Data Set 6, Part II — Appendix: Computing Methods

Minitab

Either the "Balanced ANOVA" or "General Linear Model" procedures can be used for random or mixed-model ANOVA. Their use is virtually identical except that only the former allows use of the "restricted" form of the model, and only the latter can be used with unbalanced data.

Balanced ANOVA

Procedure

Stat ➜ ANOVA ➜ Balanced ANOVA…

Then specify the Responses:, the Model:, and which are the Random factors: (If a main effect is specified as random, Minitab also treats all interactions involving the factor as random.)

To use the "restricted" form of the model, click on the Options… button and check the box in front of Use the restricted form of the model, then click on the OK button.

To have the expected means squares included in the output, click on the Results… button and check the box in front of Display expected mean squares and variance components, then click on the OK button.

Run the analysis by clicking on the OK button.

Example Output.

Analysis of Variance (Balanced Designs)

```
Factor     Type Levels Values
species  random      2     A     B
size      fixed      3  Large Medium  Small


Analysis of Variance for mnsize

Source          DF         SS         MS       F      P
species          1   0.010299   0.010299    6.99  0.012
size             2   0.014881   0.007440    4.52  0.181
species*size     2   0.003292   0.001646    1.12  0.338
Error           36   0.053064   0.001474
Total           41   0.081536


Source         Variance Error Expected Mean Square for Each Term
               component term (using restricted model)
 1 species       0.00042   4   (4) + 21(1)
 2 size                    3   (4) + 7(3) + 14Q[2]
 3 species*size  0.00002   4   (4) + 7(3)
 4 Error         0.00147       (4)
```

General Linear Model

Procedure

Stat ➜ ANOVA ➜ General Linear Model…

Then specify the Responses:, the Model:, and which are the Random factors: (If a main effect is specified as being a random factor, Minitab also treats all interactions involving the factor as random.) To have the expected means squares included in the output, click on the Results… button and check the box in front of Display expected mean squares and variance components, then click on the OK button.

Example Output.

General Linear Model

```
Factor     Type Levels Values
species  random      2 1 2
size      fixed      3 1 2 3

Analysis of Variance for mean siz, using Adjusted SS for Tests

Source         DF     Seq SS     Adj SS     Adj MS       F      P
species         1  0.0038883  0.0090051  0.0090051    9.20  0.093 x
size            2  0.0104340  0.0119932  0.0059966    6.13  0.140
species*size    2  0.0019576  0.0019576  0.0009788    1.11  0.343
Error          28  0.0246293  0.0246293  0.0008796
Total          33  0.0409092

x Not an exact F-test.

Unusual Observations for mean siz

Obs  mean siz       Fit   StDev Fit  Residual   St Resid
 16  0.500000  0.561378    0.009886 -0.061378     -2.20R

R denotes an observation with a large standardized residual.

Expected Mean Squares, using Adjusted SS

Source           Expected Mean Square for Each Term
 1 species       (4) +  4.7265(3) + 14.1794(1)
 2 size          (4) +  4.7481(3) + Q[2]
 3 species*size  (4) +  4.7481(3)
 4 Error         (4)
Error Terms for Tests, using Adjusted SS

Source           Error DF  Error MS  Synthesis of Error MS
 1 species           2.02 0.0009784    0.9955(3) +   0.0045(4)
 2 size              2.00 0.0009788  (3)
 3 species*size     28.00 0.0008796  (4)

Variance Components, using Adjusted SS

Source        Estimated Value
species               0.00057
species*size          0.00002
Error                 0.00088
```

SAS — Analyst

The "Factorial ANOVA," "Linear Models" and "Mixed Models" procedures can be used for random or mixed-model ANOVAs. The first two are quite similar, and accomodate random effects only by allowing the user to specify the denominator term to use in the *F* test for any particular term in the model; "Mixed Models" is much more flexible (and therefore more complex to use). "Factorial ANOVA" can be used only with balanced data sets; "Linear Models" and "Mixed Models" can be used with balanced or unbalanced data sets.

Factorial ANOVA

Procedure

Statistics ➜ ANOVA ➜ Factorial ANOVA…

Select the Dependent and Independent variables in the appropriate boxes.

Click on the Model button, then specify the terms to be in the model. For example, click on the Standard Models button and select the level of interactions desired, *e.g.* Effects up to 2-way interactions. Click the OK button.

To specify the proper *F* tests, click on the Tests button. Select the appropriate denominator term from the list of Effects, then click on the Error button. Then select from the list of Effects the term(s) to be tested over this denominator, and click on the Add button; the specified tests will be listed in the H=Effect E=Error term: window. Click the OK button.

After specifying any other desired options (*e.g.* graphs), click the OK button.

Example Output

```
                          The GLM Procedure

                       Class Level Information

                Class         Levels    Values
                species            2    A B
                size               3    Large Mediu Small

                                Number of observations    42

 Dependent Variable: mnsize

                           Sum of
Source           DF       Squares     Mean Square    F Value    Pr > F

Model             5    0.02847217      0.00569443       3.86    0.0066
Error            36    0.05306430      0.00147401
Corrected Total  41    0.08153647

             R-Square     Coeff Var      Root MSE    mnsize Mean
             0.349196      6.891578      0.038393       0.557098
```

output continued on next page

Source	DF	Type III SS	Mean Square	F Value	Pr > F
size	2	0.01488052	0.00744026	5.05	0.0117
species	1	0.01029927	0.01029927	6.99	0.0121
species*size	2	0.00329238	0.00164619	1.12	0.3384

Comment: The preceding *F* tests do not take into account that this is a mixed-model analysis; they are exactly the same as would be gotten for a fixed-effects ANOVA. The only part of the output specific to the mixed-model (or a random effects analysis) is the following, for the test specifically defined using the Tests button.

Tests of Hypotheses Using the Type III MS for species*size as an Error Term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
size	2	0.01488052	0.00744026	4.52	0.1812

Linear Models

Procedure

Statistics ➜ ANOVA ➜ Linear Models…

Select the Dependent variable in the appropriate box. Select the independent variables in the Class box.

Click on the Model button, then specify the terms to be in the model. For example, click on the Standard Models button and select the level of interactions desired, *e.g.* Effects up to 2-way interactions. Click the OK button.

To specify the proper *F* tests, click on the Tests button. Select the appropriate denominator term from the list of Effects, then click on the Error button. Then select from the list of Effects the term(s) to be tested over this denominator, and click on the Add button; the specified tests will be listed in the H=Effect E=Error term: window. Click the OK button.

After specifying any other desired options (*e.g.* graphs), click the OK button.

Output

Output from this procedure will be exactly like that shown above for the “Factorial ANOVA” procedure.

Mixed Models

The "Mixed Models" procedure in SAS is based on a considerably more general model than the standard ANOVA model. Two of the more important differences are that a wide variety of correlation structures among observations can be modeled (rather than assuming observations to be independent of one another), and that the model is fit and inference is done using maximum-likelihood methods rather than least squares. Many of the complexities of this procedure are beyond what we will cover in this course and will not be described here; I only show the default mixed-model analysis (using Analyst), and how the procedure can be used to give results comparable to those from classical ANOVA.

Procedure

Statistics ➜ ANOVA ➜ Mixed Models…

Select the Dependent variable in the appropriate box. Select the independent variables in the Class box.

Click on the Model button.

With the Fixed effects box active (there will be a check in the box at the front of the box caption), select the desired term(s) from the list in the Class box, then click on the Add, Factorial, Cross or other buttons to specify the fixed-effects part of the desired model.

Then activate the Random effects box (click on the check box at the front of its caption), select the desired term(s), and use the Add, Factorial, Cross or other buttons to specify the random-effects part of the desired model. Note that SAS does not assume higher-order terms are to be treated as random if any terms contained in them are random; you must specifically list such higher-order terms in the Random effects box.

Click on the OK button to close the Model window.

If you want hypothesis tests for the random effects, click on the Tests button and click on the check box in front of Tests of variance components, then click on the OK button to close the Tests window.

If you want to use a method of estimation other than the default (REML = "restricted/residual maximum likelihood"), or if you want to get estimates of the parameters, click on the Options button. The estimation method is selected by clicking on one of the six methods listed; "Type 3" will give the equivalent of classical least-squares ANOVA (using Type III SSes). Estimates of parameters for the fixed effects and/or random effects can be gotten by clicking on the appropriate check box(es) under Solutions. Click on the OK button to close the Options window.

Click on the OK button to run the analysis.

Example Output — Default Method ("REML")

The Mixed Procedure

Model Information

Data Set	ADVBIOM.BRACANOU
Dependent Variable	mnsize
Covariance Structure	Variance Components
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Satterthwaite

Class Level Information

Class	Levels	Values
species	2	A B
size	3	Large Mediu Small

Dimensions

Covariance Parameters	3
Columns in X	4
Columns in Z	8
Subjects	1
Max Obs Per Subject	34
Observations Used	34
Observations Not Used	0
Total Observations	34

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	-114.89621772	
1	2	-120.35403904	0.00000091
2	1	-120.35412041	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Estimate	Alpha	Lower	Upper
species	0.000541	0.05	0.000097	3.2246
species*size	0.000014	0.05	.	.
Residual	0.000882	0.05	0.000555	0.001615

Fit Statistics

-2 Res Log Likelihood	-120.4
AIC (smaller is better)	-114.4
AICC (smaller is better)	-113.5
BIC (smaller is better)	-118.3

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
size	2	1	5.23	0.2953

Example Output — Type 3 Estimation

Tests of variance components, and parameter solutions, were requested for this analysis (as could also have been done in the REML analysis above).

```
                          The Mixed Procedure

                          Model Information

            Data Set                      ADVBIOM.BRACANOU
            Dependent Variable            mnsize
            Covariance Structure          Variance Components
            Estimation Method             Type 3
            Residual Variance Method      Factor
            Fixed Effects SE Method       Model-Based
            Degrees of Freedom Method     Satterthwaite

                        Class Level Information

              Class      Levels    Values

              species         2    A B
              size            3    Large Mediu Small

                          Dimensions

                    Covariance Parameters             3
                    Columns in X                      4
                    Columns in Z                      8
                    Subjects                          1
                    Max Obs Per Subject              34
                    Observations Used                34
                    Observations Not Used             0
                    Total Observations               34

                      Type 3 Analysis of Variance

                     Sum of
Source        DF     Squares  Mean Square  Expected Mean Square

size           2   0.011993   0.005997  Var(Residual) +
                                          4.7481 Var(species*size) +
                                          Q(size)
species        1   0.009005   0.009005  Var(Residual) +
                                          4.7265 Var(species*size) +
                                          14.179 Var(species)
species*size   2   0.001958   0.000979  Var(Residual) +
                                          4.7481 Var(species*size)
Residual      28   0.024629   0.000880  Var(Residual)

                      Type 3 Analysis of Variance

                                         Error
Source        Error Term                 DF    F Value    Pr > F

size          MS(species*size)           2        6.13    0.1403
species       0.9955 MS(species*size)    2.0164   9.20    0.0927
               + 0.0045 MS(Residual)
species*size  MS(Residual)               28       1.11    0.3428
Residual      .                          .       .
```

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Covariance Parameter Estimates

Cov Parm	Estimate	Standard Error	Z Value	Pr Z	Alpha	Lower	Upper
species	0.000566	0.000937	0.60	0.5458	0.05	-0.00127	0.002403
species*size	0.000021	0.000220	0.09	0.9245	0.05	-0.00041	0.000453
Residual	0.000880	0.000236	3.73	<.0001	0.05	0.000554	0.001609

Fit Statistics

-2 Res Log Likelihood	-120.4
AIC (smaller is better)	-114.4
AICC (smaller is better)	-113.5
BIC (smaller is better)	-118.3

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
size	2	1	5.10	0.2987