Homework # 14 — Solutions

(1) (a) standard method:

$\hat{p} = 339 / 870 = 0.3897$

$SE_{\hat{p}} = \sqrt{\frac{\hat{p} \cdot (1 - \hat{p})}{n}} = \sqrt{\frac{0.3897 \cdot 0.6103}{870}} \approx 0.0165$

95% CI for p: $\hat{p} \pm (z^*_{0.025} \cdot SE_{\hat{p}}) = 0.3897 \pm (1.96 \cdot 0.0165) = (0.3573, 0.4221)$

plus-four method (which, for such a large n, is almost identical to the preceding):

$\tilde{p} = (339 + 2) / (870 + 4) = 0.39016$

$SE_{\tilde{p}} = \sqrt{\frac{\tilde{p} \cdot (1 - \tilde{p})}{n}} = \sqrt{\frac{0.39016 \cdot 0.60984}{870}} \approx 0.0165$

95% CI for p: $\tilde{p} \pm (z^*_{0.025} \cdot SE_{\tilde{p}}) = 0.39016 \pm (1.96 \cdot 0.0165) = (0.3578, 0.4225)$

from Minitab, if you do not request the Normal approximation (it is not an "exact" CI based on inverting the exact test, but rather is a different approximation):
95% CI for p: (0.357096, 0.422965)
[if you specify a 1-sided test (see (b) below), you get a 95% lower bound of 0.417674]

(b) $\sigma_{\hat{p}} = \sqrt{\frac{p_0 \cdot (1 - p_0)}{n}} = \sqrt{\frac{0.791 \cdot 0.209}{870}} \approx 0.0138$

$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.3897 - 0.791}{0.0138} \approx -29.1$

Hypotheses: The null hypothesis is H_0: $p = 0.791$.
Bias either for or against selection of Mexican Americans would be illegal and we would want to detect it, which would suggest using a two-sided alternative (H_a: $p \neq 0.791$).
Realistically, however, we might consider the possibility of bias <u>for</u> selection of Mexican Americans to be so unlikely that we can ignore it, which would lead to a one-sided test with H_a: $p < 0.791$. It would be cheating, though, to choose this H_a based on the data rather than our prior thinking about the question of interest.

The result ($z = -29.1$) is so extreme the P-value is essentially 0, regardless of whether a one-sided (H_a: $p < p_0$) or two-sided (H_a: $p \neq p_0$) test is performed.
The exact test provided by Minitab gives effectively the same result (P-value is reported as 0.000.)

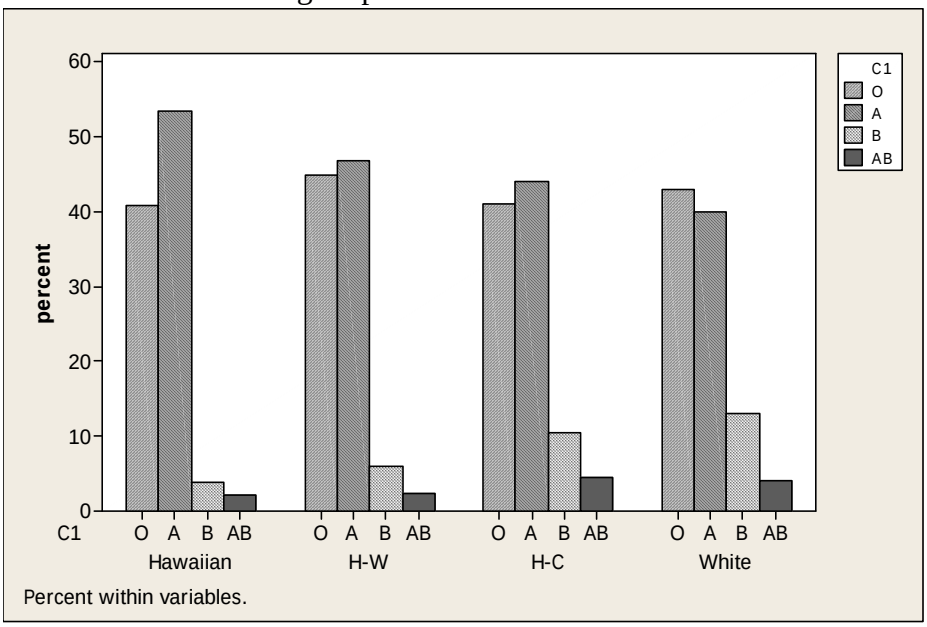
(c) There is very strong evidence that Mexican Americans are underrepresented on juries: although they constitute almost 80% of those eligible for jury duty, they are less than 40% of those selected for jury duty. This discrepancy is extremely unlikely to have occurred "by chance"—by random sampling of jurors from the pool of eligibles—and we are very

confident that the probability someone selected for jury duty is Mexican American is much less than their proportion in the pool of eligibles.

(2) A natural way to view the data is to regard the ethnic groups as different samples and examine the relative frequencies of the different blood types within the different groups, as in the following table of blood-type percentages for each ethnic group, and in the grouped bar chart below.

	Hawaiian	H-C	H-W	white	All
O	40.75	40.97	44.77	43.00	42.97
A	53.32	43.97	46.79	40.00	41.04
B	3.81	10.55	6.07	13.00	12.14
AB	2.12	4.51	2.36	4.00	3.85

Among Hawaiians, the difference between O and A is more marked than in the other groups. Also, for Whites, O is greater than A, which is the reverse of the other groups. Also Whites and Hawaiian-Chinese have a higher percent of B.



We can answer the question of whether blood type and ethnic group are related by testing the null hypothesis that they are not, specifically that the probability of a given blood type is the same for all ethnic groups:

P(blood type *i* | ethnic group *j*) = P(blood type *i*) for all combinations of *i* and *j*.

This hypothesis is tested by a X^2 test. The result is extremely significant, so we reject the null hypothesis and conclude that blood type and ethnic group are related.

The "chi-square contributions" indicate that the greatest differences between observed and expected counts are the much lower than expected proportions of Hawaiians and Hawaiian-Chinese with Type B blood, and the higher than expected frequency of Type A among Hawaiians.

Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

	Hawaiian	H-W	H-C	White	Total
1	1903	4469	2206	53759	62337
	2006.89	4289.68	2314.16	53726.27	
	5.378	7.496	5.055	0.020	
2	2490	4671	2368	50008	59537
	1916.75	4097.00	2210.21	51313.04	
	171.445	80.419	11.265	33.191	
3	178	606	568	16252	17604
	566.75	1211.41	653.52	15172.33	
	266.652	**302.556**	11.191	76.830	
4	99	236	243	5001	5579
	179.61	383.92	207.11	4808.36	
	36.179	56.989	6.219	7.718	
Total	4670	9982	5385	125020	145057

Chi-Sq = 1078.604, DF = 9, P-Value = 0.000