1. To study the impact of a decrease in expected inflation, $\pi^e$, consider a simple model of the supply and demand for t-bills. The supply curve for t-bills is upward sloping in $P$ and $Q$ space because an increase in the price of t-bills lowers their yield to maturity and induces current owners of t-bills to sell their bills, increasing the quantity of bills supplied. In contrast, the demand curve for bills is downward sloping because as the current price of a bill, $P_t$, rises relative to the bill’s face value, $FV$, the yield to maturity on bills declines, $\downarrow i = \frac{FV-P_t}{FV} = \frac{FV}{FV} - 1$, leading to a decline in the quantity of bills demanded.

![Figure 1: Supply and Demand for T-bills](image)

Assume that the t-bill market begins in equilibrium at point A in Figure 2, where bond traders are expecting an inflation rate of 2% during 2010. Recall that the Fisher equation defines the ex-ante real interest rate as

$$r^e_t = i_t - \pi^e_t.$$  \hfill (1)

From equation (1), if bond traders expect inflation, $\pi^e_t$, to decrease in 2010, then given the current yield to maturity on t-bills, $i_t$, they expect a higher real return from holding t-bills, $r^e_t$. As a result, the demand for t-bills will increase, shifting the demand curve from $D_1$ to $D_2$. At the same time, traders who already own t-bills will expect to earn a higher real return and will be less inclined to sell their existing holdings. This leads to an increase in the supply of t-bills offered for sale and shifts the supply curve from $S_1$ to $S_2$. (Note that the U.S. treasury sells new t-bills to finance government...
expenditures and rarely makes financing decisions based on changes in the expected cost of financing. Also, you can answer this question assuming no change in supply.)

The decreased supply, and increased demand for t-bills creates an excess demand $Q'' - Q'$. Excess demand drives the price of t-bills up to the new equilibrium price, $P_2$.

With an increase in the price of bills, the yield to maturity on t-bills falls (see equation 1). As shown in Figure 2, at the new equilibrium (point B), the quantity of t-bills bought and sold remains unchanged at $Q_1$ implying that the increase in the price of bills has caused an decrease in the nominal rate sufficiently large as to completely offset the decrease in expected inflation. In other words, the real rate remains unchanged. (Note that this result is not guaranteed but is a reasonable assumption. Whether or not the real rate changes depends on how far the curves shift.)

2. The pure expectations theory of the term structure begins with the assumption that government bonds with differing maturities are near substitutes. Therefore, the yield to maturity on a three year note is approximately equal to the expected yield on a portfolio that invests in three successive one year bills,

$$i_{3,t} = \frac{i_{1,t} + i_{e}^{t+1} + i_{e}^{t+2}}{3}. \quad (2)$$

In equation (2), $i_{3,t}$ and $i_{1,t}$ are the yield to maturity on a three-year note and a one-year bill today (time $t$). The expected yields on one-year bills in period period $t + k$ (for $k = 1, 2$) are written as $i_{e}^{t+k}$. The right hand side of equation (2) is approximately the expected return from investing in three successive one-year t-bills.

The intuition behind the equality in equation (2) is quite simple. Suppose that the yield on three-year bonds was higher than the expected return from holding the one-year bills so that the equality in (2) does not hold. Investors would rush to buy the three-year bond (some of them selling one year bills to do so). The increased relative demand for the longer term security would push its price up in the spot market and the lack of demand for the one year-bill would push its spot price down. As a result, the three-year yield would decrease and the return on the portfolio of one-year bills would increase until the expected returns are the same.

3. Following our answer to question 1. above, if bond traders expect inflation to continue to decline in 2011 and 2012, our model predicts that the nominal interest rate on t-bills will continue to decline in 2011 and 2012. If as is argued above, the real rate of interest remains unchanged at say 0.5% from 2010 through 2012, then as the expected rate of inflation declines to 1.5% in 2010, 1.25% in 2011, and 1.0% in 2012, the nominal interest rate would decline by the same amount from 2% in 2010, to 1.75% in 2011 and 1.5% 2012.

Assume that bond traders expect the the nominal yields to follow this pattern, then using an equation like (2), we can determine that the two year note today should have a yield of $i_{2,t} = (2 + 1.75)/2 = 1.875\%$, and the three year note should have a yield of $i_{3,t} = (2 + 1.75 + 1.5)/3 = 1.75\%$. In other words, the expectation of falling one year t-bill rates implies that today’s one-year interest rate is above the interest rates
on both two-year and three-year notes. At least on the short end of the yield curve, the curve is downward sloping.

4. In my answer to questions 1 and 3, I argued that the real rate of interest remained unchanged, i.e. $\pi^e \rightarrow$ the decline in the nominal interest rate was enough to leave real rates constant. This is not the only possibility, and may not even be the most likely outcome. Historically nominal interest rates have not moved one-for-one with changes in expected inflation. However, the assumption is plausible and greatly simplifies my answer to a question about the impact of changes in expected inflation and nominal interest rates on the value of the dollar. To study short run currency movements we make use of the interest parity condition. This is an equilibrium condition between expected returns on domestic and foreign deposits. Specifically, the interest rate on domestic deposit should be equal to the interest rate on foreign deposits minus the expected appreciation of the domestic currency,

$$i^D = i^F - \frac{E_{t+1}^e - E_t}{E_t},$$

where $i^D$ is the domestic interest rate, $i^F$ is the interest rate on foreign deposits, $E_t$ is the exchange rate (units of foreign currency per unit of domestic currency) at time $t$ (today), and $E_{t+1}^e$ is the expected value of the exchange one period in the future.

The key assumption for the interest parity model is that foreign bank deposits and domestic bank deposits are perfect substitutes. When capital is mobile and bank deposits are perfect substitutes, it must be true that there is no difference in the expected returns on domestic and foreign deposits.

In the questions above, we found that the one-year t-bill rate declined because of a decline in expected inflation, but we assumed that the real interest rate, and interest rates and inflation in the Euro remain unchanged. The decline in U.S. nominal interest rates cause the return on dollar deposits to decrease. As a result, the demand for dollar assets will decline and the demand curve shifts to the left. If the decline in nominal interest rates occurred with no change in expected inflation, then the decline in demand for the dollar would lead to an excess supply ($Q_s - Q_{D2}$) of dollar deposits, and a decline in the price of dollar deposits, i.e. a depreciation of the dollar from $E_1$ to $E_2$. As the spot exchange rate declines, for a given future expected exchange rate, $E_{t+1}^e$, the expected appreciation of the dollar ($\frac{E_{t+1}^e - E_t}{E_t}$) increases, and the expected foreign return decreases. This decline in the expected foreign return is a movement down the demand schedule from point B towards the new equilibrium where $Q_s = Q_d$ again. The end result is that lower domestic interest rates lead to a depreciation of the domestic currency.

But, because there is also a drop in expected inflation, there is an offsetting effect on the RHS of the interest parity condition. Assume that initially (at the equilibrium at point A, $E_{t+1}^e = E_t$ so that $i^D_t = i^F_t$. Then a decline in expected inflation $\pi^e \downarrow$, which, by PPP, causes an increase in the expected future exchange rate, $E_{t+1}^e \uparrow$, and results in a decline in the expected return on foreign deposits. That is, $i^F_t - \left(\frac{E_{t+1}^e}{E_t}\right) - 1$ declines. Because we are assuming no change in the domestic real interest rate, the decline in
Figure 2: Exchange Rate Determination Using Interest Parity Model

The expected foreign return must be enough to shift the demand for dollar deposits all the way back to $D_1$. This way, the spot exchange rate returns to its original level as is consistent with an unchanged real interest rate in both the U.S. and the Euro area.