Solution: Discussion #5, Male vs. Female Laysan Finch Weight and Beak Width

Summary

The analysis using only lower beak width and sex as explanatory variables is fairly straightforward. Although scatterplots suggest that the slopes of the regressions of weight on beak width differ between the sexes, this difference is not statistically significant. In a parallel-regressions model the difference in intercepts is statistically significant ($P = 0.004$). For a given beak width, males are about 2.5 g heavier on average than females. No serious problems are evident in residual plots.

Data Exploration

Males tend to be heavier than females, and to have wider beaks. All of the distributions are somewhat skewed, but not severely, and there are no outliers.

Weight and beak width are positively associated within each sex, and these relationships are roughly straight. The relationship appears to be slightly steeper in females than males, and in the range of beak widths at which the sexes overlap, it appears that males are heavier for a given beak width.

Analysis

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>lowerwid</td>
<td>1</td>
<td>488.16</td>
<td>93.57</td>
<td>93.57</td>
<td>12.38</td>
<td>0.001</td>
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<tr>
<td>sex</td>
<td>1</td>
<td>67.15</td>
<td>21.04</td>
<td>21.04</td>
<td>2.78</td>
<td>0.098</td>
</tr>
<tr>
<td>sex*lowerwid</td>
<td>1</td>
<td>17.69</td>
<td>17.69</td>
<td>17.69</td>
<td>2.34</td>
<td>0.129</td>
</tr>
<tr>
<td>Error</td>
<td>94</td>
<td>710.20</td>
<td>710.20</td>
<td>7.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>97</td>
<td>1283.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S = 2.74870$  $R$-Sq $= 44.65\%$
Test for coincidence:

The sex and sex x beak width terms can be tested together using the sequential sums of squares in the output:

\[ SS(\text{sex, sex*beakwidth} \mid \text{beakwidth}) = 67.15 + 17.69 = 84.84 \]
\[ \text{df} = 2 \]
\[ \text{MS} = 42.42 \]
\[ F^* = 42.42 / 7.56 = 5.61 \]
\[ P = 0.005 \]

We therefore reject the null hypothesis that the regressions for the two sexes are coincident.

Test for parallelism:

The output gives this test directly, with \( P = 0.129 \). At the usual levels of significance (\( \alpha \)'s), this test is not significant so we would accept the null hypothesis of equal slopes. Note, though, that the \( F \) statistic is considerably larger than 1, so we should not conclude that the slopes definitely are the same, just that the data do not provide convincing evidence the slopes differ.

Test for equal intercepts, given equal slopes:

If we follow the text’s approach and first test for coincidence, which is rejected, and then for parallelism, which is not rejected, we could go on to conclude that the intercepts differ. If we instead first test for parallelism, which was not rejected, we would then test for equal intercepts; of course this could also be done if following the text’s approach.

It would be best to use the MSE from the full model above to test for equal intercepts, rather than to combine the interaction term into a new error term for a reduced model. The reason for this is that the interaction MS is somewhat larger than the MSE, so that combining them inflates the “error” term (and suggesting the interaction MS may be estimating something more than \( \sigma^2 \)). This test can be done using the sequential SS above:

\[ SS(\text{sex} \mid \text{beak width}) = 67.15 \]
\[ \text{df} = 1 \]
\[ \text{MS} = 67.15 \]
\[ F = 67.15 / 7.56 = 8.88 \]
\[ P = 0.0003 \]

We therefore conclude that while the slopes are not significantly different, the intercepts are.
**Estimation:**

The coefficients for the parallel-but-not-coincident model (which must be run directly) are

<table>
<thead>
<tr>
<th>Term</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.695</td>
<td>7.687</td>
<td>1.00</td>
<td>0.319</td>
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<tr>
<td>lowerwid</td>
<td>30.164</td>
<td>9.504</td>
<td>3.17</td>
<td>0.002</td>
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<tr>
<td>female</td>
<td>-1.2408</td>
<td>0.4191</td>
<td>-2.96</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Since Minitab codes the indicator variable representing sex as female = 1, male = -1, the estimated models for the two sexes are

- **females:** weight = (7.695 - 1.2408) + 30.164 x beak width
  = 6.4542 + 30.164 x beak width
- **males:** weight = (7.695 + 1.2408) + 30.164 x beak width
  = 8.9358 + 30.164 x beak width

**Diagnostics**

The plot (below) of residuals vs. fits for this parallel-regressions model shows the slightly different slopes for the two sexes (producing slight trends in the two clouds of points for the two sexes). Otherwise there is no indication of nonlinearity. It somewhat looks like the spread is greater for males, but in fact the standard deviations of the residuals are very similar for the two sexes (2.668 for females, 2.818 for males). The distribution of residuals is fairly symmetric, with a few large values; with this sample size this slightly nonnormality is not a problem.

**Conclusion**

Weight increases about 30 g per 1 cm increase in width of the lower beak, for males and females alike. For a given beak width, males weigh about 2.5 g more than do females.