Homework # 6  
due Monday, 10 October

(1) from 4.90 & 4.91 Fire Insurance
An insurance company looks at the records for millions of homeowners and sees that the mean loss from a fire in a year is $\mu = $300 per person. (Most of us have no loss, but a few lose their homes. The $300 is the average loss.) The standard deviation of the loss is $\sigma = $400.

a. What are the mean and standard deviation of the average loss for 10 policies? (Losses on separate policies are independent.)

b. Can you determine the probability that the average loss for 10 policies is greater than $400? If so, do so. If not, explain why not.

(2) In each situation below, is it reasonable to use a binomial distribution for the random variable X? Give reasons for your answer in each case. If a binomial distribution applies, give the values of $n$ and $p$.

a. A random sample of students is selected for a fitness study. $X$ is the number of students in the sample with mean systolic blood pressure greater than 130.

b. You toss a coin until you get heads 5 times. $X$ is the number of times you get tails (assuming heads and tails are the only possible outcomes).

c. A manufacturer of running shoes picks a random sample of the production of shoes each day for a detailed inspection. $X$ is the number of defects in today’s sample of shoes.

d. A nutrition study chooses an SRS of 250 college students. They are asked whether or not they usually eat at least five servings of fruits or vegetables per day. $X$ is the mean daily number of servings of fruits or vegetables.

(3) A study of rush hour traffic in San Francisco records the number of people in each car entering a freeway at a suburban interchange. Suppose that this number $X$ has mean 1.5 and standard deviation 0.75 in the population of all cars that enter at this interchange during rush hours.

a. Does the count $X$ have a binomial distribution? Why or why not?

b. Could the exact distribution of $X$ be Normal? Why or why not?

c. Traffic engineers estimate that the capacity of the interchange is 700 cars per hour. What is the approximate distribution of the mean number of persons $\bar{x}$ in 700 randomly selected cars at this interchange?

d. The count of people in 700 cars is 700$\bar{x}$. Use your result from (c) to give an approximate distribution for the count. What is the probability that 700 cars will carry more than 1075 people?

(4) The mailing list of an agency that markets scuba-diving trips to Maui contains 70% males and 30% females. The agency calls 20 people chosen at random from its list. What is the probability that exactly 15 of the 20 are men? (Use the binomial probability formula. This assumes that the mailing list is much
larger than the sample of 20 who are called, so that the effect of sampling without replacement is negligible.)

(5) An experiment to compare the nutritive value of normal corn and high-lysine corn divides 40 chicks at random into two groups of 20. One group is fed a diet based on normal corn while the other receives high-lysine corn. At the end of the experiment, inference about which diet is superior is based on the difference $\bar{y} - \bar{x}$ between the mean weight gain $\bar{y}$ of the 20 chicks in the high-lysine group and the mean weight gain $\bar{x}$ of the 20 in the normal-corn group. Because of the randomization, the two sample means are independent.

a. Suppose that $\mu_X = 360$ grams (g) and $\sigma_X = 55$ g in the population of all chicks fed normal corn, and that $\mu_Y = 385$ g and $\sigma_Y = 50$ g in the high-lysine population. What are the mean and standard deviation of $\bar{y} - \bar{x}$?

b. The weight gains are normally distributed in both populations. What is the distribution of $\bar{x}$? Of $\bar{y}$? What is the distribution of $\bar{y} - \bar{x}$?

c. What is the probability that the mean weight gain in the high-lysine group exceeds the mean weight gain in the normal-corn group by 25 g or more?