Introduction to Deductive Logic

Part I: Logic & Language

There is no royal road to logic, and really valuable ideas can only be had at the price of close attention.

Charles Saunders Peirce
INTRODUCTION TO DEDUCTIVE LOGIC

PART I: LOGIC AND LANGUAGE

I. INTRODUCTION
What is Logic? ................................................................. 4
What is an argument? ....................................................... 5
Three Criteria for Evaluating Arguments ................................. 6

II. ARGUMENT ANALYSIS – PART 1
Preliminary Issues ............................................................ 9
Statements and Truth Value
Premises and Conclusions .................................................. 9
Statements vs Sentences 10
Simple vs Compound Statements 12
Recognizing premises and conclusions .................................... 13
Indicator Language ............................................................. 15

TWO TYPES OF ARGUMENTS
Inductive Reasoning/Argument ........................................... 18
Deductive Reasoning/Argument ........................................... 19

III. ANALYSIS – PART II
FORMALIZING ARGUMENT
Preliminary Issue: Formal vs. Informal Argument ...................... 24
Simple and Compound Statements ....................................... 25
Types of Compound Statements .......................................... 26
Formation Rules for Sentential ............................................ 26
DEFINING LOGICAL OPERATORS ........................................... 29
Negation ................................................................. 29
Conjunction .............................................................. 30
Disjunction ............................................................... 31
Conditional/ Material Implication ....................................... 32
Bi-conditional .............................................................. 33

APPLICATION: TRANSLATING USING LOGICAL OPERATORS ....... 35

TRUTH TABLE RULES/SUMMARY OF LOGICAL OPERATORS ........ 38

TRUTH TABLE APPLICATIONS ............................................. 39
Application I: Substitute and Calculate Method ......................... 39
Application II: Calculating with Unknown Values ..................... 41
Application III: Full Truth Table Method ............................... 42
A Short Cut Technique for statements ................................ 44
Truth Table Test for Logical Equivalence ............................... 45
Application IV: Using Truth Tables to Facilitate Reasoning ........ 47
THE NEED FOR PREDICATE

Using Predicate to Represent Categorical Statements

Preliminary Issues—Grammar

Formation Rules for Predicate

Expressing Quantity

APPLICATION: Symbolizing Categorical Statements using Predicate

Negating Standard Form Categorical Statements

Summary Chart for Standard Form Categorical Statements

Extending Predicate

A Universe of Discourse

Compound Statements in Predicate

Multi-place predicate terms

Summary Chart of Logical Language Elements

IV. Answers to Selected Exercises

IMPORTANT CHARTS: The information in the following charts should be fully understood and you should be able to competently apply it to practiced and novel situations.

Indicator Language

Argument Analysis

Types of Truth-Functional Compound Statements

Summary of Logical Operators

Truth Table Values for Logical Operators

Full Truth Table Method: Review

Summary Chart for Standard Form Categorical Statements

Summary Chart of Logical Language Elements

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WHAT IS LOGIC?

All knowledge, all sciences and arts have their beginning in the recognition that ordinary and familiar things may take on different forms. 

Susan Langer

For philosopher Rudolf Carnap logical analysis promises much. He states:

The function of logical analysis is to analyse all knowledge, all assertions of science and of everyday life, in order to make clear the sense of each such assertion and the connections between them.

He qualifies this promise, however, telling us that the focus of logic – particular statements, propositions or truth claims – is exceedingly narrow.

Logic is not concerned with human behavior in the same sense that physiology, psychology, and social sciences are concerned with it. These sciences formulate laws or universal statements which have as their subject matter human activities as processes in time. Logic, on the contrary, is concerned with relations between factual sentences (or thoughts).

A generation earlier, Bertrand Russell, one of the founders of modern symbolic logic, identified what it is about statements, or propositions, that logic is concerned with. "In every proposition and in every inference there is, besides the particular subject-matter concerned, a certain form, a way in which the constituents of the proposition or inferences are put together. . . . It is forms, in this sense, that are the proper object of philosophical logic."

Logic is essentially a set of tools for analyzing arguments. More specifically, it studies the forms of statements and the relationships between them. It concerns itself with reasoning and distinguishes good, accurate, competent reasoning from fallacious reasoning that is inaccurate and weak. Because we express our reasoning in language, through the statements we make or think, statements and their relationships to one another in argument becomes a primary focus of Logic.

What Is Reasoning?

The object of reasoning is to find out, from the consideration of what we already know, something else which we do not know. 

Charles Sanders Pierce

Reasoning is a kind of thinking that involves making inferences, or drawing conclusions. Different aspects of reasoning have been studied by different academic disciplines including Psychology, Mathematics, Linguistics, and Philosophy. Although they ask very different questions, Logic overlaps with Psychology as the study of a type of mental activity. Psychology takes up questions such as why humans reason, what leads us to successful reasoning or causes us to fall into error, and whether other types of creatures reason. Logic aims at understanding when our reasoning is valid. Its primary concern is whether or not our inferences rest on solid ground. Logic overlaps
with Linguistics because our reasoning is expressed in and through language, in words, in statements and sentences. Because reasoning presents itself through definable patterns that can be symbolized and manipulated by applying formal rules, Logic overlaps with Mathematics. It is a branch of Philosophy because it was among the ancient Greek philosophers more than 2500 years ago that Logic was first explored in a systematic way as a study of argument and reasoning. The Greeks first raised many of the questions that logicians continue to grapple with today, and the work of the philosopher Aristotle provided the first formal analysis of reasoning. His studies of Logic were the standard for the discipline for over two centuries.

Reasoning has value because it moves both ideas and policy. At its best, the power of reasoning is due to the clarity and efficiency it lends to solving problems, discovering new truths, persuading others, and clarifying what we believe and why we believe it.

What is an argument?

Arguments, like men, often are pretenders. Plato

We often use the term “argument” to refer to the way we present our ideas and opinions. We think about debating, disputing opinions or even fighting with someone we disagree with. But in this course, an argument will be understood as an expression of our reasoning. When presented clearly and explicitly, an argument is simply a set of statements that have an internal dynamic in which one or more of the statements offer reasons or evidence for why another statement is true. All arguments have a structure that consists in at least two statements – a statement of reason or evidence, which we call a premise, and a statement of inference, or a conclusion, whose truth is claimed to follow from or be grounded in the evidence of the premise(s).

When we argue in our everyday lives, we use a natural, common or ordinary languages, like English, or Spanish, Hawaiian or Chinese. Ordinary languages are flexible and allow us to express ourselves in a focused and precise way, or to move freely and associatively from one idea to another without clear or obvious connection. When we present arguments in ordinary language we can choose words and phrases that presents our reasoning in a fragmented and confusing way, or we may choose to be clear and deliberate. Thus, our arguments can show our reasoning to be clear and precise, or poorly thought through and full of irrelevancies or fallacies.

To illustrate this, compare the following found in the “Letters to the Editor” section of the local paper with a more carefully crafted argument by conservationist, Aldo Leopold.

Have you noticed? Barack Obama is NOT wearing the American flag pin that President Bush, Vice President Cheney, Sen. John McCain and other true patriots would never appear in public without. On Feb. 25, the New York Times called our attention to that fact.

Think about it! Fidel Castro never had the American flag in his lapel. Few if any terrorists ever wear it.

All civilization is basically dependent upon natural resources. All natural resources, except only subterranean minerals, are soil or derivatives of soil.
Farms, ranges, crops, and livestock, forests, irrigation water, and even water power resolve themselves into questions of soil. Soil is therefore the basic natural resource. . . . It follows that the destruction of soil is the most fundamental kind of economic loss which the human race can suffer. — Aldo Leopold, “Erosion and Prosperity”

Both these passages present someone’s reasoning, but the choices made about what forms of sentences to use, what words and phrases to use, and how to arrange them impact how each argument can be understood. Broussard chooses a combination of rhetorical question (Have you noticed?), exhortation (Think about it!), fallacious appeals (to patriotism – true patriots, and to authority – the New York Times), statement of facts, exaggeration, and innuendo. His point is vague and his inferences ambiguous. We are urged to “think,” but what are is it we supposed to think? Given the statements he presents, what inferences are we to draw? Is the writer being straight or sarcastic? How can we identify the writer’s implications or the basis on which any inference may be grounded? We can even question whether the writer presents an argument at all.

Leopold, on the other hand, presents his inferences that “soil is the most basic natural resource” and the “destruction [of soil is] the most fundamental kind of economic loss” based on factual statements. He uses clear, simple language, and sequences his thoughts so that one connects directly to the next.

In ordinary conversation, we often present our reasoning clumsily and incompletely. We can be sloppy and careless in our choice of words. Sometimes we are vague or imprecise, focusing on the point we are trying to make, more than clarifying the reasons that would give it solid grounding. We may leave out important statements, assuming that our listeners or readers are thinking the same way we are. We may combine good, solid evidence with innuendo, name-calling, unsupported generalizations, or other fallacious approaches. Rather than supporting the point we are trying to defend, we can end up undermining it.

EVALUATING ARGUMENTS

Not all reasoning is good reasoning and not all arguments are good arguments. They can be strong or weak, effective or ineffective. However, if our goal is to present good quality reasoning, if we want to get our point of view across effectively, if we want to interpret and assess the quality of the reasoning we read and hear, it can be helpful to be able to assess the quality of arguments accurately. How we distinguish when an argument is good, and what makes it good, what standards and methods of evaluation we use in assessing the quality of arguments and the reasoning they communicate, in a major focus of the study of Logic

Three criteria used to evaluate arguments

When we evaluate anything, we judge its quality. We say it is good or bad. Argument quality can be judged as good or bad from three different perspectives. While these perspectives can overlap in a final analysis of a given argument’s quality, each can be considered independently.
Because the purpose of arguments is often to persuade others of the truth of the point we are arguing, we often evaluate an argument as good or bad based on whether or not it is persuasive. **Persuasiveness** concerns whether or not an argument actually persuades someone that the conclusion is true. This issue is subjective and psychologically driven. Consider, the example of a lawyer arguing the innocence of her client. In the final analysis, the client will be happy, if the lawyer persuades the jury in her favor. How the arguments are structured or even if they contain true statements will be less important. What will matter most is whether the lawyer successfully persuades the jury that her client is not guilty. If she is successful, it is likely her arguments will be judged as good; if she fails, the opposite.

Persuasive arguments need not be logically correct. In fact, humans can be quite easily persuaded by fallacious arguments. Arguments can incorporate flaws that rely on psychological or language tricks play on the fact that we often do not think too deeply about what we hear or even think. Logical **fallacies** are studied in as a part of informal logic. Many are given names, such as false cause reasoning, argument *ad hominem*, appeal to pity, slippery slope, red herring, hasty generalizations, and strawman arguments. There are dozens of such flaws and a good reasoner should be aware of them.

Arguments can also be flawed because they contain false statements. When we consider the truth or falsity of statements in an argument, we are evaluating it from the perspective of **content**. From this perspective we want to know whether or not the statements in the argument are actually true or false. We may be unaware that the statements are false, or we may believe they are in fact true. While truth value will play a role in the evaluation of arguments, from a strictly logical perspective logicians have no unique way of knowing whether any given statement is true or false. This requires knowledge of the subject matter or experience with the issue being argued about. If we lack this knowledge or experience, we can research the facts from reliable sources or appeal to the knowledge of experts, relying on a trustworthy, authority. Sometimes it is best to simply defer judgment on the matter of content because we have no special expertise. Rather than being a sign of weakness and ignorance, deferring judgment on matters we do not know anything about is a strength in critical thinking. Formal logic deals with truth value possibilities and has developed **truth tables** to address the need to consider the truth value possibilities of statements in arguments, but it does not take up the question of whether a given statements is actually, factually true or false.

Finally, we can judge the structure or form of an argument. In deductive logic, structure is the most important aspect of an argument and the deciding feature of its quality. Here, we consider the formal relationships that link the reasons or evidence given in the premise statements to the conclusion that they are said to support. If that structure is solid we can draw the conclusion forward, literally pulling it out of the premises. Such an argument is called **valid**. If the reasons or evidence offered do not support the conclusion that is being argued for, then we say the argument or reasoning is **invalid**. This level of flaw can be difficult to detect because an argument’s structure can be easily hidden or glossed over with clever or sloppy use of language.

Each of these three different facets of an argument can be considered separately, and each appeals to different standards or criteria of evaluation. Of the three, logicians are primarily interested in structure because quality at this level determines the foundational integrity of an
argument. To understand why logicians place such importance on argument structure we can compare an argument’s structure to the foundation of a house. We may be impressed with the outward appearance of a house but its integrity will be found in its foundation. If there are serious flaws at this level – if the plumbing is corroded and the foundation termite ridden – then no matter what it looks like, the house will not be judged to be worth the investment. The same holds for an argument. We may like the argument’s facade. We may feel it makes sense. We may agree with the statements it contains or be persuaded by the force of its presentation. But, if we find that the structure of the argument is flawed, the argument fails in a critical sense and does not present an example of good reasoning.

In deductive logic our first interest is in the structure of arguments. This structure is found in the relationships between the premise and conclusion statements. The premise statements should have a relationship that is strong enough to support the conclusion. The emphasis on argument structure means we must **ANALYZE BEFORE WE EVALUATE**. This encourages us to look at what the argument presents, what statements it contains, and what structure those statements show, before we judge the argument as good or bad. Because argument structure can be difficult to see, we have to look beneath an argument’s initial presentation. Logicians have developed tools that reveal the structure of arguments, and we will be learning how to use these tools in this course.

We will work primarily with simple argument patterns, to help us learn how we can analyze arguments and assess the quality of their structure. Our first step will be to break arguments into their separate statements, and identify how those statements function as either premises or conclusion. We will then learn how to use logical languages to reveal underlying patterns in argument structure.

<table>
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<th>Quick Review:</th>
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<tr>
<td><strong>Important Distinctions (be sure you can explain these):</strong></td>
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<tr>
<td>reason vs argument</td>
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<td>premise vs conclusion</td>
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<td>form vs content</td>
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<td>analysis vs evaluation</td>
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<td><strong>Be sure you can:</strong></td>
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<tr>
<td>Explain the subject matter of Logic</td>
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<tr>
<td>Define reason(ing) and argument</td>
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<td>Explain how reasoning and argument are related</td>
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<td>Identify some benefits to studying Logic and argument</td>
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<td>Explain why arguments can be difficult to evaluate</td>
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<td>Identify and explain the three perspectives from which an argument can be evaluated</td>
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ARGUMENT ANALYSIS – PRELIMINARY ISSUES:

If logic ever discusses the truth of factual sentences it does so only conditionally, somewhat as follows: if such-and-such a sentence is true, the such-and-such another sentence is true. Logic itself does not decide whether the first sentence is true, but surrenders that question to one of the other empirical sciences.

Rudolf Carnap, “Logic” in Factors Determining Human Behavior

Statements and Truth Value

Arguments are made up of statements and all statements express a truth claim. We speak in terms of truth value, and assume a statement can have a value of true or it can have a value of false. We present this basic assumptions using a truth table, and introduce a simple convention of symbolization: We will represent a statement using a single letter (p, q, or r). In the table below we will use the letter, p, to represent any given statement. We will represent the values, TRUE and FALSE, using the capital letters T and F, respectively. We organize the truth table into a single column with two rows.

<table>
<thead>
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<th>p</th>
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<tbody>
<tr>
<td>T</td>
<td></td>
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<tr>
<td>F</td>
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This simple truth table shows the two possible truth values for the statement represented by p. Here, “p” functions as a kind of variable in that it stands for, or represents any statement whatsoever. The first row of the table shows the possibility when p is TRUE, and the second row shows the possibility when p is FALSE. The table does not assert which of these possibilities is the actual, factual case about the statement, p. It simply presents all truth value possibilities for p. We limit the values we are considering to two and this conforms with our everyday reasoning. This logic is thus a binary or two-valued logic.

Premises and Conclusions

Every argument must have at least two statements. Depending on how these statements function in the argument they are called either premises or conclusions. “Premise” and “conclusion” are relative terms. Any statement may be a conclusion in one argument and a premise in another.

Premises are statements that provide the reasons or evidence why the conclusion statement is true. They are the logical starting point of the argument. They are assumed to be true and their truth is intended to support the truth of the conclusion. This is not to say that any given premise statement is necessarily true. In fact, any statement in an argument may be false, but in the context of the argument it is presumed that it is both true and unproblematic.

Although every argument must have at least one premise, there is no limit to the number of premises an argument may have. Some arguments are relatively simple. An immediate inference, for example, contains one premise and one conclusion. From the statement, “All humans are mortal,” I can directly, or immediately conclude that “I am mortal.” Other arguments are quite
involved and contain many premises. It is also possible to have compound arguments in which the conclusion of one argument in turn becomes the premise for further reasoning. In this way, chains of reasoning can develop into sophisticated arguments of great power.

**Conclusions** are statements whose truth is justified, or validated, by reference to the (assumed) truth of the premises. Conclusion statements are the logical end point of the argument. Generally, the truth value of the conclusion is more problematic than the premises. When we argue, from a logical point of view, we provide reasons that offer evidence (empirical or sense based, or logical) to justify the truth of the conclusion, or why we should believe a conclusion statement to be true.

However, an argument does not necessarily establish the actual truth of the conclusion. An argument *attempts* to establish the truth of the conclusion by referring to the truth of the premises. However, there are two distinct ways in which an argument may fail to establish the truth of the conclusion. It may be that one or more of the premises are in fact false statements. This is a flaw in the content of the argument. Or it may be that the premises are not structured strongly enough to support the truth of the conclusion even when the premises are in fact true. This would be a flaw in the argument’s structure.

**Statements vs Sentences**

*Every sentence is significant . . . but not every sentence is a statement-making sentence, but only those in which there is truth or falsity. There is not truth or falsity in all sentences: a prayer is a sentence but is neither true or false.*

_Aristotle, De Interpretatione_

Arguments are made up of statements. Statements are truth claims and all truth claims have a truth value. This means they are either true or false. Statements are expressed through sentences. Sentences are grammatical units of meaning that begin with a capital letter and end with a punctuation mark. There are four different forms of sentences: declarative sentences, exclamations, interrogatives (questions) and imperatives (commands). Only declarative sentences present statements. In everyday arguments, we often use a variety of sentence forms – questions and even imperative forms – declarative sentences are less ambiguous, and therefore, less likely to be misinterpreted or dismissed as irrelevant, vague or confusing. For the sake of clarity, statements are best expressed using declarative sentences.

To better distinguish statements from sentences we can think about the difference between content and form. The content is the meaning. It can be expressed using different forms. We can understand the form (the sentence) as the collection of words we use to express the statement’s content, or specific meaning. For example, the sentence, “LA beat NY by 18 points,” conveys a meaning that can be communicated by different sentences, such as “NY lost to LA by 18 points,” or “LA was victorious over NY by a 18-point margin,” or “LA over NY by 18!” While the words differ, the meaning stays the same.

Declarative sentences *declare* the statements they convey are true, and it makes sense to question the truth value of such sentences. Consider the sentence: “Joshua is twenty-one.” It is
reasonable to consider whether this simple sentence conveys a true or a false statement. If Joshua is in fact twenty-one (which can be verified by looking at his birth certificate), then the statement is true, but if he is nineteen, the statement is false. It is a matter of common sense that Joshua cannot both be twenty-one and not be twenty-one at the same time. The same is true of such statements as, “Honolulu is on Oahu” and “All libertarians are anarcho-capitalists.” They are either true or false and cannot be both at the same time.

Declaring truth value is just one purpose that sentences have. Exclamations, questions (interrogatives), and commands (imperatives) are different sentence forms and each has a specific function in language. For example, when we form a sentence like, “How old is Joshua?” our intention is to request information. We do not expect someone to reply, “False.” The form of an interrogative sentence communicates its purpose. We use a different tone of voice, and in writing, different punctuation. Likewise, with a command or imperative sentence form, such as, “Hand in your homework every week,” or an exclamation such as, “Watch out!” our purpose is not to address truth value, as we do with declarative sentences, but to give direction or express emotion.

Because it is not the purpose of these types of sentences to present truth claims, responding to them with “true” or “false” makes no sense. Yet, with argument we hope to derive true conclusion statements based on premise statements that are presented as true. Being clear in our intention makes it easier to serve our purpose, but it also makes it easier for others to assess and evaluate the quality of our reasoning. Being able to understand when a clear and complete truth claim has been presented, and what it means, is fundamental to good reasoning.

**Exercise** : Identify each of the following sentences by its type.

1. Please, turn off all cell phones, pagers and any electronic devices that may disrupt the class.
2. Citizens in a democracy have a moral obligation to become informed before they vote.
3. Are you crazy?
4. Your grade point average does not transfer when you go to another college.
5. Oh, my god!
7. There is a hurricane 120 miles southeast of the Florida Keys.
8. I do not like dogs.
9. Are human beings really logical?
10. Oops!
11. Abortion is an abject evil.
12. Do your homework!
13. Hawai‘i became the fiftieth state in 1950.
14. When was the last time you traveled outside the country?
15. Hooray!
16. Turn in all paperwork by three p.m. on Friday afternoon.

**Quick Review**

**Important Distinctions:**

sentence vs statement  
premise vs conclusion

**Be sure you can:**

Explain the concept of truth value  
Explain why a declarative sentence is the best choice for presenting  
Identify the two ways in which an argument can fail.
Simple vs Compound Sentences

It is important to distinguish a sentence from a statement when we combine separate logically simple statements into a single compound statement. Both simple and compound statements have truth value. The difference is that the truth value of a logically compound statement is dependent on the truth value of the simple statements they contain.

A simple statement is a truth claim that cannot be broken down or analyzed into a simpler claim. To create compound statements we link simple statements with logical operators. In ordinary language, logically compound statements are expressed using an expression of negation (such as “not” or “never”) or one of several coordinating conjunctions (such as and, but, however, or, yet, if - then, only if, etc.). These words are used to express how we intend one simple statement to relate to another. The logical meaning of the compound, and hence, the kind of conclusion or inference that can follow from it, depends on the meaning of the choice of coordinating conjunction that links simple statements.

For example, consider the two simple statements: “I need to take Math” and “I need to take Logic.” Do not consider the truth value of these statements. Think only of the fact that these are two separate simple statements. We can link these two statement using different coordinating conjunctions to create three different compound statements, each with a distinctly different meaning.

- I need to take Math and I need to take Logic.
- I need to take Math or I need to take Logic.
- If I need to take Math, then I need to take Logic.

Each of these compound statements is composed of the same two simple statements but each has a different logical meaning. The coordinating conjunctions used give them different logical forms, and changes the situations under which the compound statements are true and false.

We use these types of compound statements in our everyday reasoning and in several common types of arguments. For example, “Joshua can get a driver’s licence only if he is 16 years old,” is a single statement that combines the two simple statements: “Joshua can get a driver’s license” and “Joshua is 16 years old.” In the sentence, “Either Joshua is 16 years old or he is 15 years old,” there are also two simple statements. By themselves these statements are not arguments. However, when we add the statement, “Joshua is not 16 years old,” we create a context that allows us to reason, or draw forward an inference.

Example 1: Either Joshua is 16 years old or he is 15 years old.
Joshua is not 16 years old.

In this context, we can draw the conclusion that “John must be 15 years old.”

Example 2: Joshua can get a driver’s licence only if he is 16 years old.
Joshua is not 16 years old.

In this context, we can draw the conclusion that “John cannot get a driver’s licence.”
Each of these pairs of statements presents enough information for us to draw a third statement forward. Based on the information in each pair, we can draw an inference, or a conclusion. When we draw an inference, we are taking in the information communicated in a statement or set of statements, and reason that another statement must “follow” as true. It is not the specific content of the statements that allows this to happen, however. It is the relationship between the two statements that draws our reasoning to a conclusion.

Our minds seem to be hardwired to recognize patterns of logical structure between statements. Some but not all of those patterns are logically valid and it can be difficult to distinguish the valid ones from the invalid ones.

**Exercise: The following pairs of statements have a structural relationship. Which do you think have a relationship that supports a specific conclusion being drawn?**

| 1. Either this statement is simple or it is compound. It is not simple. |
| 2. No dogs go to heaven. Fido is a dog. |
| 3. All dogs have floppy ears. Fido is a dog. |
| 4. If tax revenues continue to fall, then state revenues will not be able to fund social services at current levels. Tax revenues are indeed falling. |
| 5. Hawai‘i can win grants from the federal government, only if it accepts national standards. Hawai‘i is not accepting national standards. |
| 6. Every statement in an argument is either a premise or a conclusion. This statement is not a premise. |
| 7. Only if Alison passed Logic last semester, will she be able to graduate. She did not pass Logic. |
| 8. Public workers can accept 5% pay cut; alternatively, the governor can begin laying off workers. The governor did not begin public worker layoffs. |
| 9. Barack Obama will not win a second term if he doesn’t get a deal on the debt ceiling. He got a bad deal on the debt ceiling. |
| 10. Provided that you pay your bill in full by the date due, no interests, fees, or charges will be accrued to your account. You paid your bill in full. |

**Recognizing premises and conclusions**

All arguments are composed of statements. When simple arguments are clearly presented, their separate statements are easy to identify. Sometimes each statement is expressed by a single declarative sentence and there is some sort of language, a word or a phrase, that identifies its function as either a premise or conclusion. But just as often, in our day to day conversations and even in written form, this clarity is lacking. Ordinary discourse takes place within specific contexts where much is left unsaid because the speakers or readers share a common understanding. So, while practice with well-structured arguments can help us understand the nature and structure of arguments, careful attention, as well as listening and reading skills are also necessary to analyze and evaluate the kind of arguments we encounter in everyday language.

Argument analysis begins by separating out the different statements contained in an argument and identifying the role each plays as either premise (or part of a premise) or a conclusion (or part of a conclusion). One reason arguments can be difficult to analyze is that there are no specific rules for how to construct an argument. How we present our reasoning is a choice. It
is often a matter of rhetorical style or personal preference. We can present our conclusion first and then follow it with the premises we feel give support to that conclusion. But we can just as easily present the premises first, followed by the conclusion statement. A third option would be to present a premise or two, then state the conclusion, and follow it with more premises.

A second reason argument analysis can be challenging is that the relationship between premises and conclusions is a relative one. This means that the statements that make up an argument will function as premises or conclusions in relation to the given argument context. The same statement can be a conclusion in one argument and serve as a premise in another. This allows us to build chains of reasoning in which simple arguments compound to build a complex context of reasoning. The more complex the chain of reasoning, the more difficult it can be to clarify what role a given statement may be playing at any logical point in the argument.

A third reason that ordinary language arguments can be difficult to analyze is that many times we do not present them fully. Sometimes we leave out the conclusion and sometimes we leave out a premise. However, as we saw above, if the relationship between statements is strong enough, and stated clearly enough, we can infer an unstated conclusion, or even an unstated premise. For example, if I know that “All full time students must carry a minimum of twelve credits,” and I also know “Sara is a full time student,” I can easily conclude “Sara must be carrying at least twelve credits.” The relationship between the statements given is strong enough for me to draw a valid inference.

There are cases when a conclusion is stated along with one premises, while a pivotal premise is left unexpressed because the arguer assumes we will be able to fill it in. For example, if I can conclude, “Sara needs a license” from the statement “Sara will be driving.” I do not need to explicitly state the universal premise, “All drivers need a license.” It is understood.

Such argument structures are called enthymemes. They are frequently used in everyday reasoning because the relationships between the statements that are expressed allow for our minds to easily supply what is missing and complete the argument. To better understand this issue of unexpressed statements within an argument, consider the examples below where the conclusion is implied but not explicitly stated. Can you supply the missing conclusion?

A. No one who is less than twenty-one can legally buy alcohol. Joshua is less than twenty-one.

B. Anyone who runs for president of the United States must be a natural born citizen. Arnold Schwartznegger is not a natural born citizen.

C. Mary will either take the car or ride the bus, but she can take the car only if Mom doesn’t need it for work. Mom does need the car for work.

In the three examples above, the pairs of sentences have a relationship that is strong enough for us to draw forward a necessary conclusion. In other contexts a conclusion and one premise are given but the full context that supports the conclusion involves a second, unexpressed or implicit premise. The following examples illustrate this. Can you supply the missing general premise?
A. Alex is a lousy driver. Look! He doesn’t even signal!
B. You shouldn’t eat that fish. It’s rotten.
C. You are not a student. You cannot attend this meeting.

Enthymemes are ubiquitous in our everyday thinking patterns. The examples above show that we do not need to “hear” every statement in the argument to be able to identify a reasonable inference. However, the less explicitness, the more difficult it will be to complete an argument structure by supplying the missing statement and analyze the full argument structure. The more difficult it is to analyze a set of statements that presents an argument, the more it becomes to determine whether or not we have a good argument. The stronger the relationship between the statements expressed, the easier it is to complete the full reasoning context. The weaker the relationship, the more difficult it is. Dividing arguments into their parts, supplying any missing part(s), and identifying how those parts relate to each other are necessary first steps in argument analysis.

**Indicator language:**

*There are in every language some words whose sole function is to indicate form.*

Bertrand Russell, *Introduction to Mathematics*

All natural languages have terms that function to indicate how we intend to relate statements to one another. In arguments there are words and phases that indicate premise statements and conclusion statements. Words and phrases like “because,” “therefore,” “so,” “for these reasons,” tell us something about the relationship between and among a set of statements. They give us clues for interpreting how statements function in the context of an argument. We will use the term *indicator language* to identify language that can help us identify when a statement functions as a conclusion and when as a premise. Words such as “therefore,” “thus,” “so” are common conclusion indicators, while words such as “because,” “since,” “for” and phrases such as “for the following reasons” can indicate a premise will follow.

While indicator language can be helpful, we cannot rely entirely on the presence or absence of any particular word or phrase when judging the role of a statement in a passage that may be an argument. Nothing demands we use indicator language when we present an argument. If no language clearly identifies the function of a statement in an argument context, we can consider which statement is the more problematic. This statement is likely the conclusion. We validate the conclusion by appealing to the truth of statements that are more likely to be accepted as true, and offered as premises. This judgement, in turn, requires a general knowledge of the subject being argued, and if we lack knowledge, we may not be able to determine which statements are assumed to be true and which is being argued for.

Reading skill and general knowledge are important. General knowledge is important in order for us to know what is objectively true or false. The better our reading skill the more sentence patterns and conventions of language we are likely to be familiar with. This familiarity helps us anticipate the intention of a given passage. Words have meaning in context and a careful reader will recognize context and be able to judge meaning more accurately. A word like, “since,” for example, has a completely different meaning in the statement, “I haven’t seen you since last Thursday” than it has in the argument, “This meat should not be eaten, since it has been sitting on the counter all night long.” Likewise, the word “for” functions as a preposition in the sentence, “I
bought this car for eight hundred dollars” and does not indicate a premise in such a context. In addition, applied effort is required to understand how the sentences we read and hear are intended to relate to one another. Often the intention is something that can only be read “between the lines.” It is more implicit than explicit.

Below is a partial list of some common patterns using indicator language to differentiate premises from conclusion where an argument is being presented. Consider what the words mean and understand that in non argument contexts the same words may have a different meaning. Use these words to help you become a more critical reader.

<table>
<thead>
<tr>
<th>Premise Indicators</th>
<th>Conclusion Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Conclusion) because (premise)</td>
<td>Therefore, (conclusion)</td>
</tr>
<tr>
<td>Because (premise), (conclusion)</td>
<td>So, (conclusion)</td>
</tr>
<tr>
<td>(Conclusion) since (premise)</td>
<td>We can conclude . . .</td>
</tr>
<tr>
<td>Since (premise), (conclusion)</td>
<td>In conclusion . . .</td>
</tr>
<tr>
<td>(Conclusion) for the following reasons (premises)</td>
<td>Thus, (conclusion)</td>
</tr>
<tr>
<td>The reasons are (premises)</td>
<td>From (premises) it follows that (conclusion)</td>
</tr>
<tr>
<td>Given that (premise) . . .</td>
<td>We can infer (conclusion)</td>
</tr>
</tbody>
</table>

Argument Analysis:

In the exercise that follows use the models given in 1 and 2 as a method to analyze arguments throughout the course.

- First, always read the argument to the end.
- Then separate out each sentence in the argument.
- Consider if some sentences contain multiple statements that need to be separated.
- Never separate “if-then” and “either-or” statements. These are logically compound statements that are making a single truth claim.
- Present your final analysis listing each premise on a separate line, followed by the conclusion.
- Remove any indicator language and replace pronouns for clarity.

Exercise: Analyze the following simple arguments, using the procedure outlined above. 1 and 2 are presented to you as models for how to present your own work. (Answers to problems 5, 7, and 9 can be found on p. 67.)

1. If Obama was born in Kenya, then he is not a US citizen. He was not born in Kenya. Therefore, he is US citizen.
   - Premise: If Obama was born in Kenya, then [Obama] is not a US citizen.
   - Premise: [Obama] was not born in Kenya.
   - Conclusion: [Obama] is US citizen. / “Therefore” identifies the conclusion

2. Professional athletes will continue to use performance enhancing drugs because these athletes will use anything that enhances their performance as long as winning remains the goal of athletic competition. And everyone knows that winning will always be the goal of athletic competition.
Premise: [Professional] athletes will use anything that enhances their performance as long as winning remains the goal of athletic competition.

Premise: Winning is the goal of athletic competition.

Conclusion: Professional athletes will continue to use performance enhancing drugs.

“because” divides premise from conclusion

3. Using illegal drugs is immoral, because it is against the law, and breaking the law is immoral.

4. A college education is a good investment. This can be inferred from the fact that having a college degree nearly doubles one’s earnings over a life time. Furthermore, a broader education enhances individual happiness. In addition, a wider range of learning allows greater life choices and opportunities.

5. Evolution is a viable theory if it explains more of the phenomena it is intended to explain than any other competing theory. Evolution indeed explains more about the diversity of the natural world than any other theory. The conclusion is clear: evolution is a viable theory.

6. I have a duty to do whatever I want to do. The reasons are simple: I have a duty to do what is right and I have the right to do whatever I want to do.

7. If God exists as a supernatural and omnipotent being, then God can prevent all evil. If God exists as an all good being, then God would want to prevent evil. But evil exists. The only possible conclusion is that either God is not a supernatural, omnipotent being, or God does not want to prevent evil.

8. Evolution is a viable theory only if it explains every detail and question we have about the natural world. Clearly evolution cannot do this. Therefore, evolution is not a viable theory.

9. The United States needs to keep Guantanamo open. The argument is simple: We need to have a place to keep those who might threaten our interests. To bring them into the United States would allow them access to the rights accorded prisoners in our court system. This would be unacceptable. So, if no other country wants them, we are left with no other choice.

10. Global warming seems to be the result of human activity for the following reasons: Global temperatures have increased steadily since the beginning of the industrial age. In the last 50 years, concentrations of carbon dioxide have increased exponentially, corresponding with increased use of fossil fuels. At the same time, more volatile weather patterns have become more frequent.

11. If we extend health care to insure everyone, more people will be treated for heart attacks. When more people are treated, the cost of caring for them will increase. Rising health care costs will mean the cost to American taxpayers will increase. Therefore, if we extend health care to everyone, the cost to American taxpayers will go up.

Thought exercise: Argument Analysis vs Argument evaluation:
Consider which of these arguments are good arguments and which are bad arguments. What factors enter into your decision?
TWO BASIC TYPES OF ARGUMENTS

We can distinguish two types of arguments, *inductive* or *deductive*. As arguments, both contain statements where premises provide reasons in support of the truth of a conclusion. Both reflect a process of reasoning, drawing of an inference from evidence. These two types of argument differ with respect to the way the evidence of the premise(s) bears upon the certainty of the conclusion, and each type of argument has its own criteria for assessing the quality of its reasoning.

**Inductive Reasoning**

An inductive argument is an argument whose premise statements support the conclusion with some degree of *probability*. This is to say that if the premises are in fact true, then the conclusion follows as more or less likely. The degree of likelihood follows from the extent and quality of evidence presented. For an inductive argument to be considered good or strong, the evidence offered in the premises needs to ensure a high likelihood that the conclusion will follow. The reasons presented in the premises should be sufficient (give enough evidence), relevant (be directly related to the subject matter of the conclusion), and true (factually verifiable).

An inductive argument presents an open-ended context of inference. While the premises provide grounds for drawing a conclusion forward – and in a strong inductive argument that support will be able to assure the truth of the conclusion with a high degree of probability – even in a good inductive argument the structure of its reasoning will leave open the possibility that the conclusion could be false. For example, the fact that I have put the key into the ignition of my car many times, and I have been able to turn the engine over, provides me with good grounds to reason that putting the key into the ignition of my car this morning will turn the engine over. Indeed I make this inference every time I put my key into the ignition and turn it, but I cannot conclude with absolute certainty that my car will start this time, and it may be that one day I do so and my car fails to start. The failure of my car to start, (the falsehood of my conclusion in this case), does not diminish the quality of my reasoning.

This fact about inductive arguments should not lead us to conclude that inductive arguments are specious or weak. As British philosopher and logician, Bertrand Russell pointed out, “All the important inferences outside logic and pure mathematics are inductive, . . . the only exceptions are law and theology, each of which derives its first principles from an unquestionable text, viz. the statute books or the scriptures.” Law bases its arguments in stipulated rules and religion assumes universal truths. The limits provided by written law or religious beliefs, direct our reasoning within a specified scope. As long as that scope is assumed, conclusions can be drawn with the certainty required by deductive reasoning.

Scientific reasoning, on the other hand, bases its arguments on tentatively presented hypotheses and works with evidence that is empirically verifiable (available to our sense perceptions). It adapts its conclusions to newly discovered evidence and therefore demands flexibility. As an example, consider how we predict the landfall of a hurricane. Many factors enter our reasoning as premises. These include facts about air pressure, the size of the storm, the temperature of the atmosphere and the water, the movement of various currents, the presence of landmasses. The list goes on. Different computer models offer different predictions, providing probable conclusions as to where and when the hurricane will make landfall. These models adjust or change their conclusions as they factor in new information, or premises. We base our conclusions regarding when and what areas to evacuate based upon these premises. Even if we have the most accurate information possible, the nature of the phenomenon about which we are reasoning is such that our predictions, our conclusions, may be wrong. This is not to denounce inductive reasoning as inferior. It is to recognize when and why probable knowledge is the best we can achieve.

The quality of reasoning in an inductive argument is based on a scale of weak to strong. The factors we consider include the **TRUTH** of the premises, their **RELEVANCE** to the conclusion, and the **SUFFICIENCY** of the evidence they provide to support the conclusion. We can improve the quality of the argument by adding more true and relevant premises, or by deleting irrelevant ones. But a feature of inductive reasoning is that we may end up with a false conclusion from a set of true, relevant and sufficient (as possible at the time) premises.
Deductive Reasoning

Deductive reasoning is distinguished from inductive reasoning by the intended support that the premises provide the conclusion. Deductive arguments present a context of reasoning within which the premises are intended to offer certain and absolute support for the truth of the conclusion. I emphasize “intended” because there are two ways in which a deductive argument may fail its intention to present a true conclusion from a given set of premises. It may fail because the structure of the argument is flawed. Here, the relationships between the premises do not, in fact, provide sufficient support to ground the conclusion. Such an argument is considered to be logically invalid. In this context, invalid is a technical term, referring specifically to a structural flaw in the argument. It is also true that a deductive argument may have a flawless or valid structure, yet have one or more false premises. In either case, the argument is not a good one and is considered to be unsound.

The standard demanded by deductive reasoning is high. We expect the conclusion to be fully and clearly justified by reference to its premises, to follow from those premises with absolute certainty. A deductive argument is making the claim that if the premises are true, the conclusion is necessarily, undeniably true. Several values support the high standard set for deductive reasoning. They include precision, explicitness, transparency, and clarity. Because ambiguity opens up questions of interpretation and further debate, we should avoid ambiguity at all costs.

The strongest feature of a deductive argument is the formal relationship that exists between its premises. It is this relationship that determines whether or not the premises provide a solid ground to support the conclusion, and allow it to be drawn forward. In a deductive argument, the relationships between the premises, and not their content, determines whether or not an argument is valid, or structurally strong. We will consider different types of deductive argument structure to familiarize ourselves with some of these basic patterns.

**Syllogisms** are one category of deductive arguments. A syllogism is a simple deductive argument structure that has exactly two premises and a single conclusion. Different types of syllogisms are distinguished by the type of statements contained in the premises. Each type of syllogism has a clear pattern that makes it easy to analyze and identify, although not all syllogisms are valid. Their patterns must be studied carefully to distinguish those that are valid from those that are fallacious or structurally flawed.

One common type of syllogism is called a disjunctive syllogism. A disjunctive syllogism is structured around a disjunctive statement given as a premise. A disjunction is a statement that presents alternatives. In English we usually use the coordinating conjunction “or” to construct disjunctions. An example of a disjunction would be, “I will take Math or I will take Logic.” Another example would be, “I will pay for medication or for food.”

From a disjunctive statement alone, we cannot validly draw a conclusion. But if we have a second statement that relates to the disjunction statement we may be able to derive a necessary conclusion. Consider the following example:

Premise 1: *Either* I will take Math *or* I will take Logic.
Premise 2: I will *not* take Logic.
Conclusion: I will take Math.

This form of argument has a strong structure. We can recognize this form in many other arguments, or even in our own reasoning. We can abstract, or separate the content of the argument and bring the form of this type of syllogism into the foreground by focusing on logically significant language.

Premise 1: *Either* A *or* B.
Premise 2: *Not* A.
Conclusion: B.

Showing the structure of the argument in this way, lets us see the intended structure clearly. We use capital letters A and B to symbolize the statement content of the argument. The symbols we choose to represent the content are arbitrary. We can choose any symbols we like, as long as we consistently use a single symbol for each statement. We retain the words (either-or, not) that link the simple statements because it is this language that carries the logic of the argument. This method of using symbols to show structural relationships between statements in an argument is one technique logicians use to analyze arguments.
Showing form in this way will let us recognize more easily when arguments share a common form. It also lets us see when the form of one argument differs from the form of other arguments. Consider the following example:

Either the battery of my car is dead or else the regulator needs replacing. The battery is dead. So, I know the regulator does not need replacing.

We can analyze the argument as follows. First, we separate premises from conclusion:

Premise: Either the battery of my car is dead or else the regulator needs replacing.
Premise: The battery is dead.
Conclusion: The regulator does not need replacing

Then, we use letters as statement symbols to show the argument’s form:

Premise 1: Either A or B.
+ Premise 2: A.
Conclusion: Not B.

While these two arguments are similar, analyzing them and showing their form, shows clearly that they have different forms. We will learn techniques that will let us show how one of these forms is a valid form and the other is not.

Hypothetical Syllogisms

Another important category of simple syllogisms is based on hypothetical or conditional statements. We use the coordinating conjunction “if–then” to express these types of statements. Like a disjunction statement, a conditional statement is not, by itself, an argument, but when combined with another statement a conditional statement can form a hypothetical syllogism. Some of these structures present good, valid arguments and some present flawed or fallacious arguments. Logic studies these structures and uses a variety of tools and techniques to identify well structured from poorly structured arguments. Consider the following argument:

If the Lakers re-sign Lamar Odom, then they will retain their versatility and their team chemistry. If they retain their versatility and team chemistry, they will continue to be competitive and will challenge for the 2010 NBA championship. So, if the Lakers sign LO, they’ll again challenge for NBA championship in 2010.

Again, we follow the process of breaking the argument into its separate statements and labeling whether the statements function as premise or conclusion.

Premise: If the Lakers re-sign Lamar Odom, then [the Lakers] retain their versatility and their team chemistry.
Premises: If they retain their versatility and team chemistry, [then] they will continue to be competitive and contend for the 2010 NBA championship.
Conclusion: If the Lakers sign LO, [then] they’ll again be 2010 championship contenders.

Using the same method of substituting a symbol for the simple statements in the argument we can show the form of this syllogism as follows. This time we use capital letters as statement symbols:

Premise 1: If A then B.
+ Premise 2: If B then C.
Conclusion: If A then C.

Again, it is easier to see the form of the argument when we replace its specific statement content with symbols. This lets us see more easily how the premises relate to one another, and what terms in the premises come into the conclusion. But while syllogisms all present a clear structure, not all forms of syllogisms present good, structurally good arguments. Consider the following:

If we are to be successful in Afghanistan, [then] we need to increase a military presence.
If we are to control the Taliban who are moving inside of Pakistan, [then] we need to increase
our military presence.

Therefore, if we are to be successful in Afghanistan, then we need to control the Taliban moving inside of Pakistan.

This argument is similar to the first argument. All the statements in the argument are hypothetical or conditional (if-then) statements. Within these if-then statements, we can see that two of those statements come out of the premises and are joined in the conclusion. Finally, we can see that there is one statement common to both premises that does not appear in the premises. But if we replace the individual simple statements with symbols and retain the “if-then” language that gives the argument its logical structure, we can see the difference in the two argument patterns more clearly.

Premise 1: If A then B.

+ Premise 2: If C then B.

Conclusion: If A then C.

Other hypothetical syllogisms use the “if-then” statement as a primary premise. A second premise then addresses either the “if” part, or the “then” part of the hypothetical statement. The argument pattern known as Modus Ponens is a widely recognized pattern of reasoning. We can see this pattern in the following examples:

A. If I brought my keys with me, then they are in my bag. I did bring my keys with me. So, my keys are in my bag.

B. If we want universal health care for our citizens, then we will need to either pay more in taxes, or limit the range of health care we offer. We do want all our citizens to have access to health care. Therefore, we will either pay more in taxes or limit the range of health care we offer.

C. If Obama has a Hawai‘i birth certificate, then he is a US citizen. He does have such a birth certificate. Therefore, Obama is a US citizen.

These three arguments all share the form called Modus Ponens. Again, using capital letters to replace the simple statements in the argument we can show the form of Modus Ponens.

Premise 1: If A then B.

+ Premise 2: A.

Conclusion: B.

Another common form we find in our reasoning introduces a negative statement in relation to a conditional premise. Examples of this form of argument include the following:

A. If Phil 211 is going to be offered in the Fall semester, it must have at least 15 students. It does not have 15 students. Therefore, the class will not be offered.

B. If the governor can legally implement furloughs for state workers, she must negotiate them with the unions. She did not negotiate them with the union. So, she cannot legally implement them.

C. If you have the flu, you would have fever, body aches and chills. You don’t have these symptoms. So, I would argue that you don’t have the flu.

These three examples share the common form called, Modus Tolens. It is a negative form of this hypothetical argument. Abstracted the form looks like this:

Premise 1: If A then B.

+ Premise 2: Not B.

Conclusion: Not A.

These simple syllogistic forms appear frequently in our argument, but we can fail to recognize them beneath the words and rhetoric of ordinary language. This is because there are so many different ways of
expressing the “if-then” relationship in English. Also, as with other forms of syllogisms, there are similar patterns that are easily mistaken for similar argument forms that are poorly structured and therefore fallacious.

We have identified two basic types of argument, inductive and deductive, and shown how it is possible to analyze them. We have also shown that it is possible to abstract the form of arguments from their content. Finally, we have shown that different arguments can share a common form, and introduced a simple method for showing the form of simple deductive arguments.

**Exercise**: Analyze each of the following arguments, identifying premises and conclusion. Then try to show the form of the argument. (Answers to some odd problems p. 67.)

1. Jan doesn’t pay for her parking pass because if she is a faculty member, then she does not pay for a parking pass. It is known that Jan is a member of the faculty.

2. The argument is either valid or invalid. The argument is not invalid. Therefore, it is valid.

3. If whales swim in the ocean then they are fishes. If they are fishes then they have gills. So, if whales swim in the ocean, then they have gills.

4. Either the speaker has power or he has no power. If he has power then he should be able to get the votes to pass the bill. He couldn’t get enough votes. So, it seems a simple conclusion that the speak has no power.

5. Either the class fills to 50 percent capacity or it will be dropped. The class was dropped. Therefore, it did not fill to 50 percent capacity.

6. If God is a man, then God is not a god because if God is a man, then God is not supernatural and if God is not supernatural, then God is not a god.

7. If life forms are fixed and unchangeable, then the theory of evolution is false. But life forms are not fixed and unchangeable. So, evolution is true.

8. This argument is not a syllogism because if it were, then it is a deductive argument. But it is not a deductive argument.

**Three more challenging arguments:**

1. Democracy is based on principles of open public discussion and transparent government. Yet, open public discussion of our surveillance methods will leave us vulnerable to attack by terrorists because it gives them information as to what those methods are. It is a sad but necessary conclusion that democracy leaves us vulnerable to terrorist attacks.

2. In order to enroll in the college, I must first get a health clearance. I can get a health clearance only if I have had a valid TB test within the past year. In addition if I was born after 1957 I need an MMR vaccination. I was born in 1983 and although I had a valid TB test last year, I have not had an MMR vaccination. So, I can’t enroll in the college right now.

3. The United States is an empire. The evidence for this is that it maintains a large standing military presence in foreign countries to protect its national interests. If any country maintains such a presence, then it counts as an empire.
Quick Review: Do you know . . .

- Key distinctions: how are inductive vs deductive reasoning/argument distinguished?
- can you explain how they are evaluated?
- can you give an example of each?
- What is a syllogism?
- different forms of syllogism
- Can you analyze an argument?
- what specific steps are involved
- how should you present your work

**Six arguments from philosophers:** Can you see underlying deductive patterns in the following arguments?

1. Either wealth is an evil or wealth is a good; but wealth is not an evil; therefore wealth is a good.  
   Sextus Empiricus

2. If a man could not have done otherwise than he in fact did, then he is not responsible for his action. But if determinism is true, it is true of every action that the agent could not have done otherwise. Therefore, if determinism is true, no one is ever responsible for what he does.  
   W. Nesbitt and S. Candlish, “Determinism and the Ability to Do Otherwise”

3. That which is evil does harm; that which does harm makes a man worse. But pain and poverty do not make a man worse; therefore, they are not evil.  
   Seneca

_In these arguments the conclusions are not stated, but are implied. After you isolate each statement, it is easier to decide what the conclusion must be._

4. When names are not used properly, language will not be used effectively; when language is not used effectively, matters will not be taken care of; when matters are not taken care of, the observance of ritual propriety and the playing of music will not flourish; when the observance of ritual propriety and the playing of music will not flourish, the application of laws and punishments will not be on the mark; when the application of laws and punishments are not on the mark, the people will not know what to do with themselves.  
   Confucius, The Analects

5. - Either the boy has at some time acquired knowledge (of geometry) which he now has, or he has always possessed it. If he always possessed it, he must always have known. If on the other hand he acquired it some time previously, it cannot have been in this life, unless somebody has taught him geometry. . . . Has anyone taught him these things?
   - I know that no one ever taught him. . . .
   - Then, if he did not acquire them in this life, isn’t it immediately clear that he possessed and learned them during some other period.  
   Plato, The Meno

6. If concepts are not clear, words do not fit. If words do not fit, the day’s work cannot be accomplished, morals and art do not flourish. If morals and art do not flourish, punishments are not just. If punishments are not just, the people do not know where to put hand or foot.  
   Confucius, Analects, XIII,3

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23
FORMALIZING ARGUMENT

Because language is misleading, as well as because it is diffuse and inexact when applied to logic . . . logical symbolism is absolutely necessary to any exact or thorough treatment of our subject.

Bertrand Russell

[The woof and warp of all thought and all research is symbols, and the life of thought and science is the life inherent in symbols; so it is wrong to say that a good language is important to good thought, merely; for it is the essence of it.] Charles Sanders Pierce

In order to avoid the inadequacies of the natural languages for the purposes of logical analysis, it is necessary first to translate into a more exact notation.

Alonzo Church

Formal vs. Informal Argument

The distinction between formal and informal argument has to do with the way in which arguments are presented. In our day-to-day reasoning, we reason and argue informally. This simply means that we present our reasoning in ordinary, everyday, or natural language. Natural languages are also called informal, ordinary or common languages. They are the languages we speak. They are rich and complex, allowing us to express the fullest range of our thoughts and feelings. They develop organically to serve the different needs we have to communicate.

In our everyday speech it is unnecessary and even inefficient to be absolutely precise and explicit in everything we mean or say. We use natural languages to respond to novel situations that haven’t been experienced or spoken of before, accommodating new and diverse levels of expression. Both at the level of structure (syntax) and vocabulary (semantics), natural languages evolve to meet the changing needs we have to express ourselves. When we argue informally, we may not say everything we could say, or we may say more than we need to. We use our tone of voice to emphasize points. We may use vague or ambiguous language. We make use of different sentence types, and our statements are often fragmentary and implicit rather than explicitly stated.

We distinguish ordinary, natural languages from formal, artificial or symbolic languages, which are developed to represent a specific facet of a phenomenon precisely, consistently, clearly and unambiguously. The formal language of mathematics is the most common example of an artificial language. It uses a unique set of symbols (for example, the symbols of arithmetic (+, -, x, ÷ and =) to represent relationships of quantity. Programming languages offer a more contemporary example of formal languages. They serve the purpose of representing strings of code that instruct computers to respond in specifiable ways. Music notation can be seen as a kind of formal, symbolic language developed to represent organized sounds.

Like all languages, formal languages are composed of basic elements. They have a set of symbols that are given meaning and there are formation rules that indicate how symbols can be arranged to produce meaningful statements. A key advantage of artificial languages is the time saved by representing only those features that are of concern and bringing them into the foreground. Rather than writing out every statement in an argument, the logical languages we will learn are types of formal languages. They will function like a shorthand, recording only what is necessary to allow us to assess an argument’s structural quality. A further advantage is that these languages are available to speakers of many different natural languages. As a formal language, music allows musicians each speaking a different natural language, to understand a common piece of music because they are all able to understand the language in which the score is written. Similarly, scientists, logicians and mathematicians across the globe are able to work on common problems because they have knowledge of the same formal languages.

In Logic, the purpose of formal, logical languages is to bring the logical form of statements and arguments into the foreground. Formal, logical languages allow us to present the argument’s form clearly and without embellishment. They allow us to isolate the structure of the statements in the argument from the particular content and the elements of natural language that can be distracting. We can easily be distracted, for example, by the rhetorical power of natural language, by its emotive content, or by the persuasive elements of voice. Words and phrases can be used for psychological effect. We can also be distracted by whether or not we personally believe a given statement is true or false, and thereby judge the quality of the argument based on a
subjective impression rather than an objective assessment. When we react to, and base our judgments about the quality of an argument on its superficial features, we may neglect the structure that really ties the argument together, and so fail to see some inherent flaw. Once an argument’s logical structure is exposed, there are a number of tools that logicians use when they wish to test an argument’s integrity.

Beginning with Aristotle, philosophers have represented the formal features of arguments, but modern symbolic logic is a recent science, developing only during the last century. Two artificial languages, **Sentential** and **Predicate**, helped to spur important developments in the field giving logicians tools for representing and testing their ideas. **Sentential** is a basic language that allows us to represent the logical relationships that exist **between** statements. This language allows us to analyze and evaluate a wide variety of truth-functional arguments. A truth-functional argument is one in which the logic of the argument depends on the relationships between simple and compound statements, as these are expressed using coordinating conjunctions. But many arguments important arguments depend on the logical relationships that exist within simple statements and, therefore, require another level of analysis. **Predicate** provides a more powerful language tool, capable of representing a broader range of arguments in considerable detail. It shows clearly and precisely the logical relationships that exist **within** statements.

**Sentential** and **Predicate** contain only a handful of simple elements, but as with any language, simplicity builds into complexity quickly. The way the basic elements of each language are put together into meaningful expressions is determined by a simple set of formation rules. Formal rules (of inference) show when and how inferences can be drawn forward from given statements (in other words, how conclusions can be drawn out of premises). In Logic we use these languages specifically to expose the formal features of arguments in order to evaluate more easily the structural integrity of an argument. (Remember the house analogy?) As logicians we will play the role of structural engineers inspecting the underpinnings of arguments to determine if the supporting structures (the premises) are really able to hold up the conclusion.

**SENTENTIAL: Simple and Compound Statements**

We begin our study of **Sentential** by revisiting the distinction between **simple and compound statements**. A simple statement is a statement that has no other statement as a component part. For example, *John is home*, is a simple statement. It is both grammatically simple and logically simple. It is logically simple because it does not contain anything further that is asserted as a statement. Another example of a simple statement is, *Some caterpillars turn into butterflies*. This simple statement has a more involved grammar but it is still a logically simple statement because it cannot be analyzed into anything simpler that would still be a statement. Remember that every statement declares a truth value. So, “John is home” and “Some caterpillars turn into butterflies” are both presented as having a value of TRUE and there is no way to analyze any component within them as having a truth value.

Simple statements can be quite grammatically involved. For example, the statement, “The derelict car that has been parked across the street for more than a week is being towed by the City and County,” is a logically simple statement. It makes a simple claim about a car being towed. The fact that it contains a lot of words is not an issue. The only issue is whether or not it be analyzed into anything simpler that would still be a statement.

In Sentential, simple statements are distinguished from **compound statements**. Compound statements combine simple statements and logical operators. They are called **truth-functional compound statements** because their truth values are determined by the truth value of the simple statement or statements within them and how those relate, or are mediated by the **logical operators** in the compound. For example, *John is home but Mary is at work* is a compound statement. It is compounded of two simple statements: *John is home*, and *Mary is at work*. The truth value of this compound statement will depend on the truth value of the two simple statements it contains, and the logical meaning expressed by the word, “but.” *Some caterpillars turn into butterflies and some turn into moths* is also a compound statement. It can be analyzed into the two simple statements: *Some caterpillars turn into butterflies*, and *Some (caterpillars) turn into moths*. Its truth value depends upon, or is a function of, the truth value of those simple statements, and the logical operator represented by the word, “and.”

Whether or not a statement is logically simple or logically compound has to do with whether or not it can be broken down or analyzed into simpler statement components. The two basic components of a compound
statement are simple statements and logical operators. So, a statement such as *Elisha is not a student* is a compound of the simple statement, *Elisha is a student*, and the logical operator expressed by the word, *not*. The statement, *Joanna believes that she is living in a dream*, is also simple. It makes a claim about what Joanna believes. It cannot be broken into the two simple statements: *Joanna believes* and *Joanna is living in a dream*.

When analyzing statements, we need to be cautious and not divide them into units that cannot be presented as simple statements. We should also not divide them into units that are not themselves statements. Sentence fragments – noun phrases, verb phrases, and modifying phrases – are units that are too small and do not count as basic simple statements. A simple statement must have a subject term and a predicate term. It must be able to stand as a complete statement.

**Types of Compound Statements**

There are five types of compound statements: **negation**, **conjunction**, **disjunction**, **simple conditional** and **bi-conditional**. The table below shows each type of statement, along with the most common English word, or phrase, that expresses the logical operation.

<table>
<thead>
<tr>
<th>Example</th>
<th>Type of Statement: Identified by Logical Operator</th>
<th>Primary English language expression for this logical operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colette plays shortstop.</td>
<td>Simple</td>
<td>no logical operator</td>
</tr>
<tr>
<td>Colette does not play shortstop.</td>
<td>Negation</td>
<td>“not”</td>
</tr>
<tr>
<td>Colette plays shortstop and Fred is the catcher.</td>
<td>Conjunction</td>
<td>“and”</td>
</tr>
<tr>
<td>Colette plays shortstop or Fred is the catcher.</td>
<td>Disjunction</td>
<td>“or”, “either . . . or”</td>
</tr>
<tr>
<td>If Colette plays shortstop, then Fred is the catcher.</td>
<td>(simple) Conditional / Material Implication</td>
<td>“If . . . then”</td>
</tr>
<tr>
<td>Colette plays shortstop if, and only if Fred is the catcher.</td>
<td>Bi-conditional / Logical equivalence</td>
<td>“if, and only if”</td>
</tr>
</tbody>
</table>

All compound statements in natural language can be analyzed, or broken into, component simple statement(s), and a word or phrasing that indicates the logical operator. The examples in the table above use the simple statements, “Colette plays shortstop” and “Fred is the catcher.” The choice of simple statements is entirely arbitrary and there are an infinite number of statements that we could use as examples. Simple statements each have an independent truth value. When they are combined with logical operators, the truth value of the resulting compound statement will vary, depending on the truth value of the original simple statement or statements and the way in which the specific logical operator (negation, conjunction, disjunction, etc.) mediates truth value.

Let’s explain this more fully. Logical operators work to mediate the truth-value of simple statements. It is easy to see how this works in the case of negation. If we know that the simple statement “Colette plays shortstop” is a true statement, it is a matter of common sense that the negative statement, “Colette does not play shortstop,” is false. The negation mediates or alters the truth value of the original claim. However, if the simple statement, “Colette plays shortstop” is false, then the resulting negative statement “Colette does not play shortstop,” will be true. So, the truth value of the compound statement is contingent upon the value of the simple statements and the logical operator that connects them.

**Formation Rules for Sentential**

A simple statement in Sentential is symbolized by either a single capital letter (*A*–*Z*), or a lower-case letter *p, q, r*, etc. These are called **statement symbols**. Capital letters will be used to translate actual sentences from natural language, and lower-case *p, q, r*, etc. will represent the general form of a statement and will
function much like variables. Thus “p” will represent any simple statement whatsoever, whereas “A” will represent a given English language statement. Its meaning will be given by a language key.

Statement symbols combine with logical operators to form compound statements. The symbols for logical operators that we will use in this course are the following: ~ (negation), • (conjunction), v (disjunction), ⇒ (simple conditional), and ⇔ (bi-conditional). As with any language there are rules for ordering the elements of the language to produce meaningful statements. For example, in English, we cannot combine words in any way whatsoever. To say, “Car fast on drives the road the” makes no sense, whereas, “The car drives fast on the road,” is perfectly understandable. The same is true with logical languages. A properly formulated statement in a symbolic language is called a well-formed formula, or a wff, for short. The rules for combining statement symbols with operators to produce meaningful statements in Sentential are simple and direct.

### Formation Rules for Sentential

1. All single letters are wffs.
   
   In other words, all single letters (lower and upper case) will represent statements.

2. If p is a wff, then ~ p is a wff.
   
   This rule lets us form negative statements. The negation is understood to negate the statement that directly follows it. Punctuation is used to indicate the scope negation.

3. If p and q are wffs, then (p • q), (p v q), (p ⇒ q), (p ⇔ q) are also wffs.
   
   This is the rule for the formation of compound statements. Note that the operator symbols (with the exception of the negation symbol) are always placed between two statements. Compound statements can be compounded to create more complex statements.

4. Nothing else is a wff.

This simple set of formation rules allows us to form meaningful statement strings using only statement symbols and logical operators, and punctuation marks. Punctuation marks let us identify clearly the main logical operator in a complex statement, one that combines statement symbols with more than one logical operator.

It is important to understand that “p” and “q” function like variables and can be substituted with any wff whatsoever, simple or compound. Therefore, the statements “A” and “B” may be substituted for “p” and “q,” respectively, but we also substitute for “p” any compound statement such as “A v B” or “C ⇔ D.” The relationship at the level of the main logical operator (in this case the conjunction operator) is the important focal point.

The following are all different substitution instances of the conjunction, p • q. In each the conjunction symbol (the dot) is the main logical operator.

<table>
<thead>
<tr>
<th>A • B</th>
<th>Where A is substituted for p, and B is substituted for q.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A v B) • C</td>
<td>Where A v B is substituted for p, and C is substituted for q.</td>
</tr>
<tr>
<td>(A v B) • (D v E)</td>
<td>Where A v B is substituted for p, and D v E is substituted for q.</td>
</tr>
<tr>
<td>¬ ¬ A • ¬ (B ⇒ C)</td>
<td>Where ¬ ¬ A is substituted for p, and ¬ (B ⇒ C) is substituted for q.</td>
</tr>
</tbody>
</table>

In the work that you will be doing throughout this course, the concept of a substitution instance is
critical. To use this concept, a few things are important. First, every truth-functional compound can be identified by its statement type, where its statement type is identified by its main logical operator. Thus every compound statement will be either a negation, a conjunction, a disjunction, a conditional or a bi-conditional. The main logical operator will be the logical operator that is not enclosed in punctuation. A statement will only be a negation if that negation exists outside of all punctuation, or it is negating a single simple statement.

**Exercise:** Identify the type of statement for each by its main logical operator. Show the basic statement for which each is a substitution instance. (Answers to odds on p. 68)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A v B</td>
<td>11.</td>
</tr>
<tr>
<td>2.</td>
<td>~ (A v B)</td>
<td>12.</td>
</tr>
<tr>
<td>3.</td>
<td>(A v B) ⊃ C</td>
<td>13.</td>
</tr>
<tr>
<td>4.</td>
<td>~ (A v B) ⊃ C</td>
<td>14.</td>
</tr>
<tr>
<td>5.</td>
<td>(C • D) ⊃ (A v B)</td>
<td>15.</td>
</tr>
<tr>
<td>6.</td>
<td>C • D</td>
<td>16.</td>
</tr>
<tr>
<td>7.</td>
<td>C • (D ⊃ ~B)</td>
<td>17.</td>
</tr>
<tr>
<td>8.</td>
<td>~ C • (D ⊃ ~B)</td>
<td>18.</td>
</tr>
<tr>
<td>10.</td>
<td>~ (A ⊃ C)</td>
<td>20.</td>
</tr>
</tbody>
</table>

**Quick Review:** Do you know . . .

**Key distinctions:**

- formal (logical) vs informal (ordinary language) arguments?
- Sentential vs Predicate
- simple statement vs compound statement

The basic elements of the language of Sentential?
The five logical operators and the corresponding symbols used in this course?
How to form a meaningful statement (wffs) in Sentential?
How to identify statements by their main logical operator?
Do you understand the concept of a substitution instance?
DEFINING LOGICAL OPERATORS

Deductive logic intends to reason from true premises to reach conclusions that are necessarily true. Therefore, precision and clarity are required. Thus far we have introduced logical operators through ordinary language, but the degree of precision that deductive reasoning demands a more precise definition. We will use the formal, logical language called Sentential and truth table to define precisely each of the five logical operators.

The meaning of each operator will be stipulated with a unique truth table. These tables show a logically minimal meaning and do not necessarily reflect all that the English language expressions mean. The defining truth table for each operator will “fix” the meaning of the operator, by distinguishing how it mediates the truth value of simple statements. Truth tables serve the values of deductive logic by being clear and unambiguous. We begin with the negation operator, which simply changes the value of the statement that follows it.

Negation

The symbol we will use for the logical operation of negation is ~ (a symbol called a tilde). This symbol will have the meaning indicated by the truth table below. To create a truth table to illustrate the meaning of “~” we expand the simple truth table we considered earlier (see p. 8 above) by adding a column to the right for the expression, ~ p. The first column, headed by “p” shows the two possible truth values for the statement represented by “p.” Remember that “p” represents any statement whatever. The second column, headed by ~ p, shows how that original truth value of “p” changes when we place a negation operator in front of it.

<table>
<thead>
<tr>
<th>p</th>
<th>~p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

This table shows how the negation operator mediates the truth value of the statement represented by “p.” When “p” is true “~ p” is false, and when “p” is false “~ p” is true. This is what we mean when we say that negation is truth-functional. This stipulated meaning for the symbol “~” corresponds with our ordinary language use of the concept of negation.

Each of the four remaining logical operators will be defined in the same way, using a truth table to illustrate how the operator mediates truth value. This will stipulate a limited meaning for that operation, and that truth table will be considered the definition of that logical operation. We will now take up each of these operators in turn.

Conjunction

The conjunction operator corresponds closely with the English word, “and.” In ordinary language, however, there are many expressions and even modes of punctuation that we will reduce to a logical conjunction, and some expressions of “and” do not present logical conjunctions. So, we must be careful when we encounter the word “and” and let the context help us to determine whether or not we are dealing with a truth-functional conjunction statement.

Every conjunction relates two statements. These statements are called conjuncts. We use a dot symbol “•” to represent this operation. The truth table expands upon the one above to accommodate the fact that two statements are related. Each conjunct requires a column, and a third column is required to show the value of the conjunction. Not only does the table add columns, we will need more rows in order to generate all the possible truth value combinations for two separate simple statements. This can be done in exactly four rows as shown below.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p•q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
In this table $p$ and $q$ represent the two *conjuncts*. The third column shows how the conjunction operator mediates their truth value possibilities. The table shows that the conjunction can be true only when both conjuncts are true. This is illustrated in row one of the table. If either conjunct is false, the resulting conjunction will be false. This is illustrated in rows two, three and four. The logical meaning of the conjunction operator corresponds loosely with our common sense understanding of the term “and” in English. If I claim, for example, “The president of the United States must be thirty five years old and a citizen of the United States,” I mean it is true that the president of the United States must be thirty five years old, and it is true that the president must be a citizen of the United States. If I am wrong about the truth value of either or both statements, my claim is false.

It is important to understand the conjunction operator does not capture all that we mean in each and every English language usage of the word, “and,” or any of the other English terms that express logical conjunction. The dot symbol specifically represents a truth-functional conjunction and it is defined by, and means only what is represented in the truth-table above. I can claim, “I am a part-time student but Leilani is full time.” This claim reduces logically to “I am a part-time student and Leilani is a full-time student.” I choose to use the term “but” to emphasize a contrast, but this emphasis has no truth-functional significance. The English words “but,” “yet,” and “however” have more nuanced connotations than the term “and;” however, truth-functionally they all collapse into the minimal meaning conveyed by the dot symbol.

Not only does the dot symbol not capture all that we communicate in many of the words we use to express logical conjunctions, not every use of these terms implies a logical conjunction. When translating natural language statements into symbolic notation, we must always be attentive to the possibility of ambiguity. Ambiguity can be an asset in ordinary discourse. Where the context is generally understood, explicit clarity is neither required nor desired. But when the goal is to determine what does or does not follow with necessity, as is the goal of deductive reasoning, any ambiguity is a hindrance and to be avoided. Therefore, we must carefully analyze and identify the simple statements contained in any truth functional compound statement and distinguish the intended relationship between them. Focusing simply on particular words or phrases can lead to mistakes.

For example, consider the statement, “Alexis and Alex are married. The word “and” may cause us to think that it is a truth-functional conjunction. If this is so, we should be able to identify the simple statements that are its two conjuncts. When we try to break the statement apart we discover an ambiguity. Does it mean that Alexis and Alex are married to each other, or that they are married independently? A correct interpretation is often clear in context, but in isolation it can be difficult, if not impossible to know what is intended. My interpretation will be directed one way if my reasoning occurs in this context: Oh, I wonder if I should ask Alexis (or Alex) to go with me to Maui on a romantic holiday. It will be directed in an entirely different way if I am reasoning in this context: I need to find Marvin’s father. Marvin is Alexis’s son. Alexis and Alex are married. So, Alex must be Marvin’s father. How I read “Alexis and Alex” will depend entirely on the context in which this statement occurs.

Careful reading is a necessary part of the skills we must apply when we begin to work with logical languages. It takes some thought to know how to move from ordinary speech into symbolic notation. If we are rendering a statement in isolation, we will default to the minimal meaning the statement can support. If we are rendering arguments, we must consider the entire argument context first before we make a judgment about the type of statements within a given context. We cannot rely on specific words or phrases to tell us how a given statement functions, or what type of statement it may be. While certain words and phrases can give us clues, we need to slow down and look for reinforcing clues before we can be secure in our judgment, particularly where there is ambiguity or significant complexity. This requires that we read critically and carefully, and be able to justify our analysis by referencing what is stated.
Disjunction

The disjunction operator corresponds closely with the English word, “or” or the phrase “either - or.” A vee symbol, “v,” or “wedge,” is used to represent this logical operation. The precise meaning of this symbol is presented in the truth table below.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p v q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Again, in this table “p” and “q” represent different statements. The third column shows how the disjunction operator mediates the truth value possibilities of these two statements, which are called disjuncts. The table shows that a disjunction is true when at least one of its disjuncts is true. This is illustrated in rows one through three. A disjunction will only be false if, and only if both disjuncts are false. This possibility is shown in row four.

Two types of disjunction

We distinguish two types of disjunction in English, an **inclusive disjunction** and an **exclusive disjunction**. Both are expressed using the same language but they have different logical meanings. I call the exclusive disjunction, “the-soup-or-salad ‘or’” because most of us are familiar with the meaning of the statement, “You may have soup or salad with your meal.” We know this statement means that we may have soup or we may have salad but we may **not** have both. One choice excludes the other. This is the meaning of an exclusive disjunction.

If we study the truth table above, we can see that this is not the meaning conveyed by the vee symbol, which represents an inclusive disjunction. An inclusive disjunction allows for the inclusion of both p and q, as illustrated in row one of the table. Therefore, we will understand the disjunction operator, symbolized by the “v” to be an inclusive disjunction. Then, using the symbols we have at this point, we can to symbolize an exclusive disjunction. Since the exclusive disjunction means “one or the other, but not both” we can express this meaning using the “v,” with the “·” and the “¬.” The resulting formula will look like this: (p v q) · ¬ (p · q). This formula reads in English as: “p or q, but not both p and q.”

As with the conjunction we can see that when we express disjunctions in ordinary, natural languages (in this case English) ambiguity is possible. Often context clarifies the intended meaning, but sometimes this is insufficient, and we must ask for clarification. For the purposes of this course, when we encounter an “or” statement, we will assume an inclusive disjunction, if it is logically possible for both statements to be true. We will assume an exclusive disjunction only if it is obviously so. Obvious examples of exclusive disjunctions include such statements as: “The moon is either full or it is new,” “The Lakers or Golden State will win the Pacific Division title,” “Cheney was either the Vice-President or the President of the United States.” In these statements it is clear that only one of the alternatives is possible at any given point in time. To symbolize exclusive disjunctions we will use the formula, (p v q) · ¬ (p · q).

**Exercise:** Identify the following statements as inclusive or exclusive disjunctions. Assume only the minimum of whether one possibility excludes the other.

1. The moon is either full or it is new.
2. I will buy pizza or potatoies.
3. Either the Yankees or the Dodgers will win.
4. We can cut taxes or decrease revenues.
5. This flower is either a rose or a plumeria.
6. I will bake a souffle or a quiche.
7. We can go to dinner or a movie.
8. Joe will marry either Jack or Jill.
9. Today is either Tuesday or Thursday.
10. The Jets or the Steelers will win.
Conditional / Material Implication

The *conditional operator* corresponds with the English “if - then” phrasing. This operator is also called *material implication* or a *hypothetical*. The horseshoe “ $\rightarrow$ ” represents this logical operation and the meaning of this symbol is presented in the truth table below.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

We use the terms *antecedent* to refer to the left-hand statement ($p$), and *consequent* to refer to the right-hand statement ($q$). This terminology applies only to the simple conditional. The table above shows clearly that the only time a conditional statement is false is when the antecedent statement ($p$) is true and the consequent ($q$) is false. This is shown in row two of the table. In all other cases a conditional statement is true.

The meaning given for the horseshoe symbol does not correlate strongly with our common sense understanding of the ordinary language meaning contained in many “if-then” statements. Particularly, rows three and four do not correspond easily to our intuitive expectations. A conditional statement is about the relationship between the antecedent and consequent statements, and not so much about the truth value of these two components. A conditional statement asserts that when the antecedent is true, the consequent is also true. It does not assert that the antecedent is true, and to conclude so is a fallacy.

“*If*” is the operative word here, and to help us more comfortably grasp the way logicians have defined this operator, I play a game: *When Am I a Liar?*

*When Am I a Liar: A Game*

Suppose I meet four strangers A, B, C and D, and to each of these strangers I say, “If I shake your hand, I will give you $100.00. Now consider the following situations which correspond to the four rows of the truth table above:

Row 1: I greet stranger A with a handshake, and I give A a crisp new one-hundred dollar bill.
Row 2: I greet stranger B with a handshake, but I do not give B a one-hundred dollar bill.
Row 3: I do not greet stranger C with a handshake, but I do give C a one-hundred dollar bill.
Row 4: I do not greet stranger D with a handshake, and I do not give D a one-hundred dollar bill.

Clearly, strangers A and D would have no grounds to justify calling me a liar. Given the conditions stipulated in my statement, *If I shake your hand, then I will give you one-hundred dollars*, each got only what they could expect. Stranger B, would surely call me a liar because the statement I uttered is certainly false, as any witness would testify. Stranger C, I would contend, might be perplexed, but I don’t think C would call me a liar. (He’d be too happy thinking about how he could spend the money.) So, the only case in which I am undeniably a liar is represented in row 2 with stranger B. My claim, “If I greet you with a handshake, I will give you $100.00,” is clearly false in this case.

Logical operators have a narrow, specifically defined, truth-functional meaning. With respect to the conditional, many different meanings collapse into this horseshoe symbol, which reflects only a *minimal logical meaning* of “if-then” statements, namely, that they are all false when the antecedent is true but the consequent is false.
Bi-conditional

The **bi-conditional** operator presents a **logical equivalence**. A logical equivalence asserts that both statements in the equivalence have the same truth value. I call such statements “fortified conditionals.” They are expressed by the English phrase “*if, and only if.*” The three-bar logical equivalence symbol, “≡” is used to represent this logical operation, and the precise meaning of this symbol is presented in the truth table below.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ≡ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The table shows that when “p” and “q” have the same values (rows one and four) the value of the bi-conditional statement is true. Where they have different values (rows two and three) the bi-conditional is false.

The bi-conditional abbreviates the longer formula, \((p \equiv q) \cdot (q \equiv p)\), in which p and q imply each. In ordinary language as well, we often abbreviate the full meaning of the bi-conditional, using the language of a simple conditional, when the stronger bi-conditional is implied. Consider the following two statements, which share a common form, if \(p\) then \(q\) but which carry different implications.

A. *If* you are carrying 12 credit hours, **then** you are a full-time student.

B. *If* it is a dog, **then** it is a mammal.

Statement A (*If you are carrying 12 credit hours, then you are a full-time student*), relates the two simple statements (“You are carrying 12 credit hours” and “You are a full-time student”) quite differently from statement B. Even though it uses the same “if - then” phrasing, statement A carries the implication that “if you are a full-time student, then you are carrying 12 credit hours,” AND “if you are carrying 12 credit hours or more, then you are a full-time student.” Statement A implies an “if, and only if,” relationship even though it is not explicitly stated. This is not true with statement B. Statement B is a “simple” conditional. It states, “*if it is a dog, then it is a mammal.*” It does not imply “if it is a mammal, then it is a dog.” In the case of statement A, if I know you are carrying 12 credit hours, I can infer that you are full-time. And if I know you are full-time, I can infer that you carry 12 credit hours. With statement, A, if I know “it is a dog,” I can infer “it is a mammal,” but if I know “it is a mammal,” I do not necessarily know “it is a dog.”

The contrast between the meanings of the stronger bi-conditional statement and the weaker conditional, illustrates several important points. One, it reinforces the contrast between natural languages, with their inherent ambiguity, and the need to represent the logical meaning of a statement in a way that is precise and unambiguous. In ordinary, context rich situations we often do not need such precision to understand what is being said, but in some contexts, (for example, the law courts or in areas of science and math), explicitness is valued and there is a need to eliminate as much ambiguity as possible. Secondly, the example shows why these two logical operators are different, in that different conclusions can be drawn from them. Thirdly, it illustrates that the bi-conditional is an abbreviation for the conjunction of two simple conditionals in which the antecedent of one is the consequent of the other, and vice versa.

This completes the introduction of the basic elements of Sentential. The table below summarizes each of the five operators, their corresponding symbols, alternative symbols used (which you may see in other places, in other texts or on the internet), common English language expressions for each operation, and a symbolization for these basic statement forms.
### Summary of Logical Operators – Symbols and English Equivalents

<table>
<thead>
<tr>
<th>Logical Operation</th>
<th>Symbol</th>
<th>Alternate Symbols*</th>
<th>Common English Language Equivalents</th>
<th>English Examples</th>
<th>Basic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negation</td>
<td>( \sim )</td>
<td>( \neg, , \overline{\sim} )</td>
<td>not, never, none, etc.</td>
<td>I will not go.</td>
<td>( \sim p )</td>
</tr>
<tr>
<td>Conjunction</td>
<td>( \cdot )</td>
<td>( \land, , &amp; )</td>
<td>and, however, but, yet, also, both, whereas, additionally, etc.</td>
<td>I will go and you will go.</td>
<td>( p \cdot q )</td>
</tr>
<tr>
<td>Disjunction</td>
<td>( \lor )</td>
<td>( \lor )</td>
<td>or, either - or, unless, alternatively</td>
<td>Either I will go or you will go.</td>
<td>( p \lor q )</td>
</tr>
<tr>
<td>Conditional or Implication</td>
<td>( \Rightarrow )</td>
<td>( \rightarrow )</td>
<td>if - then, only if, implies, entails, means, causes is necessary for is sufficient for, causes, when, etc.</td>
<td>If I go, then you will go. I go only if you go. You go, if I go.**</td>
<td>( p \Rightarrow q )</td>
</tr>
<tr>
<td>Bi-Conditional</td>
<td>( \equiv )</td>
<td>( \leftrightarrow )</td>
<td>if and only if, is necessary and sufficient, is logically equivalent to</td>
<td>I will go if, and only if, you will go.</td>
<td>( p \equiv q )</td>
</tr>
</tbody>
</table>

* Alternate symbols may be found in other texts and on internet.

**Note the inverted form of the conditional statement in English and how it is symbolized.
APPLICATION: TRANSLATING USING LOGICAL OPERATORS

This work with truth tables has hopefully allowed you time to become comfortable using the basic elements of Sentential. Now we want to use Sentential to translate arguments whose structure depends on truth-functional relationships between simple statements.

When we translate from English into Sentential or from Sentential into English, we use a language key. A language key tells us the specific meaning of given statement symbols. By reducing the statement content to a single letter we are left with skeletal representation of the argument’s structure. Any features having to do with that content are available in the language key, but do not impose themselves in this symbolized form. Symbolizing arguments in this way allows us to focus exclusively on their form.

In presenting an argument in symbolized form, we will adopt the convention of presenting the premises first in a sequence (separated by commas if there is more than one). After the all the premises have been listed, we show the conclusion. At this point we introduce the symbol for the conclusion, which is a three dot pyramid or \( \because \). This symbol will be placed after the premises have been listed and directly before the conclusion statement.

The following example is presented as a model for how to take an argument from a basic analysis through to its complete symbolization in Sentential.

First, and always, read the argument to its end:

If Jan is faculty then Jan can park in Lot A. We know that Jan is faculty. Therefore, we can conclude that she can park in lot A.

Step 1: Analyze the argument by isolating its separate statements, and identify whether the statement is functioning as a premise or a conclusion. Use indicator language, if there is any, to ensure your analysis is correct, but do not include that language when you present the individual statements in the argument. Also, replace pronouns for clarity and accuracy.

P. If Jan is faculty, then Jan can park in Lot A.
P. Jan is faculty.
C. [Jan] can park in Lot A.

Step 2: Build a language key: Let B = Jan is a faculty member.
C = Jan pays for parking.
D = Jan parks in Lot A.

Step 3: Show the argument form. This requires abstracting the content, which is kept in the language key and using logical operators to represent the logical relationships between simple statements.

P. If B, then D
P. B
C. D

Step 4: Translate/symbolize the argument into Sentential

P. B \( \supset \) D
P. B
\( \because \) D
Here is a more complicated example. But the steps are exactly the same.

First, read the argument:

Jan must pay for parking because she is either a student or a faculty member. If she is faculty then she does not have to pay. However, if she is a student then she either pays or she doesn’t park in Lot A. Jan parks in lot A and she is not a member of the faculty.

**Step 1**: Analyze the argument, identifying which statements are premises and which are conclusions. Present each statement on a separate line with the premises coming first and the conclusion last. Replace pronouns for clarity.

P [Jan] is either a student or [Jan is] a faculty member.
P If [Jan] is faculty, then [Jan] does not have to pay.
P However, if [Jan] is a student, then [Jan] either pays or [Jan] doesn’t park in Lot A.
P Jan parks in lot A and [Jan] is not a member of the faculty.
C Jan pays for parking.

Before building a language key, it can be helpful to highlight the different simple statements in the argument and the logical language that will be symbolized with logical operators.

**Step 2**: Build a language key: Let $A = \text{Jan is a student.}$ $B = \text{Jan is a faculty member.}$ $C = \text{Jan pays for parking.}$ $D = \text{Jan parks in Lot A.}$

**Step 3**: Show the argument form: (Do this step on your own)

**Step 4**: Translate /symbolize the argument:

\[ P A \lor B \]
\[ P B \rightarrow \neg C \]
\[ P A \rightarrow (C \lor \neg D) \]  It is not necessary to translate “however” in this case. Punctuation identifies the MLO.
\[ P D \rightarrow \neg B \]
\[ \therefore C \]

This example gives you a simple step-by-step method that takes an argument from a basic analysis through to its complete symbolization. Using this method will help you produce consistent and accurate translations of English arguments into symbols.

**Exercise A**: First do a basic analysis of each argument. Then use the language key given above to translate the following arguments from English into Sentential. (Answers to evens p. 71-72)

1. Jan is either a student or she is a member of the faculty. She is not faculty. Therefore, Jan is a student.

2. Jan cannot park in parking lot A because if she is a student then she can park in lot A only if she pays. Jan is a student but she does not pay.

3. If Jan can park in parking lot A and she is faculty, then she does not have to pay for parking. Jan parks in lot A and she pays for parking. So, she is not a faculty member.

4. Either Jan pays for parking or she doesn’t. If she pays for parking then she is a student. If she doesn’t pay for parking she is either faculty or she doesn’t park in Parking Lot A. It seems an obvious conclusion that either Jan is a student or if she doesn’t pay for parking, then she is faculty.
5. Jan doesn’t pay for her parking pass because if she is a faculty member, then she does not pay for a parking pass. Jan is a member of the faculty.

6. Jan is a student if, and only if she is not faculty. If she parks in parking lot A and pays for her parking, she is a student. She parks in parking lot A but does not pay. So, Jan is faculty.

**Exercise B:** Use the same language key to translate the symbolized arguments below into English. The separate premises in the argument are separated by a comma. A backslash divides the conclusion statement from the premises.

1. \( A \lor B, \sim A, /:: B \)
2. \( B \supset D, D \supset \sim C, /:: B \supset \sim C \)
3. \( D \supset \sim C, A \lor B, (D \supset \sim C) \supset \sim A, /:: B \)
4. \( D \equiv (C \lor B), D \sim B, /:: C \)

5. \( [B \lor (A \cdot C)] \supset D, \sim D, /:: \sim C \lor \sim B \)
6. \( A \equiv \sim C, C \lor B, /:: B \supset \sim A \)
7. \( \sim A \cdot \sim B, (B \lor A) \supset D, /:: \sim D \)
8. \( C \equiv B, \sim B \supset D, D \lor \sim C, /:: A \)

**EXERCISE C: MORE PRACTICE**

Translate the following statements into English from Sentential, using the language key provided. Translate each logical operator.

**Language Key:** Let \( C = \) Slow Jo Crow goes; \( R = \) Rose grows; \( M = \) Mo goes

1. \( C \supset (R \cdot M) \)
2. \( (M \cdot R) \equiv C \)
3. \( M \supset (R \cdot C) \)
4. \( (M \lor C) \supset R \)
5. \( \sim [(M \cdot C) \supset R] \)
6. \( C \supset (\sim R \cdot \sim M)* \)
7. \( \sim R \supset (C \lor M) \)
8. \( \sim (\sim R \supset M) \)
9. \( \sim \sim M \)
10. \( R \cdot \sim M \)
11. \( \sim (R \cdot M) \)
12. \( \sim R \lor \sim M \)
13. \( (C \equiv R) \cdot \sim M \)
14. \( (M \cdot R) \supset \sim C \)

* In Logic the \( \sim \) does not distribute. You should also translate each \( \sim \) as a separate expression of negation. Example: \( \sim (\sim M \cdot \sim R) \) / Translation: It’s not true that Mo doesn’t go and Rose doesn’t grow. This applies to #6 and beyond.

**Arguments:**

15. \( (M \lor R) \lor \sim C, C \supset M, \sim M /:: R \cdot M \)
16. \( C \supset (R \cdot M), (M \cdot R) \equiv C, \sim C /:: \sim R \lor \sim M \)
17. \( \sim R \lor \sim M, \sim R \supset (C \lor M), R /:: C \equiv M \)
18. \( \sim R \supset (C \lor M), \sim (R \cdot M) /:: C \)
19. \( \sim [(M \cdot C) \supset R], C \lor R /:: \sim M \)
20. \( C \supset (\sim R \cdot \sim M), C \cdot M /:: \sim R \)
### Summary of Logical Operator Definitions and Truth Table Rules

<table>
<thead>
<tr>
<th>p</th>
<th>~ p</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>~ changes the truth value of whatever</td>
</tr>
<tr>
<td></td>
<td></td>
<td>statement comes directly after it</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ( \land ) q</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>A conjunction is true just in case both conjuncts are true.</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>OR A conjunction is false just in case at least one conjunct is false.</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ( \lor ) q</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>A disjunction is true just in case at least one disjunct is true.</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>OR A disjunction is false just in case both disjuncts are false.</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ( \rightarrow ) q</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>A conditional is false just in case the antecedent (left side) is true and the consequent (right side) is false.</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ( \leftrightarrow ) q</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>A bi-conditional is true just in case both sides have the same truth value.</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>OR A bi-conditional is false just in case both sides have a different truth value.</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>
TRUTH TABLE APPLICATIONS

Truth tables were first introduced to explain the basic assumption of our binary logic. Above we saw how they are used to define the meaning of logical operators. We will now develop techniques and methods for using truth tables to determine the truth value of given statements and to determine when different statements have the same truth value.

First, we will learn how to calculate the value of compound statements when we know the truth value of the simple statements involved. Second, we will calculate the value of statements when we know the values of some but not all the simple statements involved. We will then learn how to calculate the value possibilities for statements when the values of all simple statements are unknown using a “full truth table method.” Finally, we will learn how to determine when different statements are logically equivalent, or have the same truth value possibilities.

APPLICATION I: SUBSTITUTE & CALCULATE METHOD

When determining the truth value of statements in a binary logical system, there are only three possibilities. Either a statement can be determined to be true, or it can be determined to be false, or we lack sufficient information to determine its truth value. If we know the truth value of all the simple statements that comprise a compound statement, the value of the statement can be determined. It is a simple matter of calculation. To illustrate this we will consider compound statements made up of the simple statements A, B, C and D, where A and B are assigned a true truth value, and C and D are assigned a false value.

The truth value of the given the compound statement \((A \lor B) \land (C \lor B)\) determine be determined by simply substituting the known values and calculating down to a single value. This “substitute and calculate method” will be illustrated in stages.

Example 1:
Given the statement: \((A \lor B) \land (C \lor B)\)

Step 1: Substitute T and F for the simple statements, as directed.

\((T \lor T) \land (F \lor T)\)

Step 2: Using the truth table for the logical operators to calculate from inside the parentheses first. \(T \lor T\), which correspond with row 1 of the disjunction table, will result in a value of T, and \(F \lor T\), which corresponds with row 3 of that same table, will also result in a value of T. This gives the result:

\(T \land T\)

\(T\)

To calculate the value of the statement to a final, single value, we refer to the conjunction table because this is the main logical operator of the original statement. We look at row 1 to see that two true conjuncts result in a true conjunction. So, the final value we are able to calculate is: \(T\).

Example 2:
This second example can be a source of error because of the need to properly calculate the negation sign.

Given the statement: \(\sim (D \lor A)\).

Step 1: Substitute the given values for the statements, D and A.

\(\sim (F \lor T)\)

Step 2: The correct way to proceed is to calculate a value for the disjunction within the parentheses first.

\(\sim T\)  A single value is found for the disjunction in parentheses. The negation applies to the resulting value.
Step 3: The negation sign is now calculate to give the final result:

\[ F \]

A common error is to calculate the negation sign incorrectly by applying it to one or both of the values inside the parentheses. The general rule is that a negation sign changes the value of the statement that directly follows it. If this is a simple statement, the negation sign changes the value of the simple statement. If the statement that follows directly is a compound statement, you must calculate to a single value within the parentheses first and then calculate the negation.

\[ \sim (F \lor T) \]
\[ T \lor F \quad \text{This is an incorrect way of applying the negation} \]
\[ T \quad \text{It results in an incorrect answer.} \]

A negation outside the parentheses does NOT change the truth value of the statements within the parentheses. In Logic, the negation never moves through parentheses without a change of logical operator. We always begin calculating from inside the parentheses, working out to the main logical operator. The main logical operator will be the last value calculated. The correct was to calculate the statement above is:

\[ \sim (F \lor T) \]
\[ \sim T \quad \text{The negation applies to the value within the parentheses.} \]
\[ F \quad \text{This results in a correct answer.} \]

Exercise: First identify the main logical operator for each statement. Then use Truth Tables to determine their truth value. Assume A and B are True and C and D are false. (Answers to odds p. 38)

1. \((A \cdot B) \Rightarrow C\)
2. \(\sim (A \cdot C)\)
3. \((D \lor A) \lor (C \Rightarrow B)\)
4. \((A \lor \sim D) \Rightarrow \sim C\)
5. \((A \cdot B) \equiv C\)
6. \((A \cdot B) \equiv \sim (C \lor D)\)
7. \(\sim [(A \Rightarrow B) \equiv \sim (C \lor \sim D)]\)
8. \([(A \cdot \sim B) \equiv C] \lor [(A \cdot B) \equiv \sim (C \lor D)]\)
9. \(\sim (A \cdot C) \equiv \sim (B \lor D)\)
10. \((C \lor \sim D) \lor (\sim C \lor D)\)
APPLICATION II: Calculating with Unknown Values

There are instances where we can determine a truth value for compound statements even though we
do not have the truth value for each of the simple statements in the compound. This is because each operator
mediates the truth value of statements according to given patterns. For example, we know that a disjunction
will be true as long as one of the disjuncts is true. Therefore, if we know that one disjunct is true, the
disjunction as a whole will necessarily be true. The truth value of the other disjunct does not matter.
Similarly with the conditional, we know that any conditional statement will be true whenever its antecedent
is false, or its consequent is true.

Once we understand these truth table facts, we can sometimes deductively determine the final truth
value of a statement even if it is not possible to know the truth value of every part. In other situations,
however, the only logical judgment to make is that we do not have sufficient information to draw a definitive
conclusion about the truth value of the given statement. In such cases we simply say, “The value cannot be
determined.” For the example below we will calculate simple statements A and B as true, C as false, and D
as unknown, where unknown means we cannot calculate whether if the statement is true or false.

Example 1:
\[ \sim C \lor D \]

Step 1: \[ \sim F \lor ? \]
-We substitute F for C and ? for D
Step 2: \[ T \lor ? \]
-We can calculate \( \sim F \) which becomes \( T \)
Step 3: \[ T \]
-We can calculate \( T \lor ? \) by the rule of disjunction. This is
the final value.

Even though we do not know the value of D, because the statement is a disjunction and the first disjunct has
been calculated to be \( T \), the value of the statement must be \( T \).

Example 2:
\[ \sim C \Rightarrow D \]
\[ \sim F \Rightarrow ? \]
\[ T \Rightarrow ? \]

The value cannot be determined

Since the value of this conditional will change depending on the value of D (the consequent), we cannot
definitively determine its value. The answer is “the value cannot be determined.”

Exercise: For each statement below determine if a truth value can be known. Assume that A and B are true,
C is false but D is unknown. You may either calculate to a single value OR end your calculation at any point
where you can definitively defend your judgment. In this latter case, you should write a short, informal
argument to defend your judgment. Here is a template for such an argument:

The statement is a [specify statement type, ie a conjunction] and [specify a
determining part of the statement, ie the left conunct] has the value [T/F]. Therefore,
the value of the statement must be [T/F/ cannot be determined].

Practice using proper terminology. For example, the left side of any conditional is called an antecedent and
any right side is called the consequent. The two sides of a conjunction are called the right conjunct and left
conjunct respectively. With a disjunction we use the terminology of right disjunct and left disjunct. For a
logical equivalence, you may simple use the terms “right side” and “left side.” (Answers to odds p. 39)

1. \( A \lor (\sim B \cdot D) \) 2. \( (A \cdot B) \Rightarrow D \)
3. \( \sim (A \cdot B) \Rightarrow D \) 4. \( (A \cdot B) \equiv \sim (C \lor D) \)
5. \( \sim (D \Rightarrow A) \) 6. \( C \Rightarrow (D \lor A) \)
7. \( \sim (D \equiv C) \) 8. \( D \equiv D \)
9. \( (A \Rightarrow B) \Rightarrow D \) 10. \( \sim D \cdot \sim B \)
APPLICATION III: Full Truth Table Method – Calculating Truth Functional Statements with No Known Values

Finally, it is possible to determine the truth value of some statements even when we do not know the truth values of its component simple statements. To make this determination, however, we will need to construct a full truth table. As an example, let’s consider the truth table for the exclusive disjunction. Remember that an exclusive disjunction makes the following claim:

\[(p \lor q) \cdot \sim (p \land q).\]

All full truth tables begin by establishing columns for the simple statements. We call these “guide columns” because we will refer back to them to get the values for further columns. We then generate the appropriate number of rows so that we can account for each and every truth value possibility that can obtain given the number of simple statements we are dealing with. Again, we use the formula \(2^n\), where 2 represents the number of values in our binary logic (true and false), and superscript \(n\) represents the number of simple statements in the context. In this context, \(2^2\) will give us four rows. These columns are filled in by starting with the guide column that is furthest to the right and alternating values, beginning with T, and continuing until the column is filled in. As we move to the left we double the number of T’s over the number of F’s until all guide columns are filled in. In this way we can generate every possible combination of truth values for as many statements is required.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \lor q</th>
<th>p \land q</th>
<th>\sim (p \land q)</th>
<th>(p \lor q) \cdot \sim (p \land q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

Columns generated to the right of guide columns build with the complexity of the statement for which we want a truth value. We build from the most simple to the more complex statements until we have the full statement represented. To get a value for any column we need to identify the prior columns that show the values for the statement or statements needed and the truth tables for the appropriate logical operator. For example, to fill in columns 3 and 4 we need to refer to the values in columns 1 and 2, since p (column 1) and q (column 2) are the component simple statements in both of these compounds. For column 3 we refer to the truth table for the disjunction operator, and for column 4 we refer to the conjunction operator. The table below shows how these columns are correctly filled in.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \lor q</th>
<th>p \land q</th>
<th>\sim (p \land q)</th>
<th>(p \lor q) \cdot \sim (p \land q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

To fill in column 5 we simply reverse the values in column 4, according to the negation operator.
The final column generates the values for the complete statement. Again we refer to the conjunction truth table and the values in columns 3 and 5. This final calculation shows us a value for the exclusive disjunction.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p v q</th>
<th>p · q</th>
<th>~ (p · q)</th>
<th>(p v q) · ~ (p · q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
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<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

Since the final column shows a combination of truth values, we say that the statement is **contingent**. This means it will be true under some circumstances (those shown in rows 2 and 3) and false in other circumstances (those shown in rows 1 and 4).

**Using truth tables to identify compound statements as Contingent, Contradiction or Tautology:**

Most statements in ordinary language are **contingent** statements. This simply means that they are true under some circumstances and false under others. There are some statements, however, that are always false, regardless of the truth value of the simple statements, and others that will always be true. When this is the result of the form of the statement, and not any particular content, we call these **contradictions** (if they are formally false) or **tautologies** (if they are formally true).

Contradictions and tautologies can be easily identified using a truth table. To find out if the statement \( A \lor (B \supset \neg A) \) is a contradiction, a tautology, or if it is contingent, we set up a truth table. We first identify the simple statements. There are two, \( A \) and \( B \). These statements establish the guide columns. We set up the remaining columns by breaking the statement down and accounting for every logical operator.

The final table will look like this:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>~A</th>
<th>B ⊃ ~A</th>
<th>A ∨ (B ⊃ ~A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
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<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
When we calculate the value of this disjunction, in the final column of the table, we find only Ts. This means that the statement, \(A \lor (B \rightarrow \neg A)\), can only be true – even when its simple component statements are false. It is a **tautology**, true by virtue of its form, regardless of the truth value of its parts.

A contradiction is the opposite of a tautology. It is a statement that is always false, by virtue of its form. The following statement, \(A \land (B \land \neg A)\), is an example of a **contradiction**.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>~A</th>
<th>B \land \neg A</th>
<th>A \land (B \land \neg A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<tr>
<td>T</td>
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<td>F</td>
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<td>T</td>
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<td>F</td>
</tr>
</tbody>
</table>

This table shows that the statement, \(A \land (B \land \neg A)\), is *always false*, regardless of the truth values of A and B. Whatever their truth value, the compound statement, \(A \land (B \land \neg A)\), will be false. This is what we mean by a **logical contradiction**.

**A Short Cut Technique for Statements:**

It is possible to use a shorter method to determine the truth value of a statement when we do not know the truth value of each of its parts. This method involves listing the values for each part of the statement directly beneath the full statement itself, and then calculating the parts in turn. This method requires greater focus, as the context becomes more crowded with Ts and Fs. Below is an example of how that method would look for the statement, \(\neg (A \lor B) \equiv (B \lor \neg A)\). The table shows the statement to be **contingent**.

\[
\neg (A \lor B) \equiv (B \lor \neg A)
\]

<table>
<thead>
<tr>
<th>~ (A \lor B)</th>
<th>(B \lor \neg A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

**Exercise:** Use either the full table or the short cut method to determine the truth value character/profile of the following statements. Identify each as **contingent**, a **tautology** or a **contradiction**. (Answers to evens p. 70)

1. \((A \lor B) \lor A\)
2. \(\neg A \land (B \lor \neg A)\)
3. \((A \lor (B \land \neg A))\)
4. \(\neg (A \lor (B \land \neg A))\)
5. \((A \equiv B) \equiv (B \lor A)\)
6. \(\neg (A \lor B) \lor (\neg A \lor \neg B)\)
7. \((A \lor A) \lor (A \lor A)\)
8. \(\neg A \lor (B \land \neg B)\)
9. \((A \lor B) \equiv (A \land \neg B)\)
10. \((A \lor B) \equiv (A \land \neg B)\)
Application IV: Using Truth Tables to Test for Logical Equivalence

We can use a truth table to show that the bi-conditional \( (p \equiv q) \) has the same logical meaning, or is logically equivalent, to the statement \( (p \Rightarrow q) \cdot (q \Rightarrow p) \). Logically equivalent statements have the same truth values under the same circumstances, row for row. This means they have the same logical meaning.

To construct a table to show that two or more statements are logically equivalent, requires that we extend the basic table to create a column that will give a reading for the conjunction, \( (p \Rightarrow q) \cdot (q \Rightarrow p) \). Intermediate columns are required as the table builds from simplicity to complexity. Values are filled in by referring to previous columns and the defining truth tables for the relevant logical operators. The completed table will look like this:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \equiv q</th>
<th>p \Rightarrow q</th>
<th>q \Rightarrow p</th>
<th>(p \Rightarrow q) \cdot (q \Rightarrow p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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</tr>
</tbody>
</table>

Guide columns for \( p \) and \( q \) are filled in according to the formula \( \text{2}^{2} \). In this formula “\( 2 \)” represents the two possible values, true and false, and the superscript “\( 2 \)”, represents the two simple statements, \( p \) and \( q \). This simple formula gives us four rows, enough to generate all the possible combinations of truth value for two statements that have only two possible values. Subsequent columns are filled in by referring to relevant values in preceding columns and the defining truth tables for relevant operators. Column 3 is filled in by referring to the guide columns and the truth table for the bi-conditional. Columns 4 and 5 are filled in by referring to the same two columns and the conditional truth table. (Note the difference between rows 2 and 3. This reflects the difference between the values for the antecedent and consequent statement relative to the guide columns.) Column 6 is completed by referring to the values in columns 4 and 5 and the conjunction truth table. This column is then compared with the values in column 3.

After the truth table is filled in, we read it by comparing columns 3 \( (p \equiv q) \) and column 6 \( ((p \Rightarrow q) \cdot (q \Rightarrow p)) \). We see the same pattern of values in both columns. They have the same truth values under the same circumstances. Therefore, they are logically equivalent.

Exercise: Use either a full truth table or the short cut method to determine if the following pairs of statements are logically equivalent. (Answers to evens on p. 71)

<table>
<thead>
<tr>
<th>1. q \Rightarrow p \quad \text{and} \quad p \Rightarrow q</th>
<th>2. p \Rightarrow q \quad \text{and} \quad \sim p \lor q</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. \sim p \lor q \quad \text{and} \quad \sim (p \lor q)</td>
<td>4. \sim (p \lor q) \quad \text{and} \quad \sim p \lor \sim q</td>
</tr>
<tr>
<td>5. \sim (p \lor q) \quad \text{and} \quad \sim p \land \sim q</td>
<td>6. p \Rightarrow q \quad \text{and} \quad \sim (p \land \sim q)</td>
</tr>
<tr>
<td>7. p \equiv q \quad \text{and} \quad q \equiv p</td>
<td>8. p \Rightarrow q \quad \text{and} \quad \sim q \Rightarrow \sim p</td>
</tr>
</tbody>
</table>

To use truth tables successfully you need to know three discreet things. You need to know: 1) how to set a table up, 2) how to fill it in and 3) how to read it. Once the table is filled in completely, your reading will depend on the information you need to get from the table, or the question you are asked to answer. The box below gives a basic set of instructions for setting up a full truth table. These instructions will apply throughout this course. Some of you may want to use the short-cut method.
A wonderful mathematician and logician, Raymond Smullyan, has written a series of books containing a wide range of logic puzzles that require and illustrate the use of deductive reasoning. To show how we can use truth tables to solve reasoning problems, I’ve taken a few puzzles, taken from his book, *What is the Name of This Book?* and adapted them to a 21st century problem. Here’s the problem:

A national intelligence has received a tip that a bomb has been planted at the local airport. They are further told that the bomb has been planted either in a backpack, a wastebasket, or in a UPS box. They are also told each item has a message on it and that at most one of the messages is true.

When agents arrive at the airport, they locate the backpack, the wastebasket and the UPS box, each with a message. Given the messages they found, how can the agents determine where the bomb is?

Students have had different reactions to this puzzle. Some immediately draw a conclusion about where the bomb is. Some conclude it must be in the wastebasket. Some conclude it must be in the UPS box. Occasionally, some argue it is in the backpack. Others argue that it is impossible to know. Now, it is important to understand that in Logic it is not just a matter of getting the answer correct. The work is also about justifying your answer with an argument. So, for any given conclusion you may draw about where the bomb MUST be, it is important that you can explain why your judgment is necessarily true. This is the difficult but essential part of deductive reasoning.

Given the statements that establish the parameters of this puzzle, it is possible to determine where the bomb is, and to provide an argument that justifies the one single conclusion that solves this riddle. A truth table can facilitate the reasoning, taking some of the pain out of the effort. How can this tool be used?

First, we must remember that a truth table is a kind of data base that presents all the logically possible truth value combinations for any given number of statements. Truth tables can be extended both by rows (which is a factor of the number of simple statements we are working with) and columns (which is a factor of the complexity of the relationships within and between statements). In this context there are four statements, one limits the context of our reasoning. This means the table need only compare the truth value possibilities of three statements.

We begin by establishing a table that presents all possible truth value combinations for the statements on each of the three items in which the bomb might possibly be. Using the basic formula $2^n$ we can figure that eight rows will be required to generate all possible combinations of values for these three statements. “Guide Columns” indicate the three places. The table will look like this:

<table>
<thead>
<tr>
<th>Backpack</th>
<th>Wastebasket</th>
<th>UPS box</th>
</tr>
</thead>
<tbody>
<tr>
<td>The bomb is here.</td>
<td>The bomb is not here.</td>
<td>The bomb is not in the backpack.</td>
</tr>
</tbody>
</table>
In this table all truth value combinations are represented and no combination is repeated.

It would be tedious to go through each of these eight rows. But by using the statement, “At most only one of these statements is true,” we can eliminate rows 1-3 and row 5 because these rows have two true statements each. We are left with only four rows:

<table>
<thead>
<tr>
<th>Backpack</th>
<th>Wastebasket</th>
<th>UPS box</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
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<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

If we compare what is written on the Backpack and UPS box, we can eliminate two more rows, because these statements are contradictory. Contradictory statements cannot have the same truth value. Therefore, they cannot both be false. So, rows 2 and 4 can be eliminated.

This leaves two possibilities. Study the table above carefully. What do you see? What necessary conclusion can be drawn from the data presented?

To solve this kind of puzzle we must know what the logical possibilities are for the statements involved. This involves knowing what truth value possibilities can obtain for the set of
The bomb is not in the wastebasket.

The bomb is here.

At least two of these bombs were made by Crook.

Below are a few for you to try and figure out for BONUS BANK credit. However you are able to solve it, you will need to present a clear, cogent argument to demonstrate the correctness of your reasoning.

One final puzzle:

This puzzle is more complicated, but still solvable. This time there are three bombs, planted in the same three locations. Each has a statement attached to it by the person who planted the bomb, but only one will be detonated. But this time the bomb planters are known to the agents. Agents know 1) that the bombs were planted by either Crook or Book, and 2) that Crook always lies while Book always tells the truth. In order to press a successful case in federal court, agents need to know not only where the bomb that will be detonated is, but who planted each bomb as well. Here are the messages found on the bombs.
THE NEED FOR PREDICATE:

Categorical Syllogisms: A Model of Deductive Argument

I hold that the invention of the form of syllogisms is one of the most beautiful which the human mind has made, and even one of the most considerable. It is a kind of universal mathematics whose importance is not sufficiently known.

Leibnitz

The logician to the old man: Here is an exemplary syllogism. A cat has four paws. Isidore and Fricot each have four paws. Therefore Isidore and Fricot are cats.
The old man, to the logician: My dogs have four paws too.
The logician to the old man: Then, it’s a cat . . .
The old man, to the logician after long reflection: Therefore, logically, my dog will be a cat.
The logician to the old man: Logically, yes. But the contrary is also true. . .
The old man, to the logician: On the condition that it not be abused. . . .
The logician to the old man: Another syllogism: all cats are mortal. Socrates is mortal. Therefore, Socrates is a cat.
The old man: And he has four paws. It’s true. I have a cat named Socrates.
The logician: You see.

Eugene Ionesco, The Rhinoceros

The presentation of Sentential looked at several forms of simple syllogism, which are deductive arguments that have two premises and a single conclusion. In addition to the disjunctive and hypothetical syllogism patterns, there is another type of syllogism that we use when we reason about categories of things and the relationship of particular things to categories. Leibnitz refers to this type of syllogism in the quote above. And it is illustrated in the excerpt from Ionesco’s play. The categorical syllogism was discovered by Aristotle, who first systematically analyzed this form of argument. The study of categorical syllogisms became the basis of the discipline of Logic from his time until the 19th Century when philosophers identified specific limitations in Aristotle’s analysis. We will consider this type of argument form to extend our understanding of syllogisms and to introduce the logical language called Predicate.

Categorical syllogisms are constructed entirely from categorical statements. Categorical statements make claims about classes of things. Statements like “All gods are mammals,” “Kimo is a good student,” “Some of the candidates will not be elected,” “No students will take a Math course,” “Some cars are gas guzzlers,” “Eliana is not a mechanic,” “Some logicians are philosophers,” are all examples of categorical statements. What these statements have in common is that they make definite claims that relate two specific things (either two classes of things, or some individual to a class).

Categorical statements have one of two qualities. They are either positive/affirmative or negative. Affirmative statements claim that an individual, a part of, or an entire class (named by the grammatical subject) is or are a member of another class (named by the grammatical predicate). Examples of affirmative statements would be: “Hina is a poet,” “Some poets are good writers,” “All poets are good writers.” Negative statements deny that an individual, a part, or an entire class is a member of another class. Examples of negative statements would be: “Hina is not a poet,” “Some poets are not good writers,” “No poets are good writers.”

Categorical syllogism will be made up of only categorical statements. They will have exactly three statements. In a well structured or standard form categorical syllogisms, these three statements will relate exactly three identifiable terms. We identify these terms using a grammatical analysis that divides the grammatical subject term from the grammatical predicate term. (More or
less than three terms will result in a fallacious argument). In this type of argument, one term (the so-called the **middle term**) appears in both premises, but not the conclusion. The other two terms appear once in each premise only and then are related in the conclusion.

The following categorical syllogism will illustrate these features:

All mammals have four legs. No birds have four legs. Therefore, no birds are mammals.

“Therefore” indicates the conclusion. The first two statements are the argument’s premises. To make it easier to analyze the argument further we use the same method as before and present each statement in the argument on a separate line, clearly identifying it as either premise or conclusion.

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>All <em>mammals</em> have four legs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>No <em>birds</em> have four legs.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>No <em>birds</em> are <em>mammals</em>.</td>
</tr>
</tbody>
</table>

We can see that this argument has exactly three terms, “*mammals*,” “*birds*” and “*having four legs*.” The premises share the common term “*having four legs*.” This is the middle term. We can see that the terms “*mammals*” and “*birds*” appear in each premise and that they are related in the conclusion.

The repetition of terms within a categorical syllogism may make the argument sound redundant, as if it is “going nowhere.” But a careful look shows that the conclusion is a unique statement, that relates two terms that were not been related in either premise. In this way the categorical syllogism, like other syllogism, illustrates a key feature of deductive reasoning—namely, that the terms in the conclusion can be found in the premises.

To get closer to the structure of the argument we can abstract the content (the terms of the argument) and replace them with symbols. We will use the symbols A, B, and C to represent each of the three terms respectively. The result will look much like this:

<table>
<thead>
<tr>
<th>Premise 1</th>
<th>All A have B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise 2</td>
<td>No C have B.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>No C is an A.</td>
</tr>
</tbody>
</table>

Representing the structure in this way is analogous to “showing the form” in Sentential arguments. We can see clearly that the premise statements have a strong relationship to the conclusion, and how the terms link the premises to each other.

Showing the form of the argument is an intermediate step to showing the skeletal structure of the argument, where we strip all language away. We can understand argument form as a kind of vessel that can hold different content. We can replace these terms with any three others (say, “baseball players,” “steroid users” and “national heroes” OR “politicians,” university administrators” and “liars”) and the structure or logic of the argument will be the same.

Showing the form of the argument can also help us appreciate that like the structure of a river bed compels the water in the river to move in a given direction, with a given flow, the structure of arguments compels our minds to move (or not move) in a given direction. The force of the logic is in its structure and not its content.
**Self Exercise:** Develop 5 different categorical syllogisms, using these basic statement forms:

- All As are Bs.
- No As are Bs.
- (A specific individual is A)
- Some As are Bs
- Some As are not Bs.
- (A specific person is not A)

Make sure each example fits the structural requirements of a categorical syllogism. Then symbolize the structure of the syllogism by giving each term a unique symbol. Discuss the differences you find in the structures of the arguments you have developed.

**Using Logical Language to Represent Categorical Syllogisms**

We now want to take our formalization of a categorical argument to a full symbolization. Consider the following argument:

All libertarians are anarcho-capitalists. Ron Paul is a libertarian. Therefore, Ron Paul is an anarcho-capitalist.

For most of us, the logical form of this argument is easy to “see” and we can recognize the validity in its structure. However, if we use the language of *Sentential* to symbolize the argument, that logic will not be visible. In Sentential each simple statement is represented by a single capital letter. The follow language key shows us an assignment of statement symbols for each simple statement in the argument:

- Let A = All libertarians are anarcho-capitalists.
- B = Ron Paul is a libertarian.
- C = Ron Paul is an anarcho-capitalist

This symbolization would leave us with a representation of the argument as:

Premise 1:   A
Premise 2:   B
Conclusion:  C

In this representation the conclusion appears to have no relationship to the premises at all. It does not show how the premise statements link to one another or how they relate to the conclusion. Yet it is clear that the premises do have a substantial link to one another and that conclusion *does* follow from the premises.

Here we meet the limit of the power of Sentential to represent the logic of the argument. This is because that logic plays out the relationships between terms within the simple statements. *Sentential* is a useful logical tool when the argument being represented depends on the structure of the relationships between simple statements. However, each categorical statement is a simple statement. It cannot be broken into simpler statement components. We can, however, break a categorical statement into simpler *terms*. By dividing the statements of the argument into their grammatical subject and grammatical predicate terms, we can show the logical relationship between these terms— as we illustrated above. What we need is a logical language that will then allow us to
represent these terms, the logical difference between individuals and class or relational terms, and quantative terms such as “all” and “some” upon which the logic of categorical syllogisms depends. We need a more powerful logical language, capable of representing logical relationships within statements.

**Predicate** is a logical language that accommodates the need to express the subject-predicate relationships within statements. It uses symbols that represent class and relational terms as well as symbols that represent specific individuals. It has symbols that express quantity. We will use categorical syllogisms as a model for learning this language.

**PRELIMINARY ISSUES: GRAMMAR**

Frege’s . . . discovery of quantification [is] the deepest single technical advance ever made in logic.

Michael Dummett

The language of **Predicate** uses the same logical operators as **Sentential**. However, it analyzes simple statements in greater detail, showing how terms within the statement relate to each other. It is necessary to use this language when the logic of an argument depends on the logical relationships within a simple statement. This is true of categorical syllogisms and we will use this class of arguments to learn the fundamental elements of the logical language called Predicate.

**Subject / Predicate analysis**

To symbolize a simple categorical statement into Predicate, we need to understand the basic grammatical structure of a statement. We will be using a basic subject-predicate grammatical analysis to find the terms in the statement. Every complete declarative sentence in English can be divided into two basic parts, a grammatical subject and a grammatical predicate. (Although the grammatical analysis is not always the same as the logical analysis, the grammatical subject term will have some logical relationship to the grammatical predicate term.) We present this basic level of grammatical analysis as follows:

```
subject term | predicate term
```

This level of analysis is an important first step to translating English sentences into Predicate. To see how it applies to simple statements, we consider a few simple examples.

1. John is a teacher.

In this example the grammatical subject, “John,” is related to the grammatical predicate, “astronaut.” The analysis would be:

```
John | is an astronaut
```

2. Some butterflies are yellow.

In this example the grammatical subject, “butterflies” is related to the grammatical predicate, “yellow.” The analysis would be:

```
Some butterflies | are yellow
```
3. Only registered party members can vote in the state primary.

In this example the grammatical subject is “registered party members” and the grammatical predicate is “can vote in the state primary.” The analysis would be:

Only registered party members can vote in the state primary.

Even more complex statements display the basic subject-predicate structure.

4. All the derelict cars parked across the street for a week were towed by the City and County the day before yesterday.

In this example, the grammatical subject is “All the old derelict cars parked across the street for a week” and the grammatical predicate is “were towed away by the City and County the day before yesterday.” The analysis would be:

All the derelict cars parked across the street for a week were towed by the City and County the day before yesterday.

The most basic element in this statement reduces to: The cars were towed. Everything else qualifies the subject term (cars) and the predicate term (were towed).

Ordinary language sentences develop grammatically into more complex structures as the subject and predicate terms are qualified or as these terms compound. But we always want to get back to the most basic element, the basic subject-predicate relationship, no matter how complex the grammar of the sentence may be. It is in this relationship that we find the grounding relationship of terms in a statement.

Once we have identified the grammatical subject and predicate, we need to know how to symbolize their logical relationship. While in Sentential, the simplest unit of content was a simple statement, in Predicate, the simplest unit of meaning is a predicate function. A predicate function combines a predicate letter (capital letters A-Z) with a variable (lower case letters x, y, and z). Predicate letters will stand for class names, qualities or attributes. Predicate letters represent the content of the argument. Variables function as place holders for individual names.

“Ax” is a predicate function. “A” is a predicate symbol and “x” is a variable. Let’s say “A” stands for the class, “astronauts.” The predicate function, “Ax,” means, “x is an astronaut.” A predicate function, thus, has meaning, but it is not a statement. (Remember a statement has truth value and “x is an astronaut” does not have truth value.) We can take this predicate function, however, and create statements in one of two ways.

There are two ways to create a statement from a predicate function. One way is to replace the variable with an individual constant. Individual constants are represented by lower case letters a-w. We can use “j” to represent John. Replacing x with “j” would give us, Aj, which would mean “John is an astronaut.” We can use different individual names, or individual constants to make an infinite amount of statements by simply replacing “x” with different individual names.

A second way of creating a statement from a predicate function is to place a quantifier directly in front of it. In Predicate, there are two quantifiers. The symbol (∀x) means “all” or “for any x.”
Placing \((\forall x)\) directly before a predicate function creates a universal statement. Thus, \((\forall x)Ax\) would mean, “Everyone is an astronaut.” The symbol \((\exists x)\) means “some” or “there is an x” or “an x exists.” Placing \((\exists x)\) directly in front of a predicate function creates an existential statement. Thus, \((\exists x)Ax\) would mean, “Someone is an astronaut,” or “There are astronauts,” or “Astronauts exist.”

We can use these simple elements (quantifiers, variables, individual names or constants, and predicate terms), the same logical operators we used in Sentential, and a set of formation rules to create an infinite variety of statements in the language of Predicate. Predicate gives us a powerful tool for representing the logical relationships we find in many arguments, both categorical syllogisms and other, non-syllogistic argument forms.

**Formation Rules for Predicate:**

The formation rules for Predicate parallel those for Sentential. Again, all statements must be sensible. They must be **well-formed formula (wff)**. This can be accomplished by applying the following rules, where \(\phi\) means any predicate term:

1. All statement letters are wffs. Any predicate letter (\(\phi\)) followed by an individual name is also a wff.
   
   As with Sentential, we can still use statement letters, in conjunction with predicate terms to form *wffs* in Predicate. We use the Greek letters \(\phi\) and \(\psi\) (*phi* and *psi*) to indicate *any* predicate term. In an actual statement, predicate terms will be symbolized with capital letters A-Z. Individual names will be symbolized by lowercase letters (a-w).

2. If \(\phi\alpha\) is a wff, then \(\sim\phi\alpha\) is a wff.
   
   This rule parallels the rule for forming negations in Sentential. Negative statements are formed by placing the negation symbol directly in front of the statement to be negated.

3. If \(\phi\alpha\) and \(\psi\alpha\) are wffs, then \((\phi\alpha \cdot \psi\alpha)\), \((\phi\alpha \lor \psi\alpha)\), \((\phi\alpha \supset \psi\alpha)\), \((\phi\alpha \equiv \psi\alpha)\) are also wffs.
   
   This rule parallels the rule for forming compound statements in Sentential.

4. If \(\phi\alpha\) is a wff that contains any name \(\alpha\), but does not contain any quantifier, then the resulting expression that is obtained by replacing all occurrences of \(\alpha\) in \(\phi\alpha\) by a variable \(x\), and attaching "\((\forall x)" or "(\exists x)" to the front, is also a wff.
   
   This rule explains how we can form generalized statements in Predicate. It will be illustrated more specifically below.

5. Nothing else is a wff (*a well-formed formula*).

These rules are presented abstractly to apply to the widest range of situations. To show how we use them to form *wffs* in Predicate, we will work with specific English statements into Predicate.
Expressing Quantity

Much of our reasoning relies on speaking about things in general, and drawing general conclusions. Predicate symbolizes generality using quantifiers and variables. We use $x$, $y$, $z$, just as we do in Math, to symbolize variables. Variables are simply place-holders, and will be used when we want to express a generalized statement. Variables will always be paired with quantifiers.

*Predicate* uses two quantifiers, represented by composite symbols ($\forall x$) and ($\exists x$). The symbol ($\forall x$) means “given any x” or “for all x’s.” A statement such as “Everyone is happy,” is symbolized as ($\forall x$) $Hx$. The variable x, refers to “all” unknown or unspecified members in the class.

The existential quantifier ($\exists x$) means “for some x” or “there are some x’s” or “there exists an x.” When we say “Someone is not happy,” symbolized as ($\exists x$) $\neg Hx$, we do not refer to a specific individual but some unidentified individual or individuals. Again, we use the variable x as a place holder for individuals.

To review: Specific, named or otherwise identified individuals are symbolized by lowercase letters (a-w). Lowercase letters $x$, $y$, and $z$ are used as variables. Individual constants replace variables under certain circumstances. Why we move from variable to individual and vice versa will be explained later. Upper-case letters (A-Z) represent class terms (such as mammals, swimmers, happy people, bugs, dogs, cats, politicians, idiots, etc.) or qualities (big, red, nice, pretty, famous etc.) or relations (next to, in front of, married to, between, around the corner from, etc.).

Adding quantifiers and variables expands the types of statements we can symbolize. For example, we can symbolize claims such as “Everyone is happy,” “No one is a student,” “Everyone is a teacher,” and “Someone is not happy,” where the quantifier is the grammatical subject term. We can also symbolize classic standard form categorical statements such as “All students are happy,” “Some teachers are students,” “No dogs are cats,” which require logical operators.

**Application: Symbolizing Standard Form Categorical Statements**

As a case study for how Predicate works, we will look at categorical statements. There are six basic forms of categorical statements. Two of these forms make claims about individuals in relation to a specific class (such as I am a teacher, you are a student, John is happy), and four are generalized statements that relate two classes (such as All students are teachers, No students are teachers, Some students are teachers and Some students are not teachers.

Each type can be categorized by its form and by its type, as illustrated in the chart below:

<table>
<thead>
<tr>
<th>Standard form categorical statements</th>
<th>Statement form</th>
<th>Statement type</th>
</tr>
</thead>
<tbody>
<tr>
<td>All humans are mortal.</td>
<td>All H are M</td>
<td>Universal Affirmative</td>
</tr>
<tr>
<td>No humans are mortal.</td>
<td>No H are M</td>
<td>Universal Negative</td>
</tr>
<tr>
<td>Some humans are mortal.</td>
<td>Some H are M</td>
<td>Existential Affirmative</td>
</tr>
<tr>
<td>Some humans are not mortal.</td>
<td>Some H are not M</td>
<td>Existential Negative</td>
</tr>
<tr>
<td>Socrates is human.</td>
<td>s is H</td>
<td>Individual Affirmative</td>
</tr>
<tr>
<td>Socrates is not human.</td>
<td>s is not H</td>
<td>Individual Negative</td>
</tr>
</tbody>
</table>
The center column of the chart above shows the first level of abstraction, reducing the content of each statement to a predicate symbol (capital letter). The individual statements in rows 5 and 6 use lower case letter “s” to represent the individual “Socrates.”

From this intermediate level of abstraction we can move easily to a full symbolization using a predicate function Hx to symbolize “x is a human” and Mx to symbolize “x is mortal.” We then link these terms with the logical operator that best conveys the relationship that is asserted between “being human” and “being mortal” in each of these forms of categorical statements. That logical operator will be determined by the logical force of the quantifier terms, “all” and “some,” respectively. In the case of individuals, no operator is required.

To determine what operator is necessary in any Universal statement we must think what operator is best to convey the meaning behind “all humans are mortal,” The concept of “all” carries the logical meaning of the universal statements. Relating two classes in this way suggests that the conditional operator best expresses what is mean by all this is that. Hx ⊃ Mx provides a compound predicate function that means, “if x is human, then x is mortal.” We place the universal quantifier directly in front of this compound function. The resulting formula is translated in Predicate as (∀x)(Hx ⊃ Mx), and in English as All humans are mortal.

Parentheses are required. They enclose the function to make clear that it is the relationship that is being quantified. We use the phrase “scope of the quantifier” or “range of the quantifier.” The parentheses bind the x’s of H and of M are bound to the quantifier. The literal logical meaning can be understood as “Given any x, if x is human, then x is mortal.”

This kind of precision is also in the Existential Affirmative statement. When we make the claim, “Some humans are mortal,” we are stating that “there exist some things” that are “human” “and” those x’s are “mortal.” Each of the separate elements – the quantifier, the predicate terms, the logical operator and the parentheses – is essential to communicate the precise meaning of the statement. The resulting formula will be: (∃x)(Hx • Mx).

Negating Standard Form Categorical Statements

Translating negative statements into a logical language always requires extra thought. This is because we need to be clear about what precisely is being negated. As with quantifiers, we need to consider the “range” or “scope” of the negation. A given expression of negation in English may have range over the entire proposition, or it may negate a given term within the statement. Errors made at this level are critical. If we are not correct in the placement of the negation in logic, a true statement may be rendered as a false one, or vice versa, or the meaning can be changed entirely.

While the scope of the negation can be ambiguous and difficult to identify in ordinary language, in our symbolic languages it is absolutely precise. The general rule for placing negation symbols into a given formula (either Sentential or Predicate) is that a negation is placed directly to the left of whatever is being negated. It is never placed to the right. And its scope is indicated by using parentheses or other forms of punctuation. So, whenever we want to use a negation, we must think carefully about what is being negated and place it properly.

The existential negative statement claims, Some (subject term) are NOT (predicate term). This means, “There are (members of the subject class) and they are not (members of the predicate class).” Expressed in this way it is easy to see where the negation symbol must be placed. Using the terms
“human” and “mortal” as we have been using in our examples, we can see that the resulting formula for an Existential Negative type statement will be: $(\exists x)(Hx \cdot \sim Mx)$.

In the case of the Existential Negative statement, the placement of the negation symbol ($\sim$) maps directly onto the syntax of the English language statement. It is a bit more difficult to think through the meaning of a Universal Negative type statement. Again, we must think about what exactly is being negated when we make a claim like, No humans are mortal. The most common mistake is to place the negation sign directly in front of the universal quantifier, in a way that maps the English directly against the Predicate formula to get $\sim (\forall x)(Hx \equiv Mx)$. But this formula reads: NOT ALL humans are mortal, and this, however, is not what we mean when we state, “NO humans are mortal.”

“No humans are mortal,” makes a stronger claim. When we say, “No humans are mortal,” we are saying something about ALL humans, namely that all things in the class named by humans do not have the quality of mortality. So, the correct understanding of the Universal Negative statement is: for any $x$, if $x$ is human, then $x$ is not mortal. This meaning is expressed unambiguously in the formula, $(\forall x)(Hx \equiv \sim Mx)$.

**Standard Form Categorical Statements**

A generalized categorical statement that is in standard form, will begin with a quantifier. The table below summarizes the standard forms for all six forms of categorical statements. In this chart, “S” represents the grammatical subject term, and “P” represents the grammatical predicate term of the statement. Study the patterns and look for common features and obvious differences to learn how to correctly symbolize these basic statement forms. This is true as well of the standard form English statement. The exception may seem to be the universal negative type. But if we understand that the word “no” implies “all + negative” we can see that this type too begins with a quantified term.

Categorical statements are everywhere in ordinary language. Yet, we often don’t recognize them because they can be expressed in a variety of ways. For example, All dogs go to heaven, a simple universal affirmative statement, can be expressed in English using any of the following sentences:

- Every dog goes to heaven.
- Any dog goes to heaven.
- If it’s a dog, then it’ll go to heaven.
- If it doesn’t go to heaven, then it’s not a dog.
- Dogs go to heaven.
- It’s not true that some dogs don’t go to heaven.

Any natural language allows the flexibility to express the same truth claim in different ways. Different sentences, different combinations of words, can communicate the same truth claim. However, in Logic we work we are concerned with the underlying meaning which can be expressed in a standard form and all the sentences expressed above communicate the meaning of the standard form of a universal affirmative statement: All dogs go to heaven.

The chart below summarizes these six forms of statements:
Summary Chart for Standard Form Categorical Statements:

<table>
<thead>
<tr>
<th>Statement example in Standard Form</th>
<th>Statement form</th>
<th>Statement type</th>
<th>Predicate formula in Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>All (subject term) is/are (predicate term).</td>
<td>All S is P</td>
<td>Universal Affirmative</td>
<td>$(\forall x)(Sx \supset Px)$</td>
</tr>
<tr>
<td>No (subject term) is/are (predicate term).</td>
<td>No S is P</td>
<td>Universal Negative</td>
<td>$(\forall x)(Sx \supset \sim Px)$</td>
</tr>
<tr>
<td>Some (subject term) is/are (predicate term).</td>
<td>Some S is P</td>
<td>Existential Affirmative</td>
<td>$(\exists x)(Sx \cdot Px)$</td>
</tr>
<tr>
<td>Some (subject term) is/are not (predicate term).</td>
<td>Some S is not P</td>
<td>Existential Negative</td>
<td>$(\exists x)(Sx \cdot \sim Px)$</td>
</tr>
<tr>
<td>A named individual has a given predicate.</td>
<td>s is P</td>
<td>Individual Affirmative</td>
<td>Ps</td>
</tr>
<tr>
<td>A named individual does not have a predicate.</td>
<td>s is not P</td>
<td>Individual Negative</td>
<td>$\sim Ps$</td>
</tr>
</tbody>
</table>

We will symbolize categorical statements using these basic formula. When statements are not in standard form we must first find the core logical meaning and then we look to find the standard form that best expresses that meaning.

Building a Language Key in Predicate:

In Predicate the language key will give the premise meaning for all individual constants and predicate functions that are used in the argument. We will build our language keys based on the predicate functions and any individuals named in the argument.

Let: $Ax = x$ is a mammal $Bx = x$ has a backbone $Cx = x$ is viviparous $Dx = x$ is a fish $Ex = x$ has sweat glands $j = Judy$

Exercise A. Use the chart above, and the language key below to translate the following into standard form categorical statements. Beginning with #8 you have to think harder.

1. All mammals have backbones.  
2. $(\exists x)(Dx \cdot Cx)$
3. No fish have sweat glands.  
4. $(\forall x)(Dx \supset \sim Ax)$
5. Some mammals are not viviparous.  
6. $(\exists x)(Cx \cdot \sim Ax)$
7. $(\forall x)(Ex \supset Bx)$  
8. No mammals are fish.
9. Whatever has a backbone, has sweat glands.  
10. Some that have backbones are viviparous.

Moving beyond standard form: For these you have to think beyond the form to what the statement is saying. Paraphrasing to find standard form may help.
11. All fish have backbones but not sweat glands.
13. Only mammals have sweat glands.
15. There are viviparous fish.
17. \( (\forall x) \left[ (Bx \cdot Ex) \Rightarrow Ax \right] \)

12. \( (\forall x) [Ax \Rightarrow (Bx \cdot Ex)] \)
14. Both mammals and fish have backbones.
16. Not all mammals are viviparous.
18. There is no fish without a backbone.

Some individual statements:

19. Judy has a backbone.
20. Judy is not a mammal.
21. Judy has a backbone and is a mammal.
22. Judy is either a fish or a mammal.
23. It is not true that Judy is both a fish and a mammal.

Exercise B: Translating Categorical Syllogisms: Read each of the following arguments. First analyze the argument using the method of dividing premises and conclusions. Then, construct a Language Key. Then translate the complete argument into Predicate

1. All angels have wings. All birds have wings. Therefore, all birds are angels.
2. Some birds are not angels because all birds have wings but some that have wings are not angels.
3. No angels are devils. Therefore, some birds are not devils because no birds are angels.
4. No devils fly because no devils are birds and all birds fly.
5. All angels have wings. All winged things fly. Therefore, all angels fly.
6. Some devils are not birds. Some birds are not angels. So, some angels are not devils.
7. Because no devils fly, no devils are angels, since all angels fly.
8. Satan doesn’t fly because no devils fly and Satan is a devil.
9. Harry is a devil. But Harry is also a bird. All birds fly. So, Harry flies.
10. Some devils are angels because some devils are birds and all birds are angels.
11. All angels have wings but some that have wings don’t fly. Therefore, some angels don’t fly.

The following are intended to make you think:

12. Only angels fly because some angels are not devils and some devils fly.
13. Only angels have wings. Satan is not an angel. Therefore, Satan does not have wings.
14. Mortimer is an angel but does not have wings. Whatever doesn’t have wings cannot fly. Whatever doesn’t fly must be a devil. Therefore, Mortimer is a devil and an angel.
Exercise C: Translation from Predicate:
Use the following language key and translate the following arguments into English.

Let \( A_x = x \) is toxic  
\( B_x = x \) is poisonous  
\( C_x = x \) occurs in nature

\( a = \) arsenic  
\( b = \) vitamin C

1. \( (\forall x) (B_x \supset A_x) , (\exists x) (B_x \cdot C_x) \)  
\( \therefore (\exists x) (A_x \cdot C_x) \)

2. \( \sim (\forall x) (B_x \supset C_x) , (\forall x) (B_x \supset A_x) \)  
\( \therefore (\exists x) (A_x \cdot \sim C_x) \)

3. \( (\exists x) (B_x \cdot C_x) , (\exists x) (A_x \cdot C_x) \)  
\( \therefore (\exists x) (A_x \cdot B_x) \)

4. \( A_a \supset B_a , (\forall x)(C_x \supset \sim B_x) \)  
\( \therefore \sim C_a \)

5. \( A_a \cdot C_b , (\forall x)(C_x \supset \sim B_x) \)  
\( \therefore \sim (\forall x)(A_x \supset B_x) \)

6. \( \sim (\exists x) (B_x \cdot C_x) , (\forall x)(A_x \supset B_x) , A_a \)  
\( \therefore \sim (\forall x) C_x \)

Extending Predicate:
A Universe of Discourse:

When we argue in ordinary language, we assume a context. The assumption of such a context allows us to hold common assumptions about what statements or terms are relevant to the premises we will use and the conclusions that we draw from them. In deductive logic, we call this context a universe of discourse. A universe, or domain of discourse, contains all those, and only those elements, that belong to the formal context within which our reasoning intends to take place. Placing explicit, formal parameters on our reasoning by stipulating a universe of discourse, closes the context of our reasoning and allows us to draw forward conclusions with the kind of certainty that uniquely characterizes deductive reasoning.

To illustrate this concept we can, for example, establish a universe of discourse by limiting our reasoning to the objects on the desk, the people in this room, the items on the menu in the cafeteria, members of the US congress, etc. In this way we recognize and explicitly express the limits within which we draw inferences from our premises. Within the parameters of a universe of discourse, we establish a language key that expresses how we will symbolize the terms we will be arguing about, and from these terms we then establish statements and construct our arguments.

Self Motivated Exercise:
Establish a simple universe of discourse (using no more than three classes of objects and three individuals) and make up a total of six (6) statements. Have someone read them back to you. Correct and develop questions to clarify your understanding of how Predicate works.

Here is an example:
Universe of discourse: My refrigerator.

Let $A_x = x$ is fresh $a = \text{the apple}$
Let $B_x = x$ is rotten $b = \text{the last beet in the box}$
Let $C_x = x$ was bought last week $c = \text{the chicken wing from last night}$

Symbolization  | English translation
--- | ---
1. $A_a$ The apple is fresh.  
2. $C_c$ The chicken wing from last night was bought last week.  
3. $\sim A_c$ The chicken wing from last night is not fresh.  
4. $B_c$ The chicken wing from last night is rotten.  
5. $B_b$ The last beet in the box is rotten.  
6. $\sim B_a$ The apple is not rotten.

Translating other types of statements using Predicate:

**Compound Statements in Predicate**

Using the five logical operators we expand the range of statements we can create. For example, we can link simple statements with the “and,” “or,” and the conditional operators, and we can negate statements using the negation operator. Study the following examples.

<table>
<thead>
<tr>
<th>Predicate formula</th>
<th>English meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim S_n$</td>
<td>“$n$” is not a student.</td>
</tr>
<tr>
<td>$T_l \lor S_l$</td>
<td>“$l$” is either a teacher or a student.</td>
</tr>
<tr>
<td>$T_n \land T_m$</td>
<td>“$n$” is a teacher and “$m$” is a piece of chalk. OR “$n$” and “$m$” are both teachers.</td>
</tr>
<tr>
<td>$\sim T_a \Rightarrow S_b$</td>
<td>If “$a$” is not a teacher, then “$b$” is a student.</td>
</tr>
<tr>
<td>$T_a \equiv \sim S_a$</td>
<td>“$a$” is a teacher if, and only if “$a$” is not a student.</td>
</tr>
</tbody>
</table>

**Self motivated Exercise:** Again, establish another simple universe of discourse (don’t use the same one as above) and make up 6 compound statements of your own. Have someone else read them back to you. Develop questions to clarify your understanding of how to build compound statements using Predicate.

**Multi-place predicate terms**

We been working with standard categorical statements. These statements use one-place predicates. But often, when we reason, our statements express relationships between two or more things. In Predicate we can express relationships “to the right of” or “to the left of,” “shorter than,” “is the mother of,” or “happier than” by using **multi-place predicate terms**. To symbolize these we continue the convention of using a capital letter to represent the terms “to the right of,” “to left the left of,” “is shorter than,” and “is longer than” but, since these terms relate two individuals, we place two individual symbols to the right of the predicate term. For example, if we would symbolize, “$a$ is to the left of $b$,” using the formula “$L_a$. As with one-place predicate terms, we can combine relational formula with logical operators to create an ever greater variety of logically compound statements.
Examples:

~ Lam “a” is not to the left of “m.”
~Lam ⊃ Sa If “a” is not to the left of “m,” then “a” is a student.
Rbn • ~ Tb “b” is to the right of “n” and “b” is not a teacher.
Scb v Sca “c” is shorter than “b” or “c” is shorter than “a.”

Exercise A: Use the following language key to translate the statements below

Predicate terms
Ax = x is an administrator
Sx = x is a student
Tx = x is a teacher
Lxy = x is to the left of y
Bxyz = x is between y and z

Individuals
a = Alice
b = Bob
c = Carlos
d = Deviana

1. Ab v Tb
2. If Deviana is a student, then she is not an administrator.
3. ~ Lcd
4. It is not true that if Alice is a teacher, then she is not an administrator.
5. ~(Lca ⊃ ~ Bedb)
6. Carlos is between Deviana and Bob, if and only if Alice is a teacher.
7. Tb ⊃ (Sa v Ad)
8. Either Alice is to the left of Bob or Carlos is a student and Deviana is a teacher.
9. (Babc • Ldc) • (Tb v Ta)
10. If Alice is a student, then Carlos is an administrator and Bob is a student.
11. Lca • Lad
12. If Alice is a student, then she is between Deviana and Bob.
13. Bob is between Deviana and Alice if, and only if he is not to the left of Carlos.
14. (Sa ⊃ Babc) • (Ta ⊃ Lad)
15. (~ Sa • ~ Ad) ≡ (~ Tb v Lad)

Exercise B.
Use the language key below to translate the statements given. Then, use the same language key to make up at 6 statements of your own. Give both Predicate and English translations.

Universe of discourse = A company election

Vxy = x has more votes than y
Vx = x becomes vice-president
Px = x becomes president
Tx = x becomes Treasurer
a = Alice
b = Bob
c = Chris

1. Vab
2. If Alice has more votes than Bob then Alice becomes president and Bob becomes vice-president.
3. Anyone who gets more votes than anyone else, will become president.
4. (∃x) Px • (∃x) Vx
5. If Chris becomes Treasurer then either Bob will become president or vice-president.
6. (∀x)(Px ⊃ ~ Vx)
7. \((\forall x)(\forall y)(Vxy \supset \sim Tx)\)
8. Whoever does not become president or vice president, will become treasurer.
9. \([\neg (P \lor V \land Vc) \supset \neg \sim Vba]\)
10. If Alice has more votes than Bob then Alice becomes president and Bob becomes vice-president. Alice does have more votes than Bob. Therefore, Bob will be the vice-president.

**Challenge: Analyze, then translate the arguments below, using the same language key above.**

11. Whoever becomes president gets more votes than anyone else. Alice got more votes than anyone else. So, Alice gets to be president.

12. If someone gets more votes than Alice, then Alice does not become president. Alice does become president. Therefore, no one got more votes than Alice.

13. Either Alice gets more votes than Chris or Chris becomes Vice-president. If no one gets more votes than Bob, then Bob is the President. Alice does not get more votes than Chris and no one gets more votes than Bob. Therefore, Chris is Vice-president and Bob is President.

**BONUS BANK: Categorical syllogisms from philosophers**

1. Since to fight against neighbors is an evil and to fight against the Thebans is to fight against neighbors, it is clear that to fight against the Thebans is an evil.  
   *Aristotle*

2. Since morals influence action, it follows that morals cannot be derived solely from reason; and that is because whatever is derived solely from reason cannot influence action.  
   *David Hume*

3. All babies are illogical. No one is despised who can manage a crocodile. Illogical persons are despised. Therefore, no baby can manage a crocodile.  
   *Lewis Carroll*

The following arguments on God and faith from the *Summa Theologica* by St. Thomas Aquinas

1. Mercy is a kind of sorrow. There is no sorrow in God, and, therefore, there is no mercy in God.

2. God says, “I regret that I have made man.” Whoever regrets has a changeable will. Therefore, God has a changeable will.

3. Sacred doctrine is not science. For every science proceeds from self evident principles. But sacred doctrine proceeds from articles of faith and article of faith are not self evident. Therefore, sacred doctrine is not a science.

4. It would seem that the reason cannot be overcome by a passion, against its knowledge. For the stronger is not overcome by the weaker. Now knowledge, on account of its certitude, is the strongest thing in us. Therefore it cannot be overcome by a passion, which is weak and soon passes away.

5. It would seem that angels do not assume bodies. For there is nothing superfluous in the work of an angel, as there is nothing of the kind in the work of nature. But it would be superfluous for the angels to assume bodies, because an angel has no need for a body, since his own power exceeds all bodily power. Therefore an angel does not assume a body.
**Summary Chart of Logical Language Elements**

<table>
<thead>
<tr>
<th>Logical Language</th>
<th>logical operator symbols</th>
<th>Uppercase letters A-Z</th>
<th>Lowercase letters a-w</th>
<th>Quantifiers ((\forall x), (\exists x))</th>
<th>Variables</th>
<th>Parenthesis ( ) Brackets [ ] Braces { }</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sentential</strong></td>
<td>Used to represent logical operations</td>
<td>Used to represent simple, complete declarative sentences or truth claims.</td>
<td></td>
<td></td>
<td></td>
<td>p, q, r . . . Used to represent statement forms; statement variables. Can be replaced with A-Z and compound statements, as substitution instances.</td>
</tr>
<tr>
<td><strong>Predicate</strong></td>
<td>Used to represent logical operations</td>
<td>Used to represent predicate terms</td>
<td>Individual constants. Used to represent individuals, named or otherwise specified. Reserved symbols: “t” = temporarily named individual, “r” = randomly selected individual</td>
<td>Used to represent quantities “all” and “some” The existential quantifier, (\exists x), implies existence.</td>
<td>x, y, z</td>
<td>Used to delineate scope or range of quantifiers and other logical operators.</td>
</tr>
</tbody>
</table>

**Quick Review: Do you know**
- How to use a predicate function as a base for creating statements in Predicate?
- How to correctly place a negation sign to correctly express what is meant?
- How to express quantity in Predicate?
- How to translate basic categorical statements into Predicate formula?
Answers to selected exercises

PAGE 17 Exercise: Analyze the following simple arguments, identifying each statement as either a premise or a conclusion. Then, identify any indicator language.

5.  
Premise: Evolution is a viable theory, if it explains more of the phenomena it is intended to explain than any other competing theory.  
Premise: Evolution indeed explains more about the diversity of the natural world than any other theory.  
Conclusion: Evolution is a viable theory.  
Indicator language: “The conclusion is clear:” identifies the conclusion

7.  
Premise: If God exists as a supernatural and omnipotent being, then God can prevent all evil.  
Premise: If God exists as an all good being, then God would want to prevent evil.  
Premise: Evil exists.  
Conclusion: Either God is not a supernatural, omnipotent being, or God does not want to prevent evil.  
Indicator language: “The only possible conclusion is that” identifies the conclusion

9. This is a more challenging argument to analyze. Note how statements have been paraphrased, and implied premises added.  
Premise: We need to have a place to keep those who might threaten our interests.  
Premise: To bring them into the United States would allow them access to the rights accorded prisoners in our court system.  
Premise: It would be unacceptable [to accord those who threaten us the rights accorded prisoners in our court system.]  
Premise/conclusion: If no other country wants them, we are left with no other choice.  
(Implied premise: No other country wants them.)  
Conclusion: The United States needs to keep Guantanamo open.  
Indicator language: “The argument is simple:” identifies the premise set. “So” identifies a provisional conclusion that then becomes a premise for the final conclusion of the argument.

PAGE 22. Exercise: Analyze each of the following arguments, identifying premises and conclusion. Then try to show the form of the argument and identify that form.  

1.  
Premise 1: [I]f [Jan] is a faculty member, then [Jan] does not pay for a parking pass.  
Premise 2: Jan is a member of the faculty.  
Conclusion: Jan doesn’t pay for her parking pass.  
Form: If A then not B  
A  
\[ \therefore \]  
not B  
Indicator language: “because” separates premise from conclusion

66
3. Premise: If whales swim in the ocean then [whales] are fishes.  
Form: If A then B  
Premise: If they are fishes then [whales] have gills.  
If B then C.  
Conclusion: [I]f whales swim in the ocean, then [whales] have gills.  
.: If A then C  
Indicator language: “So”

5. Premise: Either the class fills to 50 percent capacity or [the class] will be dropped.  
Form: Either A or B  
Premise: The class was dropped.  
B  
Conclusion: [I]t did not fill to 50 percent capacity.  
.: Not A  
Indicator language: “Therefore”

9. This argument is challenging but not unlike ordinary reasoning. It make the context clear, some paraphrasing and implications were added.

Premise: In order to enroll in the college, I must first get a health clearance.  
Premise: I can get a health clearance only if I have had a valid TB test within the past year.  
Premise: [I]f I was born after 1957, I need an MMR vaccination, [if I am to get a health clearance].  
Premise: I was born in 1983 and although I had a valid TB test last year, I have not had an MMR vaccination.  
Conclusion: I can’t enroll in the college right now.  
Indicator language: “In addition” “So”

Form: [If] A, then B  
B only if C  
If D, then [if B, then] E.   
D and C and not E. (Interpreting “I was born in 1983” as D)  
Not A

**PAGE 28 Exercise:** Identify the type of statement for each by its **main logical operator.** Show the basic statement form of which each is a **substitution instance.**

1. A v B  
form: p v q  
11. ~ A v C  
form: ~p v q

3. (A v B) ⊃ C  
form: p ⊃ q

13. ~[(A v B) ⊃ C]  
form: ~p

5. (C • D) = (A v B)  
form: p = q

15. (A v B) • [~(D v C) v ~A]  
form: p • q

7. C • (~D = ~B)  
form: p • q

17. ~A • ~(B v C)  
form: ~p • q

9. A ⊃ C  
form: p ⊃ q

19. D = (A = B)  
Bi-conditional  
form: p = q
Exercise A: Use the language key below to translate the symbolized arguments below into English.

Let $A =$ Jan is a student. 
$B =$ Jan is a faculty member. 
$C =$ Jan pays for parking. 
$D =$ Jan can park in Lot A.

Note: Translations below present one way of translating these statements into English.

2. $B \supset D, D \supset \sim C, \therefore B \sim C$

P If Jan is a faculty member, then Jan can park in Lot A.
P If [Jan] can park in Lot A, then [Jan] does not pay for parking.
Therefore, if Jan is a faculty member, then [Jan] does not pay for parking.

4. $D = (C \vee B), D \sim B, \therefore C$

P Jan can park in Lot A if, and only if either [Jan] pays for parking or if a faculty member.
P Jan can park in Lot A and [Jan] is not a faculty member.
Conclusion: [Jan] must pay for parking.

6. $A = \sim C, C \vee B, \therefore B \sim A$

P Jan is a student if, and only if [Jan] does not pay for parking.
P Either [Jan] pays for parking or [Jan] is a faculty member.
Conclusion: If Jan is faculty, [Jan] is not a student.

8. $C = B, \sim B \supset D, D \sim C, \therefore A$

P Jan pays for parking if, and only if [Jan] is faculty.
P If [Jan] is not faculty, then [Jan] can park in Lot A.
P Either [Jan] cannot park in Lot A or [Jan] pays for parking.
Therefore, Jan is a student.

Exercise B: First do a basic analysis of each argument. Then use the language key above to translate the following arguments from English into Sentential.

2. Argument analysis:
Premise: If [Jan] is a student, then [Jan] can park in lot A only if [Jan] pays.  
P $A \supset (C \supset D)$
Premise: Jan is a student but [Jan] does not pay.  
P $A \land \sim C$
Conclusion: Jan cannot park in parking lot A  
$\therefore \sim D$

Indicator language “because”
Note: “only if” is a simple and not a bi-conditional.

4. Argument analysis:
Premise: Either Jan pays for parking or [Jan] doesn’t [pay for parking].  
P $C \lor \sim C$
Premise: If [Jan] pays for parking, then [Jan] is a student.  
P $C \supset A$
Premise: If [Jan] doesn’t pay for parking, then [Jan] is either faculty or [Jan] doesn’t park in Parking Lot A.  
P $\sim C \supset (B \lor \sim D)$
Conclusion: Either Jan is a student or if [Jan] doesn’t pay for parking, then [Jan] is faculty.

Translation:
P $C \lor \sim C$
P $C \supset A$
P $\sim C \supset (B \lor \sim D)$
$\therefore A \lor (\sim C \supset B)$
6. Argument analysis:
Premise: Jan is a student if, and only if she is not faculty.
Premise: If Jan parks in parking lot A and pays for her parking, (then) Jan is a student.
Premise: Jan parks in parking lot A but Jan does not pay.
Conclusion: Jan is faculty

Translation
\[ P \land A \equiv \neg B \]
\[ P \land (D \land C) \equiv A \]
\[ P \land D \land \neg C \]
\[ \therefore B \]

p. 37 EXERCISE C: Translate the following statements into English from Sentential, using the language key provided. Some times two options are offered which reflect the flexibility of natural language.

Language Key: Let C=Slow Jo Crow goes; R=Rose grows; M=Mo goes

Notes: Translations below present one or more ways of translating these statements into English.

1. \[ C \supset (R \land M) \]
   If Slow Jo Crow goes, then both Rose grows and Mo goes.

3. \[ M \supset (R \land C) \]
   If Mo goes, then Rose grows and Slow Jo Crow goes

5. \[ \neg [(M \land C) \supset R] \]
   possible translations:
   It is not true that if Mo go and Slow Jo Crow both go then Rose grows.
   It is not true that Mo go and Slow Jo Crow both go only if Rose grows.

7. \[ \neg R \supset \neg (C \lor M) \]
   possible translations:
   If Rose doesn’t grow, then neither Slow Jo Crow nor Mo go.
   If Rose doesn’t grow, then it’s not true that Slow Jo Crow goes or Mo goes.
   Rose doesn’t grow only if neither Slow Jo Crow nor Mo go.

9. \[ \neg \neg M \]
   It is not true that Mo does not go.
   Note: To translate this statement as: Mo goes, is to translate an inference and not what is symbolized.

11. \[ \neg (R \land M) \]
    It is not true that both Rose grows and Mo goes.

13. \[ (C \equiv R) \land \neg M \]
    Slow Jo Crow goes if, and only if Rose grows, but Mo doesn’t go.
**Arguments:** Analyze BEFORE you translate.

15. \((M \lor R) \lor \neg C, C \supset M, \neg M \therefore R \cdot M\)
   - P Either Mo goes or Rose grows, or Slow Jo Crow doesn’t go.
   - P If Slow Jo Crow goes, then Mo goes.
   - P Mo doesn’t go.
   - Conclusion: Rose grows and Mo does go.

17. \(\neg R \lor \neg M, \neg R \supset \neg (C \lor M), R \therefore C \equiv M\)
   - P Rose does not grow or Mo does not go.
   - P If Rose does not grow, then neither Slow Jo Crow nor Mo go.
   - P. Rose does grow.
   - Conclusion: Slow Jo Crow goes if, and only if Mo goes.

19. \(\neg [(M \cdot C) \supset R], C \lor R \therefore \neg M\)
   - P It is not true that if both Mo and Slow Jo Crow go, then Rose grows.
   - P Either Slow Jo Crow goes or Rose grows.
   - Conclusion: Mo doesn’t go.

---

**Exercise:** First identify the main logical operator for each statement. Then use Truth Tables to determine their truth value. Assume A and B are True and C and D are false.

1. \((A \cdot B) \supset C\) – **conditional**
   - \((T \cdot T) \supset F\)
   - \(T \supset F\)
   - \(F\)

3. \((D \lor A) \lor (C \supset B)\) – **disjunction**
   - \((F \lor T) \lor (F \supset T)\)
   - \(T \lor T\)
   - \(T\)

5. \((A \cdot B) \equiv C\) – **bi-conditional**
   - \((T \cdot T) \equiv F\)
   - \(T \equiv F\)
   - \(F\)

7. \(\neg [(A \supset B) \equiv \neg (C \supset \neg D)]\) – **negation**
   - \(\neg [(T \supset T) \equiv \neg (F \supset \neg F)]\)
   - \(\neg [ (T \supset T) \equiv \neg (F \supset T)]\)
   - \(\neg [ T \equiv \neg T ]\)
   - \(\neg [ T \equiv F ]\)
   - \(\neg F\)
   - \(T\)

9. \(\neg (A \cdot C) \equiv \neg (B \lor D)\) – **bi-conditional**
   - \(\neg (T \cdot F) \equiv \neg (T \lor F)\)
   - \(\neg F \equiv \neg T\)
   - \(T \equiv F\)
   - \(F\)

---

**Exercise:** For each of the statements below determine if a truth value can be known if A and B are true, C is false but D is unknown. You may either calculate to a single value OR end your calculations at any point where you can definitively defend your judgment. In this latter case, you should write a short, informal argument to defend your judgment. Here is a template for such an argument:

1. \(A \lor (\neg B \cdot D)\)
   - \(T \lor (\neg T \cdot ?)\)
   - \(T \lor (F \cdot ?)\)
   - The conjunction is F because one conjunct is F
   - \(T \lor F\)

3. \(\neg (A \cdot B) \supset D\)
   - \(\neg (T \cdot T) \supset ?\)
   - \(\neg T \supset ?\)
   - \(F \supset ?\)
The value of the statement is true because the statement is a conditional and the antecedent is false.

5. \( \neg (D \Rightarrow A) \)

\( \neg (\neg \top) \) The conditional is True

\( \neg \top \) because the consequent is \( \top \)

\( \neg \top \) The value of the statement cannot be determined because the statement inside the parentheses is a bi-conditional and the value of only one side is given.

9. \( (A \Rightarrow B) \Rightarrow D \)

\( (T \Rightarrow T) \Rightarrow ? \)

\( T \Rightarrow ? \) The value of this statement cannot be determined because the statement is a conditional and its antecedent is true while the value of the consequent is not known. The value of the statement will be determined by the value of the consequent.

**Exercise:** Use a truth table method to determine the truth value possibilities of the following statements. Identify each as contingent, tautology or contradiction. A full table is shown for 2 & 4, short cut for 6, final answer only for 8 and 10.

2. \( \neg A \cdot (B \lor \neg A) \) This statement is CONTINGENT

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>\neg A</th>
<th>B \lor \neg A</th>
<th>\neg A \cdot (B \lor \neg A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

4. \( \neg [A \Rightarrow (B \cdot \neg A)] \) This statement is CONTINGENT

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>\neg A</th>
<th>B \cdot \neg A</th>
<th>A \Rightarrow (B \cdot \neg A)</th>
<th>\neg [A \Rightarrow (B \cdot \neg A)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
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<td>T</td>
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<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
6. $\neg(A \lor B) \Rightarrow (\neg A \lor \neg B)$ This statement is CONTINGENT

The short cut method is used.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$\neg (A \lor B)$</th>
<th>$\neg A \lor \neg B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

8. $\neg A \lor (B \land \neg B)$ This statement is CONTINGENT

You can construct a table using either method to demonstrate this.

10. $\neg [(A \Rightarrow B) \equiv (A \land \neg B)]$ This statement is a CONTRADICTION

You can construct a table using either method to demonstrate this.

---

**Exercise:** Use either a full truth table or the short cut method to determine if the following pairs of statements are logically equivalent. The short cut method is used in 2 and 4.

2. $p \Rightarrow q \quad$ and $\quad \neg p \lor q$ The statements are LOGICALLY EQUIVALENT

<table>
<thead>
<tr>
<th>$p \Rightarrow q$</th>
<th>$\neg p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

4. $\neg (p \lor q) \quad$ and $\quad \neg p \land \neg q$ The statements are NOT LOGICALLY EQUIVALENT

<table>
<thead>
<tr>
<th>$\neg (p \lor q)$</th>
<th>$\neg p \land \neg q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
6. $p \supset q$ and $\sim (p \cdot \sim q)$ The statements are LOGICALLY EQUIVALENT

8. $p \supset q$ and $\sim q \supset \sim p$ The statements are LOGICALLY EQUIVALENT

---

PAGE 59  Exercise A. **Using the chart above**, and the language key below to translate the following into standard form categorical statements.

**Language key:**

Let:
- $Ax = x$ is a mammal
- $Bx = x$ has a backbone
- $Dx = x$ is a fish
- $Ex = x$ has sweat glands
- $Cx = x$ is viviparous

**Answer:**

1. All mammals have backbones. ($\forall x (Ax \supset Bx)$)
2. No fish have sweat glands. ($\forall x (Dx \supset \sim Ex)$)
3. Some mammals are not viviparous. ($\exists x (Ax \cdot \sim Cx)$)
4. All that have sweat glands have a backbone. ($\forall x (Ex \supset (Ax \cap Bx))$
5. All mammals have a backbone and sweat glands. ($\forall x [Ax \supset (Bx \cap Ex)]$
6. All fish have backbones but not sweat glands. ($\forall x [Dx \supset (Bx \cdot \sim Ex)]$
7. Only mammals have sweat glands. ($\forall x (Ex \supset Ax)$)
8. There are viviparous fish. ($\exists x (Dx \cdot Cx)$)
9. All those with backbones and sweat glands are mammals. ($\forall x [Bx \cap Ex \supset Ax]$)

**P 60: Exercise: Read each of the following arguments. Construct a Language Key and then translate them into Predicate**

**LANGUAGE KEY:**

Let $Ax = x$ is an angel
- $Dx = x$ is a devil
- $Wx = x$ has wings
- $Bx = x$ is a bird

1. All angels have wings. All birds have wings. Therefore, all birds are angels.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Symbolization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise: All angels have wings.</td>
<td>($\forall x (Ax \supset Wx)$)</td>
</tr>
<tr>
<td>Premise: All birds have wings.</td>
<td>($\forall x (Bx \supset Wx)$)</td>
</tr>
<tr>
<td>Conclusion: [A]ll birds are angels</td>
<td>$\therefore (\forall x (Bx \supset Ax)$</td>
</tr>
</tbody>
</table>

5. All angels have wings. All winged things fly. Therefore, all angels fly.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Symbolization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise: All angels have wings</td>
<td>($\forall x (Ax \supset Wx)$)</td>
</tr>
<tr>
<td>Premise: All winged things fly.</td>
<td>($\forall x (Wx \supset Fx)$)</td>
</tr>
<tr>
<td>Conclusion: [A]ll angels fly.</td>
<td>$\therefore (\forall x (Ax \supset Fx)$</td>
</tr>
</tbody>
</table>

10. Some devils are angels because some devils are birds and all birds are angels.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Symbolization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise: Some devils are angels.</td>
<td>($\exists x (Ax \cdot Wx)$)</td>
</tr>
<tr>
<td>Premise: Some devils are birds.</td>
<td>($\exists x (Dx \cdot Cx)$)</td>
</tr>
<tr>
<td>Conclusion: [A]ll devils are angels.</td>
<td>$\therefore (\exists x (Ax \cdot Ax)$</td>
</tr>
</tbody>
</table>
**Analysis Symbolization:**

Premise: [S]ome devils are birds

Premise: [A]ll birds are angels

Conclusion: Some devils are angels.

\[ \therefore (\exists x)(Dx \cdot Ax) \]

**Exercise C**

Let \( Ax = x \) is toxic \( Bx = x \) is poisonous \( Cx = x \) occurs in nature

\( a = \) arsenic \( b = \) vitamin C

1. \((\forall x)(Bx \supset Ax), (\exists x)(Bx \cdot Cx) \) \( /: (\exists x)(Ax \cdot Cx) \)
   
   translation: All poisonous things are toxic. Some things are poisonous and occur in nature. So, Some toxins occur in nature.

3. \((\exists x)(Bx \cdot Cx), (\exists x)(Ax \cdot Cx) \) \( /: (\exists x)(Ax \cdot Bx) \)
   
   translation: Some poisonous things occur in nature and some toxins occur in nature. Therefore, some toxins are poisonous.

5. \( Aa \cdot Cb, (\forall x)(Cx \supset \sim Bx) \) \( /: \sim (\forall x)(Ax \supset Bx) \)
   
   translation: Arsenic is toxic and vitamin C occurs in nature. Nothing that occurs in nature is poisonous. So, it’s not true that all toxins are poisonous.

**PAGES 63-64 Exercises** Use the following language key to translate the statements below

1. \( Ab \vee Tb \)
   
   Bob is an administrator or a teacher.

3. \( \sim Lcd \)
   
   Carlos is not to the left of Deviana.

5. \( \sim (Lca \supset \sim Bcdb) \)
   
   It is not true that if Carlos is to the left of Alice, then he is not between Deviana and Bob.

7. \( Tb \supset (Sa \vee Ad) \)
   
   If Bob is a teacher, then either Alice is a student or Deviana is an administrator.

9. \( (Babc \cdot Ldc) \cdot \sim (Tb \vee Ta) \)
   
   Alice is between Bob and Deviana is to the left of Carlos, but Carlos and neither Bob nor Alice is a teacher.

11. \( Lca \cdot Lad \)
   
   Carlos is to the left of Alice and Alice is to the left of Deviana.

13. Bob is between Deviana and Alice if, and only if he is not to the left of Carlos.

15. \( (~ Sa \cdot \sim Ad) \equiv (~ Tb \vee Lad) \)
   
   Alice is not a student and Deviana is not an administrator if, and only if either Bob is not a teacher or Alice is to the left of Deviana.