Data Set 2:   Medfly Mating Success

Statistical setting

This is an example of an observational study involving **two variables of primary interest** and several **supplementary variables**; these supplementary variables are ones that might affect the response variable and might also be correlated with the explanatory variable. The intent of the statistical analysis therefore is to produce the best possible description of the relationship between the two interesting variables, taking into account possible interactions with or confounding from the other variables. The analysis involves partial- and multiple-partial-*F* tests for interactions and polynomials, and assessment of confounding.

Background and Data

Tim Whittier (ex-Entomology grad student) was trying to understand why some males of the "medfly" (*Ceratitis capitata*) have greater mating success than do others. One possible explanation is that males somehow affect female fecundity (the number of viable eggs a female can lay), perhaps by providing more or better sperm, and that females can identify and prefer males which are better in this way. To test this explanation, Tim studied the relationship between the mating success of males and the fecundity of their mates, in a laboratory study.

Methods

Tim provided several groups of 16 individually marked males with 5 new females each day for 10 days. All insects were unmated prior to use in the study; males were 7 days old and females were 10-11 days old. The mating exposure took place in 1 cu. ft. cages.

The number of times each male mated was observed; at most one mating could be obtained per day. The first female to mate with each male was provided fresh oviposition sites (peaches) every five days until she died; the peaches were kept until all adult offspring had emerged from them, and these were counted.

Data

The two variables of direct interest were:
- number of matings: the total number of times a given male mated, and
- progeny**:** the total number of adult progeny produced by the first female with which the given male mated.

Supplementary variables which might affect progeny number also were recorded:
- female longevity: how long the female lived (in days, from time of mating),
- female size: the size of the female (dry weight in mg), and
- copulation duration: the duration of copulation (in minutes).

There were 86 observations for males which obtained at least one mating; males which were entirely unsuccessful could not be used because there were no female variables to associate with them.

Data Exploration

Descriptive Statistics

Since this study is about the relationships among several quantitative variables, the most pertinent summary statistics are correlations. The two variables of primary interest are weakly correlated, but <u>negatively</u> rather than positively as was hypothesized.

	progeny
number of matings	**-0.163**

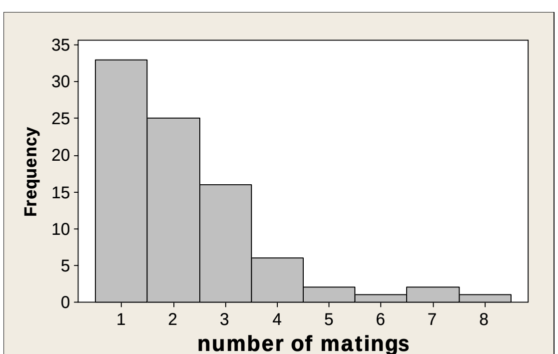
The two primary variables also are have moderate correlations with a couple of the supplementary variables: not surprisingly, progeny number is positively correlated with female longevity, and interestingly, number of matings was somewhat negatively correlated with female size (males whose first mate was large tended to have fewer subsequent matings). In addition there was a weak negative correlation between female longevity and copulation duration; while interesting, this doesn't seem relevant to the question at hand. Contrary to expectation, there is little correlation between female size and progeny number.

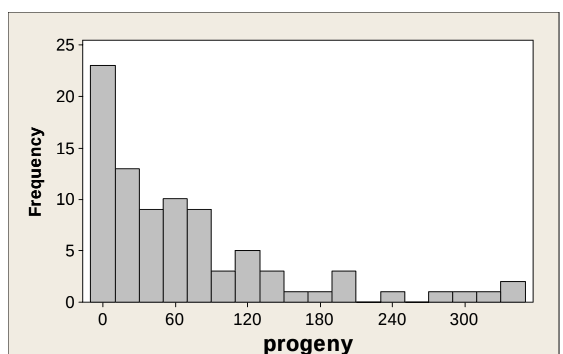
	number of matings	progeny	longevity	size
female longevity	0.095	**0.339**		
female size	**-0.161**	0.057	0.086	
copulation duration	-0.017	0.067	<u>-0.132</u>	-0.037

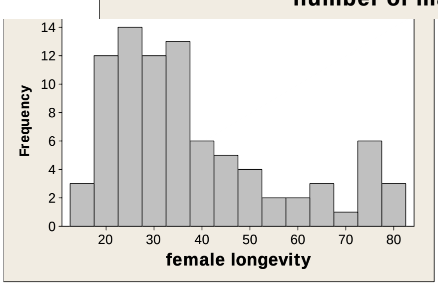
Graphical

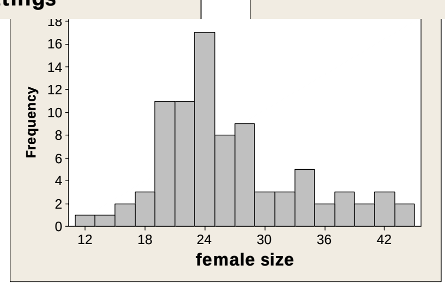
<u>Distributions</u>

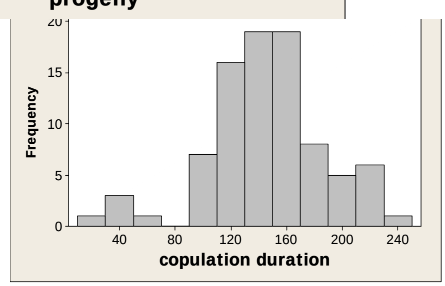
Most of the variables, including the two of primary interest, have skewed distributions with a few unusually large observations. These features would be somewhat reduced by log or square-root transformations, but the basic results of the subsequent analyses were not affected by such transformations.





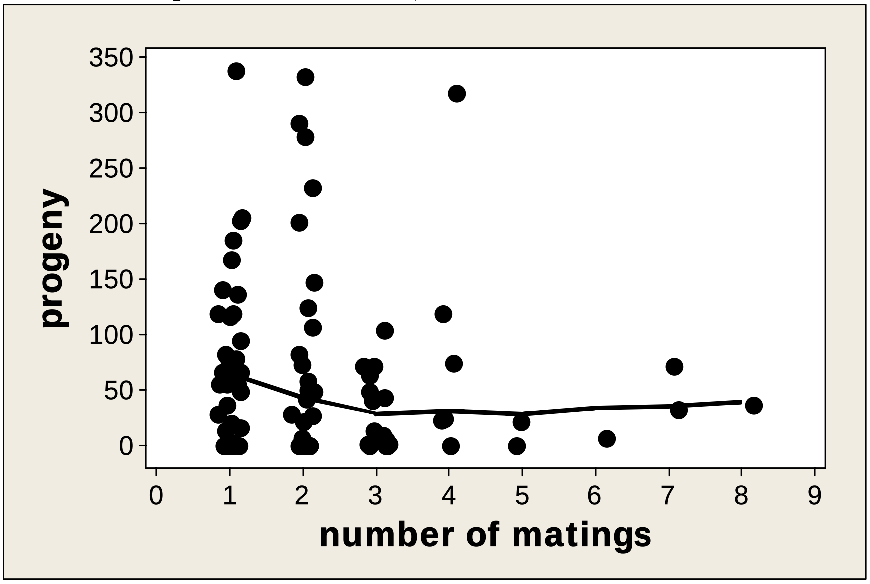




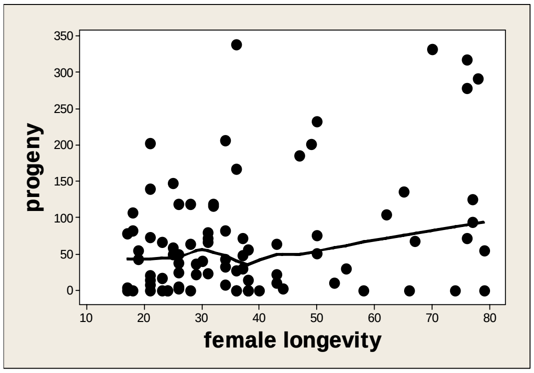


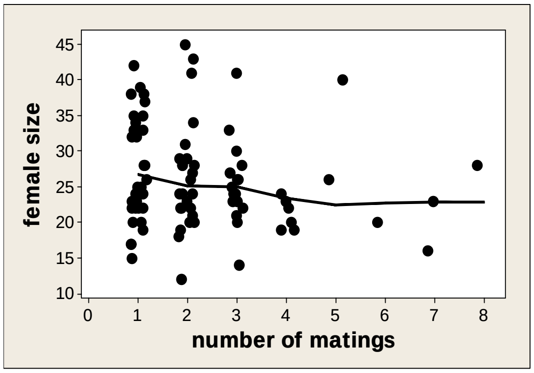
Relationships

Examination of a scatterplot indicates that the primary variables, progeny and number of matings are weakly associated, but negatively, not positively as expected. The negative association is primarily in the range from 1 to 3 matings; the relationship is flat or perhaps even slightly positive at higher numbers of matings. (In the scatterplot horizontal "jitter" has been added so that the individual points can be seen.).



The two correlations between primary and supplementary variables mentioned above — between progeny and female longevity and between number of matings and female size — can also be seen in scatterplots





Analysis

Overall Approach

The analysis presented here uses regression, with progeny singled out as the response variable and number of matings as the primary explanatory variable. There in fact cannot be a direct causal relationship between these two variables; presumably any observed relationship would be due to some underlying factor of male quality affecting both variables. It therefore would be equally reasonable to use multiple correlation analysis, or indeed to use number of matings as the response variable in the regression. I have done the analysis the way I did, however, because the mechanics of regression (especially diagnostics) are easier than for multiple correlation, and because the available supplementary variables will affect progeny rather than number of matings.

The analysis will be presented in stages: tests for interactions, assessment of confounding, tests for the number of matings effect and possible curvilinearity, and finally detailed diagnostics.

Analysis of interactions

The primary independent variable, number of matings, was multiplied by each of the supplementary independent variables to allow tests for two-factor interactions. Higher order interactions and interactions only involving the supplementary variables were not considered, to keep the total number of variables considered at a manageable level. Interactions were tested in two ways: a multiple-partial test of all three interactions together, and by a sequence of deletions of single interactions. In the latter process the interaction deleted in each step was the one with the smallest added-last SS at that stage (terms were deleted from the bottom up in the following table). The base model included first-order terms for all the independent variables, and the *MSE* for the fullest model (i.e. including all first-order terms and all three interactions) was used as the denominator for the *F** statistics.

Source	DF	Added-in-order SS	MS	F*	P
number of matings, female longevity, copulation duration, female size	4	95887			
number of matings x copulation duration	1	8508		1.42	0.24
number of matings x female longevity	1	7137		1.20	0.28
number of matings x female size	1	2159		0.34	0.56
Model	7	113691			
Error	78	465398	5967		
all three interactions	3	17804	5935	0.99	0.40

This analysis was repeated with a quadratic term for number of matings in the model also, and the results were essentially the same as presented above.

From these tests I concluded that there is no evidence of two-way interactions between number of matings and any of the other variables, and proceeded to examine the effects of the variables in models without interaction terms.

Analysis of confounding

Confounding – the effects of other variables on the modeled relationship between progeny and number of matings – was investigated not by formal statistical tests, but rather by comparing a sequence of models in terms of the slope estimate for number of matings and its standard error, as well as the standard error of predicted mean progeny. The sequence of models was determined again in a backwards elimination process, starting with a model containing number of matings and all three possible confounders, and removing at each step the confounder with the smallest added-last SS at that stage. The standard error of predicted mean progeny was calculated for number of matings equal to 2 and all other variables equal to their mean values.

variables in model	$b_{\text{number of matings}}$	**std error of** $b_{\text{number of matings}}$	**std error of predicted mean progeny**
number of matings, female longevity, copulation duration, female size	-10.916	5.745	8.45
number of matings, female longevity, copulation duration	-10.890	5.626	8.39
number of matings, female longevity	-10.921	5.634	8.40
number of matings	-9.037	5.977	8.95

The estimated slope for number of matings is substantially different — about 20% steeper — when at least one of the other variables is in the model than when none are: there is some confounding occurring. To analyze the "unique" relationship of number of matings and progeny — *i.e*. the part of the relationship that could not be explained by effects of other correlated factors I therefore needed to include at least some of these confounding variables in my models. Both the slope estimate and the predicted response also are more precise (have smaller standard errors) when one or more of the confounding variables is in the model.

Among the models with one or more of the other variables there is very little difference: the slope estimates differ by at most 2.5%, the slope standard errors by slightly less, and the estimated response standard errors by less than 1%. Based on precision of the estimates, the best of these models includes both female longevity and copulation duration as confounders, while parsimony would suggest using only female longevity. I will use both these models in the following analyses of the effect of number of matings.

This exploration was repeated with a quadratic term for number of matings in the model, and the results were essentially the same as shown here.

Analysis of possible curvilinear relationship

Because the scatterplot (above) suggested that the relationship between number of matings and progeny was not straight — that it was negative at small numbers of matings but flat or even positive at higher numbers of matings — I investigated whether a quadratic model for the "effect" of number of matings would be preferable to a linear model. This was done both by testing the significance of a quadratic term, and by lack-of-fit tests.

<u>Tests of quadratic term</u>

To do the quadratic regression number of matings first was "centered" by subtracting 2.0 from each observation (this is the median and modal value; the mean actually is 2.244); these centered values (called mate_c in the output below) were squared to give the quadratic term (mate_sqd). The quadratic regression was run twice: with both female longevity and copulation duration in the model, and with only female longevity. The quadratic term was tested first, and after it was found to be not significant the linear term was tested. (These two tests were done in one run by using added-in-order partial-*F* tests with the linear term added next-to-last and the quadratic term added last.)

With female longevity and copulation duration as confounders:

Source	**DF**	**Added-in-order SS**	**Mean Square**	**F***	**P**
female longevity	1	66445.5		11.37	0.0011
copulation duration	1	7358.9		1.26	0.2651
mate_c	1	22078.3		3.78	0.0554
mate_sqd	1	9773.8		1.67	0.1996
Model	4	105656.5	26414.1	4.52	0.0024
Error	81	473432.4	5844.8		

With only female longevity as a confounder:

Source	**DF**	**Added-in-order SS**	**Mean Square**	**F***	**P**
female longevity	1	66445.5		11.31	0.0012
mate_c	1	22204.2		3.78	0.0554
mate_sqd	1	8488.4		1.44	0.2329
Model	3	97138.1	32379.4	5.51	0.0017
Error	82	481950.8	5877.4		

In either model the quadratic term is not significant at any sensible level of significance: despite the nonlinear appearance of the smoother in the scatterplot, a curved model does not fit the data substantially better than does a linear relationship.

Lack-of-fit tests

Since there were some observations with the same values of number of matings and female longevity, it was possible to do lack-of-fit tests for the linear and quadratic models with only female longevity as a confounder. (A similar tests was barely possible if copulation duration also was included in the model, but it had only 3 degrees of freedom for the "pure error" so I didn't consider it reliable.)

Lack-of-fit test for quadratic model:

Source	DF	Added-in-order SS	Mean Square	F*	P
regression	3	97138			
lack-of-fit	61	363122	5953	1.05	0.47
pure error	21	118828	5658		

Lack-of-fit test for linear model:

Source	DF	Added-in-order SS	Mean Square	F*	P
regression	2	88650			
lack-of-fit	62	371611	5994	1.06	0.46
pure error	21	118828	5658		

These tests indicate there is no important nonlinearity: the clearly non-significant lack-of-fit test with the quadratic term in the model suggests there are no higher-order nonlinearities, and the very similar result without the quadratic term reinforces the result from the significance test above that the quadratic term itself is not significant.

Inference about the **number of matings** *effect*

The results needed to test the hypothesized relationship between a male's number of matings and the number of progeny produced by his mate are contained in the tables presented above in the investigations of confounding and possible curvilinearity:

• the test of the null hypothesis of no relationship between the number of matings and progeny is of marginal statistical significance ($P = 0.055$)

• the estimated value of slope of this relationship is −10.9 (s.e. 5.63).

The data thus are equivocal as to whether there is a relationship between the two variables of interest, but clearly if there is any relationship, it is negative, not positive. The data, therefore, clearly do **refute the research hypothesis** that males who are successful at getting matings are so because females can somehow detect that they will have more progeny by mating with those males.

Model Diagnostics

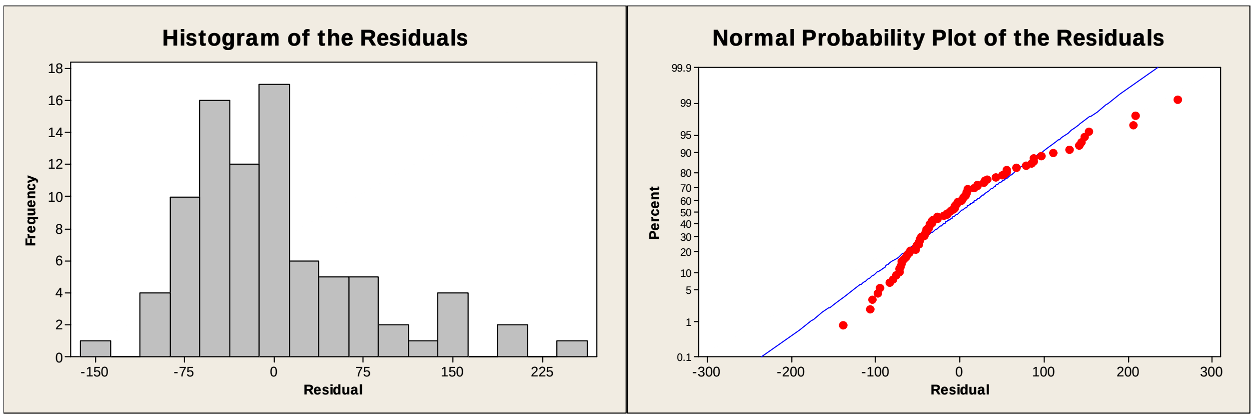
The diagnostics presented here are for the model with mating success (number of matings) and female longevity as independent variables, without any higher order terms.

The diagnostic measures used are the usual basic residual plots, followed by: partial regression plots; measures of leverage, outliers, and influence; assessment of influence by omitting individual observations; and both VIF and eigenstructure collinearity diagnostics.

Residual plots

<u>Distribution of residuals</u>

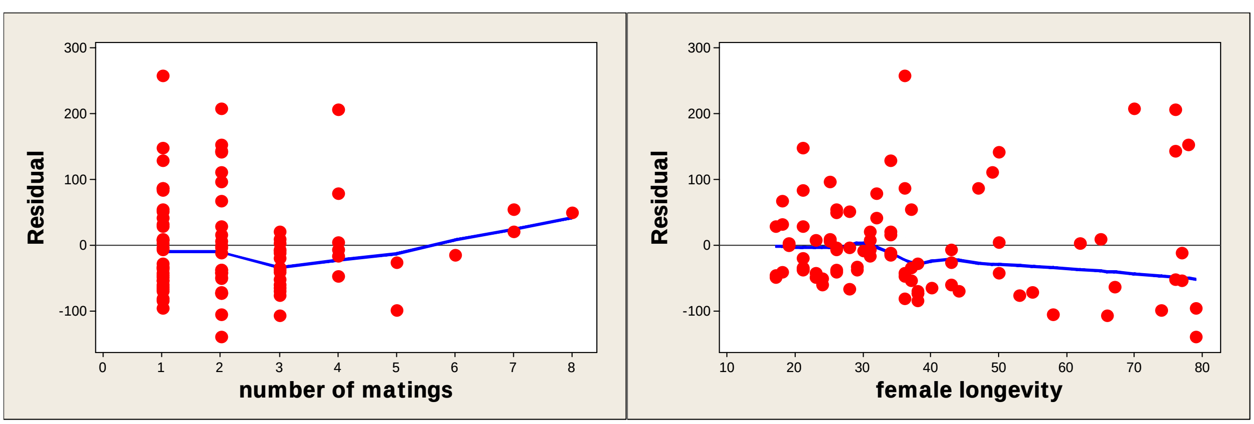
The residuals have a distinctly skewed distribution, with a long right tail.



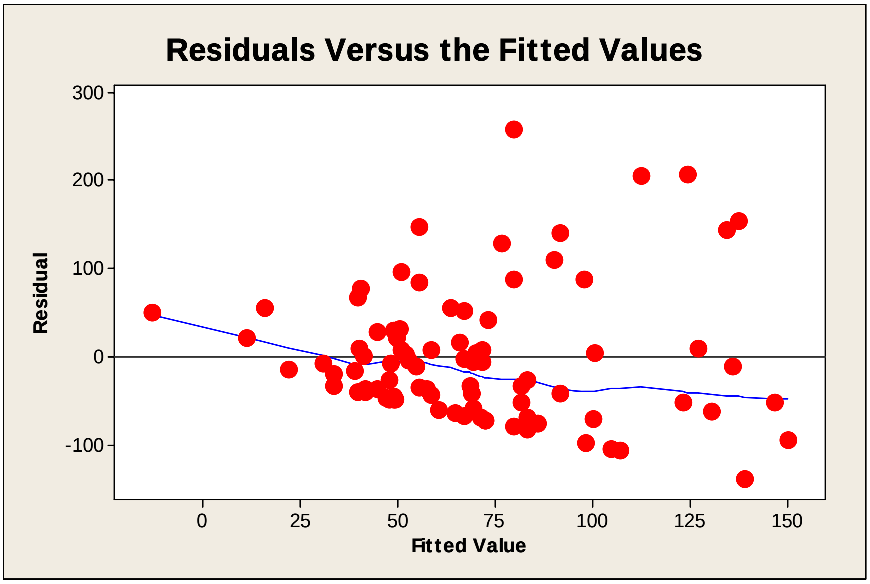
I believe, though, that with n = 86, we can invoke the Central Limit Theorem and have confidence in the results of the analysis despite this non-normality.

<u>Residuals *vs* fits and explanatory variables</u>

The plots of the residuals against the two explanatory variables do not show any major problems. The plot with number of matings shows the slight nonlinearity already seen in the initial scatterplot (and later determined to not be statistically significant). The smoother in the plot with female longevity shows a downward trend, which actually is the result of the distribution becoming broader and more skewed at high levels of longevity (so that while the mean is close to 0, the median and the non-parametric smoother are somewhat negative).



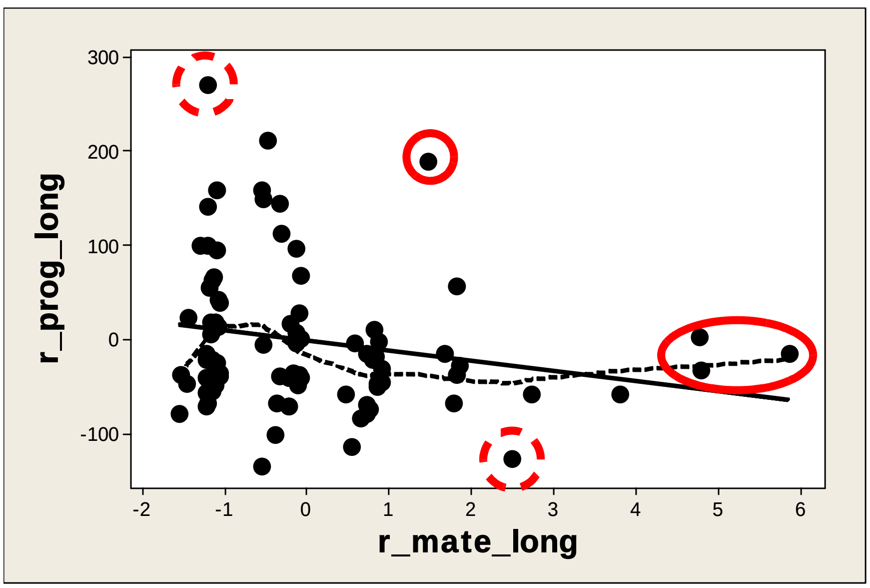
The plot of residuals *vs*. fits, however, shows a clear problem: the variability is very uneven, increasing with increasing fits. In addition the smoother in this plot again shows a downward trend, due to the increasing spread and skew of the residuals at larger values of the fits.



This violation of the assumption of constant residual variance does mean that inferences from the regression analysis cannot be considered as being very precise. The regression estimates, though, will still be unbiased, and the conclusions should be qualitatively correct.

Partial regression plots

Number of matings:



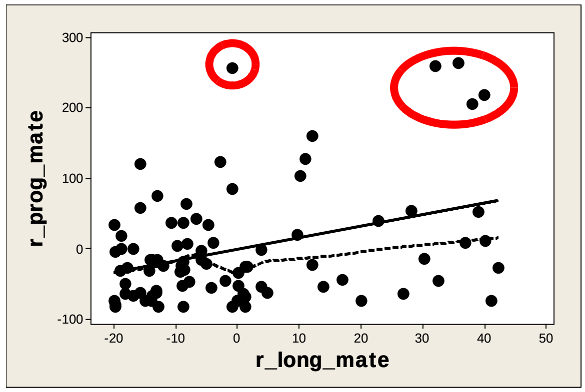
This plot shows a generally decreasing trend as well as several points at the right which do not fit the overall pattern well: observations with large number of matings residuals (basically, high values of number of matings) and unexpectedly high progeny residuals (again, basically from high values of progeny). These "aberrant" points would be expected to have high leverage, large residuals, and quite possibly high influence. These points could be interpreted as suggesting a curvilinear relationship, but as seen above, a quadratic effect for number of matings was not significant. (These patterns also are apparent in the raw scatter plot of number of matings and progeny, but not as clearly as in the partial regression plot.)

One other observation appears somewhat exceptional in this plot: the one with number of matings residual about 1.5 and progeny residual about 200.

The partial regression plot also shows (more clearly than the raw scatter plot) that the variability in progeny is uneven: being greater, and somewhat positively skewed, at low values of number of matings.

Finally, the two points shown by the dashed circles (observations #5 and #27), while standing out only mildly in the partial regression plot, will be found below to have considerably influence on the regression.

<u>Female longevity</u>



This partial regression plot shows several noteworthy features. First, there is an overall appearance of an upward trend. This trend, however, is largely due to a few observations, particularly the four at the upper right; notice that the smoother is considerably lower than the regression at the right end, reflecting the greater effect of these high outliers on the regression. The increasing trend also is accompanied by increasing variability at large female longevity. Finally, there is at least one outlying observation, with female longevity residual about 0 and progeny residual about 260. There is nothing in this plot to suggest a nonlinear relationship.

Indices of leverage, outliers, and influence

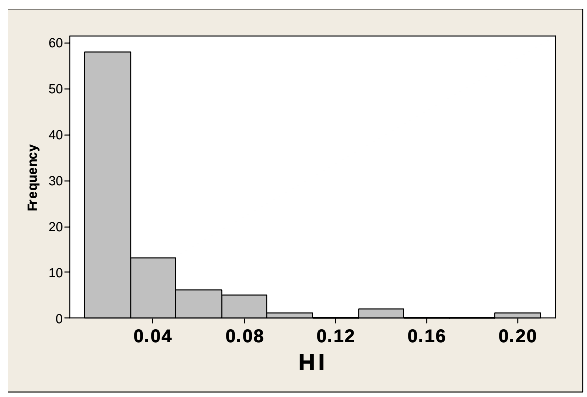
The values of these diagnostic indices for each observation are listed below and on the next page and then assessed in the following text.

obs	Y	X1	X2	TRES	HI	COOK	DFIT	Dfbeta0	Dfbeta1	Dfbeta2
1	36	1	29	-0.42441	0.022122	0.001372	-0.063835	-0.0541	0.0368	0.0204
2	14	3	21	-0.25621	0.026673	0.000606	-0.042412	-0.0233	-0.0170	0.0285
3	75	4	50	0.06715	0.031409	0.000049	0.012092	-0.0051	0.0083	0.0040
4	104	3	62	0.04775	0.033510	0.000027	0.008891	-0.0045	0.0020	0.0067
5	**318**	**4**	**76**	**2.90318**	0.074086	0.206330	**0.821213**	-0.5754	0.3233	0.6474
6	206	1	34	1.72097	0.020170	0.019853	0.246915	0.1806	-0.1542	-0.0302
7	0	2	18	-0.51944	0.026505	0.002471	-0.085711	-0.0733	0.0033	0.0635
8	167	1	36	1.14762	0.019896	0.008878	0.163510	0.1101	-0.1042	-0.0062
9	185	1	47	1.15075	0.023574	0.010615	0.178804	0.0536	-0.1120	0.0709
10	73	2	21	0.37356	0.022473	0.001081	0.056640	0.0475	-0.0030	-0.0388
11	58	2	25	0.09160	0.018111	0.000052	0.012440	0.0100	-0.0010	-0.0073
12	148	2	25	1.27858	0.018111	0.009975	0.173647	0.1394	-0.0133	-0.1013
13	**338**	**1**	**36**	**3.63324**	0.019896	0.077876	**0.517656**	**0.3485**	**-0.3298**	-0.0196
14	10	3	53	-0.99553	0.021571	0.007284	-0.147818	0.0528	-0.0473	-0.0836
15	**278**	**2**	**76**	1.96819	0.064160	0.085564	**0.515346**	**-0.2419**	-0.0802	**0.4649**
16	**332**	**2**	**70**	**2.89038**	0.048957	0.131684	**0.655787**	**-0.2733**	-0.1067	**0.5702**
17	203	1	21	1.98298	0.029016	0.037832	0.342790	0.3254	-0.1636	-0.1925
18	107	2	18	0.88792	0.026505	0.007173	0.146511	0.1252	-0.0056	-0.1086
19	233	2	50	1.88189	0.017127	0.019959	0.248417	-0.0061	-0.0466	0.1366
20	**291**	**2**	**78**	**2.11513**	0.069808	0.107419	**0.579434**	**-0.2803**	-0.0890	**0.5275**
21	40	3	30	-0.10712	0.017736	0.000070	-0.014394	-0.0054	-0.0065	0.0060
22	116	1	32	0.55885	0.020733	0.002223	0.081317	0.0637	-0.0495	-0.0166
23	119	4	32	1.03700	0.030567	0.011292	0.184141	0.0053	0.1393	-0.0530
24	119	1	28	0.68483	0.022730	0.003660	0.104443	0.0904	-0.0591	-0.0371
25	0	2	38	-0.94391	0.011945	0.003595	-0.103787	-0.0476	0.0168	0.0002
26	49	3	25	0.11690	0.021976	0.000104	0.017523	0.0085	0.0074	-0.0101
27	**0**	**5**	**74**	-1.34437	0.090307	0.059229	-0.423576	**0.3069**	**-0.2559**	**-0.2757**
28	3	1	17	-0.60477	0.034202	0.004351	-0.113807	-0.1105	0.0486	0.0737
29	0	2	24	-0.64595	0.019093	0.002726	-0.090120	-0.0733	0.0063	0.0551
30	0	2	58	-1.38902	0.026380	0.017232	-0.228638	0.0547	0.0410	-0.1691
31	82	1	18	0.41470	0.032797	0.001963	0.076364	0.0738	-0.0336	-0.0479
32	0	4	36	-0.61900	0.028725	0.003806	-0.106451	0.0080	-0.0817	0.0165
33	136	1	65	0.11797	0.048511	0.000239	0.026637	-0.0041	-0.0129	0.0205
34	0	1	28	-0.87773	0.022730	0.005990	-0.133861	-0.1159	0.0757	0.0475
35	0	1	36	-1.04967	0.019896	0.007446	-0.149555	-0.1007	0.0953	0.0057
36	29	1	37	-0.68716	0.019868	0.003211	-0.097835	-0.0628	0.0628	-0.0005
37	140	1	21	1.11882	0.029016	0.012431	0.193406	0.1836	-0.0923	-0.1086
38	201	2	49	1.46460	0.016296	0.011684	0.188507	0.0018	-0.0354	0.0974
39	68	1	67	-0.83199	0.052731	0.012892	-0.196298	0.0367	0.0920	-0.1550
40	125	2	77	-0.14379	0.066948	0.000500	-0.038517	0.0184	0.0060	-0.0349
41	**0**	**2**	**79**	-1.90747	0.072740	0.092210	**-0.534250**	**0.2620**	0.0815	**-0.4884**
42	55	1	79	-1.29558	0.084143	0.050987	-0.392697	0.1305	0.1549	-0.3432
43	72	3	76	-0.68603	0.063752	0.010751	-0.179017	0.1098	-0.0240	-0.1571
44	0	3	66	-1.42860	0.040701	0.028506	-0.294264	0.1609	-0.0577	**-0.2353**
45	94	1	77	-0.71141	0.078183	0.014394	-0.207182	0.0648	0.0839	-0.1789
46	71	3	31	0.27610	0.017105	0.000447	0.036424	0.0126	0.0166	-0.0137
47	63	3	43	-0.08340	0.015192	0.000036	-0.010358	0.0007	-0.0044	-0.0019
48	66	1	23	0.09645	0.026857	0.000087	0.016024	0.0149	-0.0081	-0.0082
49	14	1	38	-0.90693	0.019912	0.005582	-0.129272	-0.0788	0.0834	-0.0061
50	**32**	**7**	**34**	0.29154	**0.135514**	0.004491	0.115430	-0.0350	0.1101	-0.0185
51	50	2	50	-0.54527	0.017127	0.001742	-0.071977	0.0018	0.0135	-0.0396
52	2	3	44	-0.90406	0.015504	0.004300	-0.113452	0.0113	-0.0475	-0.0263
53	21	5	43	-0.35300	0.052132	0.002309	-0.082785	0.0289	-0.0723	-0.0033
54	29	2	55	-0.93184	0.022366	0.006632	-0.140945	0.0240	0.0258	-0.0962
55	119	1	26	0.72840	0.024164	0.004404	0.114621	0.1028	-0.0620	-0.0483
56	0	3	40	-0.84398	0.014692	0.003553	-0.103059	-0.0046	-0.0463	-0.0041
57	**37**	**8**	**26**	0.72435	**0.201022**	0.044257	0.363333	-0.0906	**0.3476**	-0.0926
58	5	3	26	-0.48015	0.020983	0.001662	-0.070294	-0.0328	-0.0303	0.0386
59	49	2	26	-0.04710	0.017202	0.000013	-0.006231	-0.0049	0.0005	0.0034
60	0	1	24	-0.79265	0.025887	0.005591	-0.129216	-0.1191	0.0667	0.0623
61	2	3	26	-0.51954	0.020983	0.001945	-0.076059	-0.0355	-0.0328	0.0417
62	0	2	23	-0.62485	0.020147	0.002696	-0.089598	-0.0738	0.0058	0.0572

obs	Y	X1	X2	TRES1	HI1	COOK1	DFIT1	Dfbeta0	Dfbeta1	Dfbeta2
63	24	4	26	-0.08913	0.035506	0.000099	-0.017102	-0.0029	-0.0123	0.0078
64	55	1	19	0.03782	0.031464	0.000016	0.006816	0.0066	-0.0031	-0.0041
65	7	2	21	-0.49064	0.022473	0.001862	-0.074393	-0.0624	0.0040	0.0509
66	78	1	17	0.38373	0.034202	0.001756	0.072211	0.0701	-0.0308	-0.0467
67	16	1	23	-0.56000	0.026857	0.002909	-0.093032	-0.0867	0.0468	0.0475
68	42	2	19	0.01046	0.025089	0.000001	0.001678	0.0014	-0.0001	-0.0012
69	20	1	21	-0.46490	0.029016	0.002173	-0.080365	-0.0763	0.0383	0.0451
70	0	3	21	-0.44011	0.026673	0.001787	-0.072856	-0.0401	-0.0291	0.0489
71	0	1	17	-0.64452	0.034202	0.004938	-0.121289	-0.1177	0.0518	0.0785
72	0	1	38	-1.09315	0.019912	0.008074	-0.155815	-0.0950	0.1005	-0.0074
73	56	1	38	-0.35412	0.019912	0.000858	-0.050475	-0.0308	0.0325	-0.0024
74	27	2	36	-0.54623	0.012097	0.001228	-0.060443	-0.0325	0.0091	0.0068
75	0	2	38	-0.94391	0.011945	0.003595	-0.103787	-0.0476	0.0168	0.0002
76	48	1	37	-0.43755	0.019868	0.001306	-0.062296	-0.0400	0.0400	-0.0003
77	**71**	**7**	**37**	0.76771	**0.133704**	0.030472	0.301602	-0.1030	**0.2881**	-0.0339
78	82	2	34	0.21280	0.012538	0.000194	0.023978	0.0146	-0.0033	-0.0052
79	7	6	34	-0.20287	0.089434	0.001363	-0.063581	0.0161	-0.0590	0.0111
80	64	1	28	-0.03652	0.022730	0.000010	-0.005569	-0.0048	0.0032	0.0020
81	66	1	31	-0.07426	0.021124	0.000040	-0.010909	-0.0088	0.0065	0.0027
82	43	3	34	-0.15290	0.015648	0.000125	-0.019278	-0.0049	-0.0089	0.0048
83	10	3	43	-0.77682	0.015192	0.003118	-0.096483	0.0062	-0.0413	-0.0179
84	79	1	31	0.09565	0.021124	0.000067	0.014051	0.0113	-0.0084	-0.0034
85	23	4	31	-0.20919	0.031209	0.000475	-0.037547	-0.0020	-0.0282	0.0120
86	21	2	29	-0.47619	0.014909	0.001155	-0.058583	-0.0433	0.0060	0.0261

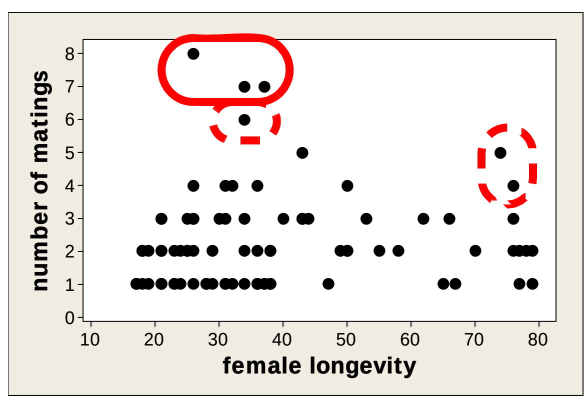
Leverage:

The leverage ("HI") values shown in the preceding list and the histogram to the right show that the one observation (#57) with number of matings = 8 has fairly high leverage (0.20, compared with $2p/n = 0.07$), and the two observations (##50 and 77) with number of matings = 7 have moderate leverage (about 0.13). Several other observations (one with number of matings = 6 and several with high values of female longevity) also have slight leverage.



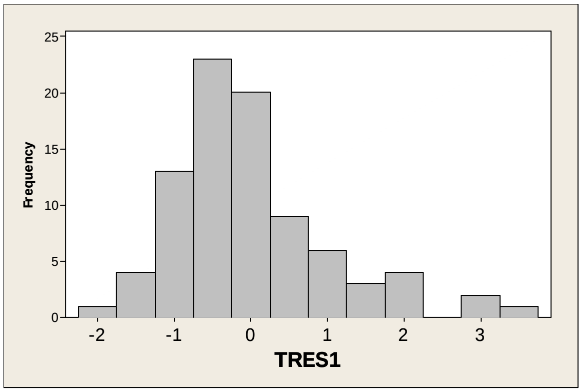
These points all stand out to some extent as unusual in the scatter plot of the two independent variables (to the right); this of course is the reason they have leverage.

These observations (especially the first three) will need to be examined for possible influence on the regression.



Outlying Y observations

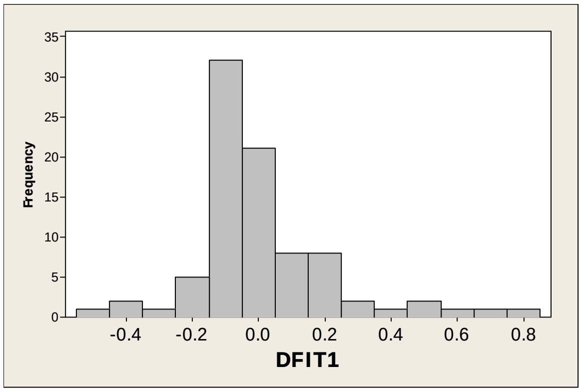
The "studentized deleted residuals" ("TRES") shown in the preceding list and the histogram to the right show that three observations have unusually large values, considerably above what would be expected. The largest of these (#13, TRES = 3.63) is in the 0.0005 (two-sided) tail of the *t* distribution with *df* = 82, while the other two (#5, TRES = 2.90; #16, TRES = 2.89) are in the 0.005 tail. These observations all have very high values of progeny, with low to moderate number of matings and high female longevity.



These outliers clearly could affect the regression, so their influence will need to be checked. It is perhaps fortunate that these *Y* outliers are not the same as the high-leverage observations.

Influence on fitted values – DFFITS:

One observation has a DFFITS value approaching 1 (#5 again, DFFITS = 0.82), and several have absolute values of DFFITS well above $2\sqrt{p/n} = 0.37$.; the largest of these are #16 (DFFITS = 0.66) and ## 13, 15, 20, and 41 (all with DFFITS between 0.52 and 0.58). Notice that these possibly influential observations include all the observations with outlying *Y*s, plus one other point (#15) also with high progeny, and one (#41) with very low progeny.

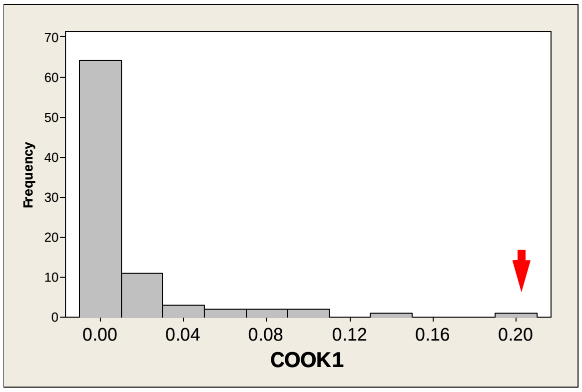


Note also that while the high-leverage points do not have the highest DFFITS, two of them do have moderately large DFFITS (## 57 and 77, DFFITS = 0.36 and 0.30).

Influence on regression coefficients – Cook's Distance

:Although one observation (#5, yet again) does stand out as having a considerably larger effect on all the coefficients than do the others, its Cook's D of 0.206 is at only the 11th percentile of the *F* distribution with 3 and 83 *df*, which suggests that it is not substantially influential.

However, since we are more interested in the coefficient for number of matings than that for female longevity, it probably is more useful to examine *DFBETA*s, and in particular those for $\mathbf{b}_1$, rather than Cook's D.



Influence on regression coefficients – DFBETAs:

Using the criterion $2/\sqrt{n} = 0.216$, a number of observations appear fairly influential on at least one of the coefficients. The most noteworthy observation again is #5, which has large DFBETA for each of the three coefficients (-0.58, 0.32, and 0.65; the latter is the largest of all the DFBETAs). Four other observations were influential on the coefficient for number of matings: two of the high-leverage points, ##57 and 77 (DFBETA = 0.35 and 0.29), the largest outlying *Y* (#13, with very high progeny and low number of matings), and one observation with 0 progeny but high number of matings (#27, DFBETA = -0.26).

The outlying observations ## 16 and 20 and the somewhat similar observation #15 are influential on the coefficient for female longevity. This is as expected from the partial regression plot, in which these three observations (along with #5) are the primary basis for the appearance of a positive relationship. Several other observations (## 41, 42, 44) with low values of progeny also are influential on $\mathbf{b_2}$, in the opposite direction.

Several observations with unusually high or low values of progeny also were moderately influential on the intercept.

Influence on inferences:

The regression was re-run omitting (one at a time) each of the five observations (## 57, 13, 5, 77, and 27) which appeared most influential on the coefficient for number of matings. The coefficients and standard errors for the full data set and with each variable removed are shown in the following table.

	$\mathbf{b_1}$	se of $\mathbf{b_1}$	P	$\mathbf{b_2}$	se of $\mathbf{b_2}$
all observations	-10.921	5.634	0.056	1.630	0.463
without #5	-12.666	5.431	0.022	1.343	0.454
without #13	-9.186	5.282	0.086	1.638	0.432
without #27	-9.486	5.707	0.100	1.757	0.470
without #57	-12.884	6.267	0.043	1.673	0.468
without #77	-12.584	6.032	0.041	1.646	0.464

These results are as would be expected. Without either of the high-leverage observations which suggest upward curvature at high number of matings (## 57 and 77), the regression would be more steeply downward. Similarly, without observation 5, which had moderately high number of matings and very high progeny, the regression would be steeper. In all three cases this would increase the statistical significance of the number of matings effect.

In contrast, removal of observations # 13 or 27 produces a somewhat less steep, and considerably less significant, slope. It would not be unfair to say that the suggestion that there is a negative relationship between progeny and number of matings is due solely to these two observations; as seen in the partial regression plot, they largely define the downward trend.

Collinearity

The two independent variables, number of matings and female longevity, are correlated (see section on data exploration, and the scatterplot in the previous section), but only slightly so: the correlation coefficient is only 0.095. As a result these is effectively no collinearity in these data. The variance inflation factors for the regression coefficients were effectively 1.0.

Conclusion from diagnostics

The uneven variability shown in the plot of residuals *vs*. fits, and the considerable influence of a few unusual observations, raises concerns about first, the quantitative accuracy of the inference, and second about the overall reliability and repeatability of the results. Neither of these problems, or the lesser ones of non-normality and slight non-linearity, though, close to being severe enough to call into question the overall finding stated in the following conclusions.

Conclusions

The data clearly **do not support the hypothesis** that the males which get more matings somehow cause or allow their mates to have higher fecundity. To the contrary, **fecundity of a female appears to be <u>negatively</u> related to her mate's success at getting matings**. There is, though, a slight hint of an upwards trend in fecundity among the most successful males.

The fecundity of a female also, and unsurprisingly, is positively related to how long she lived.

These conclusions, unfortunately, are strongly affected by a few unusual observations. I know of no biological reason for omitting any of these observations. Their strong effects cast considerably doubt on any conclusion which might be drawn from these data. As was clear from a simple scatter plot of progeny and number of matings, much more data is needed for males with high mating success: do their mates have high fecundity (as for ## 50, 57, and 77) or low fecundity (as for # 27)?