Reconstruction Deconstruction:
A Brief History of
Building Models of Nonlinear Dynamical Systems

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The Problem
The Goal:
Artificial Science

Can we automate scientific discovery and theory building?
The Guess:
Yes!

Recent advances in nonlinear dynamics
+
Recent advances in machine learning
+
Lots of computational power
The Guess:
Yes!
Recent advances in nonlinear dynamics:
Why?
Better sense of the manifestations of nonlinearity (info. gen., self-similarity, hierarchy, ... )
The Guess:
Yes!
Recent advances in machine learning:
Why?
Information-theoretic view of learning
+
Algorithms: Structure v. error
The Guess:
Yes!
Lots of computational power:
Why?
We don’t yet know how the brain does it so quickly
Agenda

- Nonlinear Dynamics:
  - History & Geometry
- Reconstruction and Nonlinear Model Building
  - Continuous-time & -value
  - Discrete-time & -value
Nonlinear Dynamics:  
Selected Historical Highlights

- Poincare (1892): Discovery of deterministic chaos
- van der Pol (Nature 1927): Expt’l discovery of chaos
- Soviet school (1910-1960): Lyapunov and students
- Kolmogorov (1957): Info theory + dynamical systems
- Modern era: 1960s onwards ...
  - Ulam-Lorenz (1962-63): Experimental mathematical studies
  - 1970s-1980s: Simple systems can be complicated (deterministic chaos)
  - 1980s-1990s: Complicated systems can be simple (pattern formation)
Nonlinear Dynamics: Mathematical Tools

- Statistical mechanics; incl. phase transitions
- Pattern formation: Center manifold theory
- Nonlinear dynamics:
  - Qualitative dynamics
  - Bifurcations: Singularity theory
Nonlinear Dynamics: Geometric Review

Dynamical System:
State space
Dynamic
Initial condition

The Attractor-Basin Portrait: The Big Picture
Basins, Attractors, Separatrices

\[ B(\Lambda^0) \]

\[ B(\Lambda^1) \]

\[ \partial B \]
Nonlinear Dynamics: Review ...

- Bifurcations: The Big, Big Picture
  - Catastrophe theory (Thom, 1960s)
  - Singularity theory (Arnold et al, 1970s on)

What happens when you change control parameters?
Reconstruction:

How to build a model of nonlinear system?

- Goals:
  - What are the states?
  - What are the equations of motion?
- Two cases:
  - Continuous measurements
  - Discrete measurements
Reconstruction: Continuous Dynamics

- **Time series**

  \[ x(t) \]

- **Reconstructed state space?**

  - **Derivative embedding:** \( \vec{y}(t) = (x(t), \dot{x}(t), \ddot{x}(t), \ldots) \)
  
  - **Time-delay embedding:**
    \[
    \vec{y}(t) = (x(t), x(t - \tau), x(t - 2\tau), \ldots) \]

  - **Embedding dimension:** Number of active degrees of freedom


Reconstruction: Continuous Dynamics

- Fixed point
- Limit Cycle
- Chaotic Attractor

Times Series

State Space
Reconstruction: Continuous Dynamics

- Equations of motion? $\dot{\vec{y}} = \vec{f}(\vec{y})$
- Find $f : \vec{y}(t) \rightarrow \vec{y}(t + dt)$

Reconstruction: Symbolic Dynamics

Simplest Dynamical System:

State space:
\[ x \in [0, 1] \]

Equations of motion:
\[ x_{n+1} = f(x_n) \]

Binary instrument:
\[ \mathcal{P} = \{0 \sim x \in [0,d], 1 \sim x \in (d,1]\} \]

Decision point: \( d \in [0,1] \)
Reconstruction: Symbolic Dynamics

One time step

\[ f(x) \]

\[ f^{-1}(d) \]

\[ f^0(d) \]

\[ s = 0 \]

\[ s = 1 \]
Reconstruction: Symbolic Dynamics

Two time steps

s = 1

s = 0
Reconstruction: Symbolic Dynamics

When are partitions good?
When symbol sequences **encode** orbits

Diagram **commutes**:

\[ T(x) = \Delta \circ \sigma \circ \Delta^{-1}(x) \]

**Good kinds of instruments:**
- Markov partitions
- Generating partitions
Measurement Channel

Goals: (i) States? (ii) Dynamic?
Computational Mechanics

How to circumvent representation choice?
Is there a preferred representation?

J. P. Crutchfield and K. Young, “Inferring Statistical Complexity”,

J. P. Crutchfield, “The Calculi of Emergence: Computation, Dynamics, and Induction”,


N. Ay and J. P. Crutchfield, Reductions of Hidden Information Sources”,
Space of Histories Partitioned
Causal States

- $\leftarrow S = S \leftarrow S$
- $\leftarrow S \sim \leftarrow S' \iff \Pr(\leftarrow S | \leftarrow S) = \Pr(\leftarrow S | \leftarrow S')$
- $\mathcal{E}$-Machine = \{S, T\}
- Theorems: Unique, minimal, & optimal predictor
- Form = $\mathcal{E}$-Machine semigroup
Stored Information (1989)

\[ C_\mu = - \sum_{S \in S} Pr(S) \log_2 Pr(S) \]

Information Production (1948)

\[ h_\mu = - \sum_{S \in S} Pr(S) \sum_{S' \in S} Pr(S'|S) \log_2 Pr(S'|S) \]
Excess Entropy (1982)
(all-point mutual information)

\[ E = I(\overleftarrow{S}, \overrightarrow{S}) \]
(Thm: \( E \leq C_\mu \))

Sync-Transient Information (2005)

\[ T = \sum_{L=0}^{\infty} [E + h_\mu L - H(L)] \]

Entropy Hierarchy

Entropy growth $H(L)$
One Lesson: Structure v. Randomness

Boredom  Delight  Confusion

Variatio Delectat
Concluding Remarks

- Yes, there is a preferred representation:
  - Machine: Causal architecture
- Structure can be formalized (and quantified)
- Intrinsic computation:
  Structure = how a system stores & processes info
- Question:
  How does all this affect modeling nonlinear systems?
Background


Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers by R. Hilborn (2001)

Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering by S. H. Strogatz (2001)
Thanks!