom’s first-year profits are equal to \( \pi = Y - I \). The present value of second-year profits is equal to \( \pi' = \frac{Y' - (1 - d)K'}{(1 + r)} = \frac{Y' - (1 - d)K'}{2} \). These calculations are given in the column \( V \), below.

(d) The net marginal product of capital is equal to \( MP'_{K} - d = MP'_{K} - 0.1 \). These calculations are also included in the table below.

<table>
<thead>
<tr>
<th>Future Trees</th>
<th>Future Output</th>
<th>Required I</th>
<th>V</th>
<th>( MP'_{K} - d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>155.0</td>
<td>-3</td>
<td>267.25</td>
<td>—</td>
</tr>
<tr>
<td>16</td>
<td>162.0</td>
<td>-2</td>
<td>270.20</td>
<td>6.9</td>
</tr>
<tr>
<td>17</td>
<td>168.0</td>
<td>-1</td>
<td>279.65</td>
<td>5.9</td>
</tr>
<tr>
<td>18</td>
<td>173.0</td>
<td>0</td>
<td>274.60</td>
<td>4.9</td>
</tr>
<tr>
<td>19</td>
<td>177.0</td>
<td>1</td>
<td>276.05</td>
<td>3.9</td>
</tr>
<tr>
<td>20</td>
<td>180.0</td>
<td>2</td>
<td>277.00</td>
<td>2.9</td>
</tr>
<tr>
<td>21</td>
<td>182.0</td>
<td>3</td>
<td>277.45</td>
<td>1.9</td>
</tr>
<tr>
<td>22</td>
<td>183.8</td>
<td>4</td>
<td>277.80</td>
<td>1.7</td>
</tr>
<tr>
<td>23</td>
<td>184.8</td>
<td>5</td>
<td>277.75</td>
<td>0.9</td>
</tr>
<tr>
<td>24</td>
<td>185.2</td>
<td>6</td>
<td>277.50</td>
<td>0.3</td>
</tr>
<tr>
<td>25</td>
<td>185.4</td>
<td>7</td>
<td>276.95</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(e) Tom’s optimal level of \( V \) is equal to 277.80. To earn this amount of profit, Tom needs to plant 4 new trees. Note that at \( I = 4 \), \( MP'_{K} - d = 1.7 > r = 1.0 \). Planting the 4th tree is therefore profitable. However, at \( I = 5 \), \( MP'_{K} - d = 0.9 < r = 1.0 \). Planting the 5th tree is not profitable. The maximum \( V \) is therefore attained at the last tree for which \( MP'_{K} - d > r \).

Question 3 on page 358. (10 points)
Answer:
The costs of the output subsidy and the investment subsidy would each require an increase in other (lump-sum) taxes to satisfy the government budget constraint with unchanged government purchases. This increase in taxes reduces consumer wealth and so labor supply shifts to the right and output supply also shifts to the right. This effect tends to increase output and decrease the real interest rate.
In the case of the output subsidy, the decrease in the real interest rate increases both consumption spending and investment spending to match the increase in output. In the case of the subsidy to investment, there is also a shift to the right in the output demand curve. This effect provides an additional increase in output. Also the increase in the real interest rate (or the smaller-sized decrease in the real interest rate) reduces consumption spending so that more of the increase in output goes to investment spending and less goes to consumption spending. Therefore, the investment subsidy is likely to be more effective in increasing investment.
Question 8 on page 359. (10 points)

Answer:
Labor supply shifts to the right, so output supply also shifts to the right. Consumption demand also increases, so the output demand curve must also shift to the right. Output must increase although the real rate of interest may rise or fall. In light of the increase in output, equilibrium employment must increase. A higher level of employment, in the absence of a shift in the labor demand curve, assures us that the real wage rate must also fall. Investment rises if the real rate of interest declines, and investment falls if the real rate of interest increases. Because output has increased, consumption will rise as long as investment remains the same or declines. Consumption falls only in the case of a decline in the rate of interest of sufficient size to increase investment by more than the increase in output.

(a) To summarize: $Y \uparrow, N \uparrow, w \downarrow, r \uparrow, I \uparrow, C \uparrow$, but more likely increases.

(b) As one possibility, at low levels of nutrition, it may be infeasible for the consumer to work very much (a very high $MRS_{c,L}$). In this case, an increase in nutrition would make the consumer more willing (and able) to work more and consume more. One could also imagine some change in the technology of using leisure that is more goods intensive. In this case the value of leisure is low without a lot of consumption goods.

Question 9 on page 359. (10 points)

Answer:
A temporary increase in $z$ increases output and employment, raises the real wage, and lowers the real rate of interest. Consumption and investment both increase. An increase in future total factor productivity, $z'$, shifts the current-period output demand curve to the right. Current output and employment increase, and the real interest rate increases. Since the current-period labor demand curve does not shift, the shift in labor supply due to the lower real interest rate causes the real wage rate to decline.

A permanent increase in total factor productivity simply combines the effects of the temporary and permanent changes in $z$. Current output and employment unambiguously increase. The real wage rate may either rise or fall. The real interest rate may either rise or fall. As long as the direct effect of the increase in $MP'_K$ outweighs any indirect effect due to a possible increase in the real interest rate, then investment will increase. As long as the direct effects of the increases in current and future income dominate any indirect effect of a possible rise in the real interest rate, then consumption will also increase.

Question 11 on page 359. (10 points)

Answer:
A temporary increase in the price of energy is best modeled as a reduction in current-period total factor productivity. Such a disturbance shifts output supply to the left. Therefore, output falls and the real interest rate increases. We can show that a larger value for the marginal propensity to consume implies a flatter output demand curve. In the figure below, we show the shift in output supply with two alternative output demand curves. When the marginal propensity to consume is high, the output demand curve is flat and the reduction in $z$ results in a large reduction in output and a small increase in the real interest rate. When the marginal propensity to consume is smaller, there is a smaller
reduction in output, and a larger increase in the real interest rate. Intuition: remember the multiplier for the economy is $1/(1-MPC)$. Higher MPC implies higher multiplier. Given the same negative shock, higher multiplier means that more profound impact on GDP.

**Question 12 on page 359. (10 points)**

**Answer:**
A hurricane destroys a significant amount of capital. This disturbance may be analyzed as an exogenous decrease in the stock of capital. The production function shifts downward. Labor demand shifts to the left. These effects result in a leftward shift in the goods supply curve. The loss in capital also increases the expected marginal product of capital, and so the goods demand curve shifts to the right. The figures below depict the case in which equilibrium output decreases.
(a) The analysis of the effects of the hurricane suggests that it is reasonable to expect a decrease in national income. However, because the model is based upon maximizing principles, it is likely that the reduction in national income represents an optimal response to the reduction in the capital stock. There is therefore no presumption that policy will improve the situation.

(b) An appropriate-sized increase in government spending can restore the economy to the original level of output, $Y_1^*$. A temporary increase in government spending generates a negative wealth effect shifting the labor supply curve to the right. The temporary increase in government spending also shifts the output demand curve to the right. The figure below depicts a case in which the increase in government spending exactly returns the economy to the original level of output.
Output is unchanged. The real interest rate increases. Employment must increase. In order to produce the same amount of output an increase in employment is needed to substitute for the lost capital.

(c) A more sensible rationale for an increase in government spending would be based upon needs to replace government-provided capital. That is, the government might want to increase spending to replace roads, sewer systems, and other infrastructure.