# The bid–ask bounce effect and the spread size effect: Evidence from the Taiwan stock market

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## Abstract

This paper examines stock return behavior on the Taiwan stock exchange which employs a call market trading mechanism. We show that, similar to the stock returns generated under a continuous trading mechanism, the call market stock prices bounce between the bid and ask price. Furthermore, we illustrate that, in a call market, bid–ask errors can induce two types of measurement errors: (1) the bid–ask bounce and (2) the spread size error. Each of these errors is likely to cause negative autocorrelation in observed returns. For stock return behavior analysis, the optimal return series, which is characterized by minimum bid–ask errors, can be defined as the returns calculated by averaging the bid and ask prices. © 1997 Elsevier Science B.V.

## 1. Introduction

The predictability of security returns is a fascinating subject, both from the view point of the academician and the practitioner. More specifically, a securities market may initially 'overreact' to new information, which will lead to a subsequent price reversal. Systematic price reversals will induce negative autocorrela-

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tion in security returns over a holding period (DeBondt and Thaler, 1985; Shefrin and Statman, 1985; De Long et al., 1990; Bremer and Sweeney, 1991). Niederhoffer and Osborne (1966) and Roll (1984) show that, given a continuous market trading mechanism, the observed security returns will be negatively correlated due to transaction prices bouncing between bid and ask prices. Also, short-run security returns of small and large firm portfolios can be positively cross-serially correlated (Lehmann, 1990; Lo and MacKinlay, 1990). Boudoukh et al. (1994) provide a reconciliation of the different schools of thought in the theory of autocorrelation for short-horizon security returns. They suggest that institutional factors (different market structures and trading/nontrading periods) are the most likely source of return autocorrelation patterns. By focusing on measurement errors in stock returns due to institutional factors, Kaul and Nimalendran (1990) examine NASDAQ stock returns and find that the predominant source of price reversals in the short run is the bid–ask error. They extract measurement errors (bid–ask spreads) in stock prices by using the bid-to-bid return series and show that there is little evidence of market overreaction.

In this study, we first examine the characteristics of stock prices on the Taiwan stock exchange (TSE) to confirm that the transaction prices generated from a call market trading mechanism also bounce between the bid and ask price. Then, we show that whenever a price is generated from a trading mechanism which causes the transaction price to bounce between the bid and ask price, the return will suffer from two types of measurement error: (1) the bid–ask bounce error and (2) the spread size error. The former has been widely studied but the latter has received little attention. As shown in Roll (1984), if the bid–ask bounce error is the only random component of the error term, except for the true return innovation, in the (observed) return series and the errors (the true return innovations) are independent and identically distributed over time, then this bid–ask bounce error will induce negative autocorrelation at only one lag in the observed returns. In order to eliminate this bid–ask bounce error caused by the trading mechanism, a return series derived from either bid-to-bid, ask-to-ask, or the mid-point of bid and ask prices should be used to generate more reliable inferences (Blume and Stambaugh, 1983; Hasbrouck and Ho, 1987; Keim, 1989; Kaul and Nimalendran, 1990). However, we show that even by taking this approach, the latter error component, the spread size error, will still exist in the return series. That is, the spread size error exists in both bid-to-bid and ask-to-ask return series even those prices do not bounce. Furthermore, due to spread asymmetry issue, the spread size error is also likely to be found in the mid-point of bid/ask return series, but perhaps to a lesser degree due to the offsetting effect in the price averaging process.

The paper is organized as follows. Section 2 introduces the price determination procedure on the Taiwan stock exchange, including the basic market clearing process and the nature of the bid and ask price under a call market trading mechanism. In Section 3, we provide an analysis of the bid–ask bounce and the spread size effect on the stock return behavior. Section 4 describes the character-

istics of the data set (provided by the TSE) and the methodology. Section 5 details the results and their implications. Section 6 contains our concluding remarks. Specific analyses utilized in the paper are described in detail in the appendices.

## 2. Price determination on the Taiwan stock exchange

The Taiwan stock exchange (TSE) is an order-driven call market. It does not utilize designated market-makers and all limit orders (market orders were suspended from trading to facilitate price determination process) are submitted by public traders. The TSE uses a periodical batch process mode to determine the market clearing price (Chang et al., 1996). Thus, for any particular transaction there exists only one price which will satisfy both sides of the market.

The market opening procedure on the New York stock exchange (NYSE) takes place under the call market trading mechanism. However, on the NYSE, for each stock there exits a market-maker (specialist) who *determines* the opening price with the assistance of the opening automated reporting system (OARS). Most of the trades at open are trades between investors. The specialist will most often participate to offset an *order imbalance* (Stoll and Whaley, 1990). The significance of the opening price relative to those of the rest of the trading day is that "[t]rading is performed at a *single* price which clears the market and which applies to all orders executed at the opening. Clearly, there is no difference between the buying and selling price, i.e. prices do not fluctuate between the bid and the ask, as is the case during the day" (Amihud and Mendelson, 1987). Hence, there is a similarity between the TSE and NYSE in terms of market opening price. However, in an order-driven call market trading system, a market open clearing price free of bid–ask errors does not imply that the market clearing prices for the remainder of the trading day will be immunized from bid–ask errors. We wish to ask two questions, on the TSE, after market open:

(1) Where do bid and ask prices come from?

(2) Do transaction prices fall *randomly* in between the bid and ask price inclusively as they do in a continuous market, such as NYSE?

By confirming the answers for these two questions, it will give us some clue whether the bid–ask spread on the TSE carries the same meaning as that from a continuous market. Now we need to know the market clearing procedures on the TSE. For the opening price determination:

(i) The market clearing price is the price that maximizes the total tradingvolume.

(ii) All buy orders with prices higher than the market clearing price and all the sell orders with prices lower than the market clearing price will be filled.

(iii) Either buy orders or the sell orders with prices equal to the market clearing price must be filled.
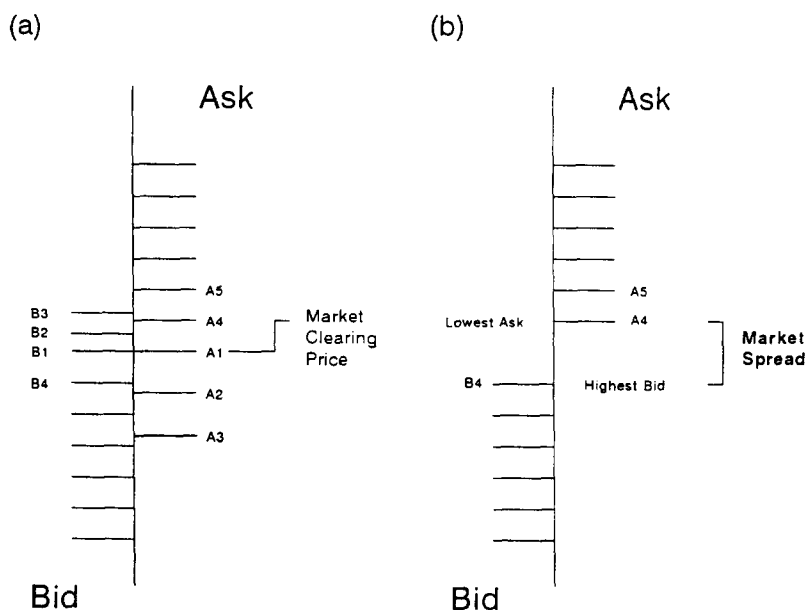
Fig. 1. The bid/ask quotes and market clearing in the Taiwan stock market.

In addition to the aforementioned criteria, there exists one additional market clearing criterion applicable to the remainder of a trading day:

(iv) If there is a bid ($P^B$) and an ask ($P^A$) price, then the market clearing price must fall between the bid ($P^B$) and the ask ($P^A$) price (inclusive).

Fig. 1 illustrates a simple market clearance scenario. In Fig. 1(a), assuming that both buy and sell orders submitted are of equal size (same number of shares), the market clearing price is set at the price $A1$ ($= B1$). At this price, all the buy orders with bid prices higher than the market clearing price and all the sell orders with the ask prices lower than the market clearing price (i.e. $B2$ and $B3$) will be filled. Furthermore, both the buy and sell limit order at the market clearing price (i.e. $A2$ and $A3$) will be filled, since they have the same order size. After market has been cleared, the highest bid price is at $B4$ and the lowest ask price is at $A4$. The distance between $A4$ and $B4$ is the market spread and the TSE discloses $A4$ and $B4$ as the ask and the bid price, respectively.

Fig. 2 represents a scenario in which, at the market clearing price, the size of some order on the demand side are not equal to the size of the orders on the supply side (i.e. $A2 = A3 = B2 = B3$, but $B1 > A1$). All the orders (from both sides) not at the market clearing price are of equal size. Thus, all the sell orders with prices equal to or below the market clearing price will be filled at the market clearing price (i.e. $A2$ and $A3$) and all the buy orders with prices above the
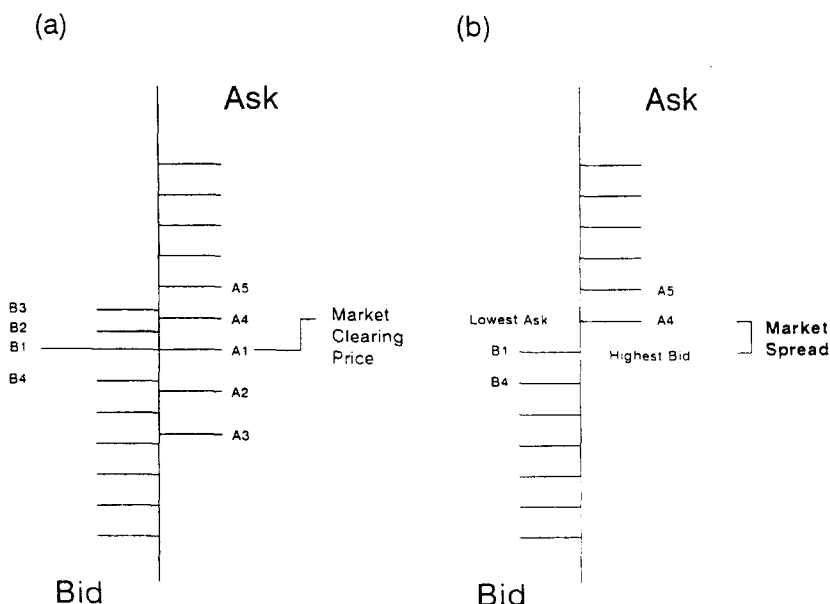
Fig. 2. The bid/ask quotes and market clearing in the Taiwan stock market.

market clearing price will be filled at the market clearing price (i.e. $B2$ and $B3$). However, the buy orders with a price equal to the market clearing price can only be partially filled. Thus, there exists excess demand at price $B1$, the market clearing price. The remaining buy orders at the market clearing price will remain on the limit order book. Hence, after the market has been cleared, the lowest ask price is $A4$, but, in contrast to Fig. 1, the highest bid price is $B1$ which is equal to the market clearing price. A similar situation exists in the case of excess supply at the market clearing price. In either case, the TSE discloses the highest bid and the lowest ask price. According to TSE trading rule (iv) (mentioned above), the next transaction price must fall in between the disclosed bid and ask price. Hence, we have illustrated that on the TSE where do bid and ask prices come from and a transaction price must fall between the bid and ask price.

What's left that we need to show is that the transaction price falls between the bid and ask price *randomly*. As stated above, the TSE trading rule *i* stipulates that the market clearing price is the price that maximizes the total trading volume. If order flow is random, to maximize trading volume, transaction prices must be random. Appendix A provides us an empirical implication of random transaction prices. That is, because there is an additional random variable, $1 + \lambda_t$, which sets the transaction price to maximize the total trading volume, the transaction return variance (Eq. (A.6), Appendix A) should be greater than ask-to-ask (or bid-to-bid)

return variance. We shall provide some empirical evidence in the later section. In the next section, we discuss that in addition to the bid–ask bounce effect, the spread size effect alone will also cause negative return autocorrelation.

## 3. The bid–ask bounce effect and the spread size effect

To show the bid–ask bounce effect and the spread size effect on stock return behavior, we utilize an approach that closely resembles the model structure in Kaul and Nimalendran (1990). A simple formulation is described in Appendix B. The intuition, however, can be explained by using Figs. 3 and 4. The conditions in Figs. 3 and 4 are basically the same, except the 'true' price in the former cases do not carry any new information, but it does in the latter cases. We define the ask spread as the distance between the ask price, the uppermost point on the vertical line and the 'true' price, the corresponding point on the horizontal line. The bid spread as the distance between the 'true' price and the bid price, the lowermost point on the vertical line. Spread symmetry means that the ask spread is equal to the bid spread at any given time point. Spread uniformity means that the spreads have a constant size (i.e. non-stochastic) over time. However, spread uniformity does not imply uniformity in the ask spreads and bid spreads over time. In Fig. 3, case (a) illustrates the case where spreads are symmetric and uniform. The return series constructed from the bid-to-bid or ask-to-ask prices parallels that of the 'true' return series. Hence, the ask-to-ask or bid-to-bid return variance and autocovariance are identical to those of 'true' return series (also for a mean return which is equal to zero). Furthermore, the spread symmetry condition here alone guarantees that the mid-points of bid–ask prices are the same as the 'true' prices. Thus, if the symmetry condition holds, a return series derived from the mid-points of bid–ask price sequence is equal to the return free of bid–ask errors.

If the spread is symmetric but non-uniform over time, as shown in case (b) of Fig. 3, the returns derived from mid-points of bid–ask prices is equal to the 'true' return series, but the return series derived from ask-to-ask or bid-to-bid prices no longer retains the same properties as those of the 'true' prices. In this case (with constant zero 'true' return series), for instance, both ask-to-ask and bid-to-bid return volatility will be larger than that of the 'true' return volatility.

In Fig. 3 case (c), spreads are uniform but asymmetric. The ask spreads and the bid spreads are uniform over time, the variance and the autocovariance of the return series derived from bid, ask, or mid-points of bid–ask prices are the same as those from the 'true' return series. The return series derived from the mid-points of bid–ask price is still the same as the 'true' return series (a series of zeros), even though the mid-points *price* sequence is not the same as that of the 'true' *price* sequence.

In Fig. 3 case (d), which is the general version of case (c), the spread size is uniform over time, but spreads are randomly asymmetric. Thus, none of the return
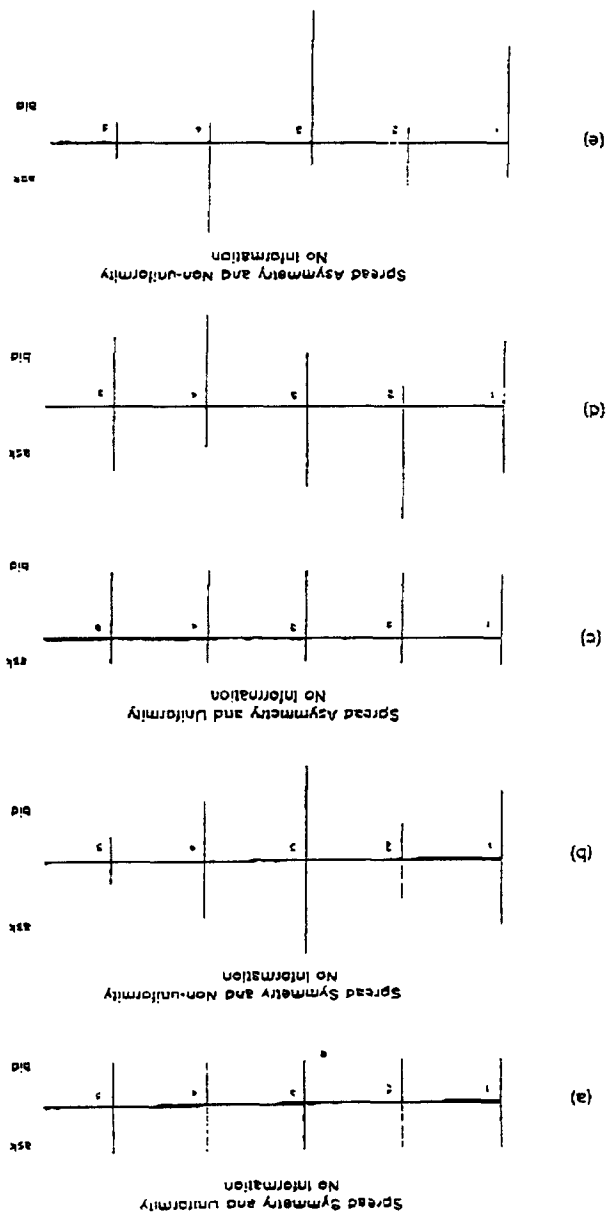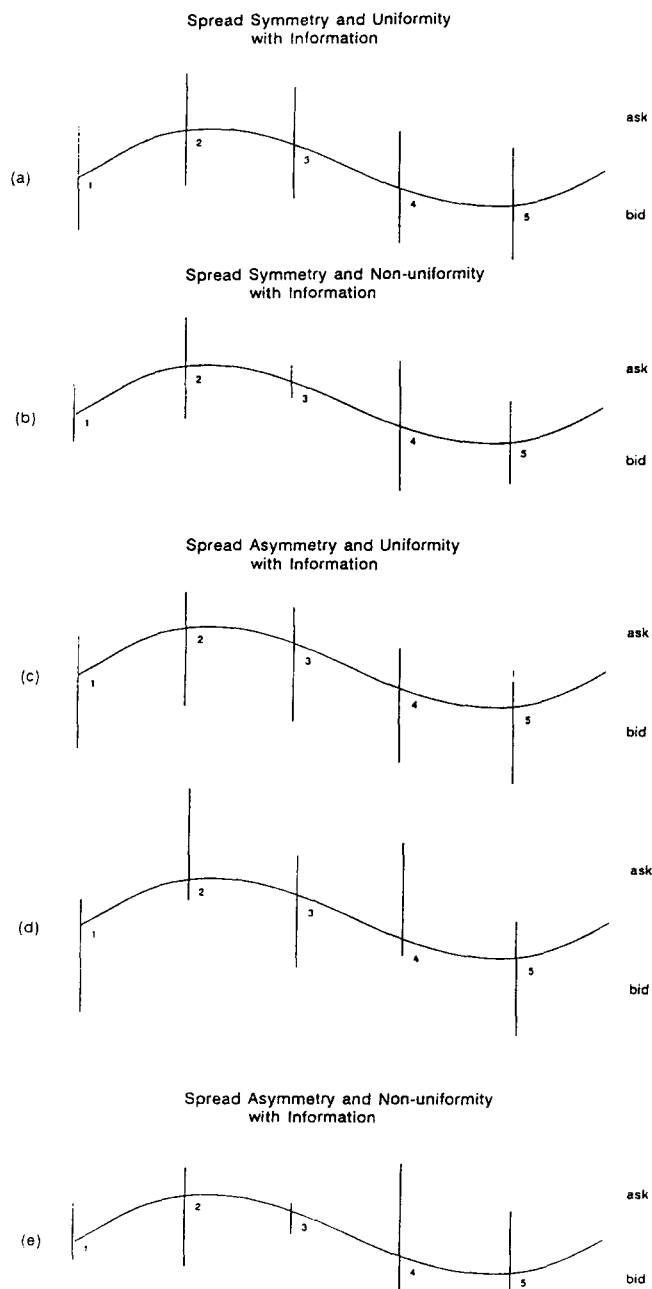
Fig. 4. Spread symmetry and asymmetry: with information.

series derived from bid, ask, or mid-points of bid–ask prices necessarily carries the same properties as that of the 'true' price sequence. The return series derived from bid-to-bid or ask-to-ask prices does not bounce between ask and bid prices, but still can be more volatile than that of the 'true' returns (so can be the mid-point return series). This is due to spread asymmetry which causes randomness in the ask spread and the bid spread. Holding everything else constant, the larger the asymmetry in spreads, the larger the volatility in the bid-to-bid and the ask-to-ask return series. If spreads are asymmetric and non-uniform over time, such as in Fig. 3 case (e), the 'true' return series cannot be easily constructed simply from bid, ask, or mid-point of bid–ask price sequence without incurring measurement errors. The reason is similar to that in Fig. 3 case (d), only reinforced by spread non-uniformity (randomness in the size of the whole bid–ask spreads).

Fig. 4 illustrates the cases of information contained in the 'true' price sequence. Hence, price changes are not necessarily equal to zero over time. Even if the spreads are symmetric and uniform over time, such as in case (a), the return series (either simple returns or continuously compounded returns) measured by ask-to-ask or bid-to-bid price sequence will not be identical to the return series measured by the 'true' price sequence. This is mainly due to non-zero price changes and the difference in measurement bases (the denominators). However, if the spread is symmetric, the mid-point of the bid and ask price sequence is still the same as the 'true' price sequence.

In Fig. 4 case (b), where information is contained in the 'true' price sequence and the spreads are symmetric but non-uniform, the return series derived from the ask-to-ask or bid-to-bid price sequence contains two components of randomness, information and the size of spreads. The difference in variance and autocorrelation between return series depends upon the correlation structure between these two random variables. For instance, if the correlation between the two random variables, information and the size of spreads, is zero or positive, the variance of the bid-to-bid return series can be greater than that of the 'true' return series. Nonetheless, the mid-point return series is the same as the 'true' return series, due to the spread symmetry property.

Since the difference between the cases illustrated in Fig. 3 and those illustrated in Fig. 4 is the information, which compounds the spread size effect, it is clear that the bid–ask bounce error is not the only component of bid–ask errors. The size of the spread is the other source of the errors in transaction prices. In fact, the bid–ask bounce error can be trivial if the size of spread is minimal (intuitively, the case where transaction prices bounce randomly between bid and ask price, but the spread is so small which makes the difference between bid and ask prices unnoticeable). The size of spread contributes to the measurement error contained in transaction prices through spread asymmetry and/or non-uniformity. The bid–ask bounce effect can be eliminated by using bid-to-bid, ask-to-ask, or the mid-point of bid–ask return series. However, none of these three series can avoid

the spread size error inherited from the trading mechanism, not to mention the transaction return series.

The bid–ask spread induces measurement errors in bid and ask prices through its size of spread which magnifies the bid–ask bounce effect in transaction prices. Hence, bid–ask errors contained in any observable return series is less likely to be completely avoided without knowing the 'true' prices.

## 4. Data and methodology

### 4.1. Data

We use transaction data generated by the Taiwan stock exchange. Our transaction data set contains transaction prices and bid and ask prices for the thirty component stocks of the composite stock price average (CSPA). These 30 stocks are the Taiwan 'blue-chips'. Their combined market value is approximately 27% of the total market capitalization on the TSE and they generated one quarter of the total trading volume during the study period. The data was gathered from the time period of January 5 through April 30, 1994, representing a total of eighty eight trading days. In order to make an intraday analysis meaningful, we exclude the fifteen Saturdays which only have a 2-hour trading period. Thus, we are left with a total of seventy three trading days with the regular 3-hour trading.

### 4.2. Methodology

In the empirical work, it contains two parts. The first part is to show that in a call market trading environment, such as the TSE, the transaction prices still bounce between bid and ask prices. Due to this additional randomness, bouncing between bid and ask prices, transaction return series should have a higher volatility (measured by variance) than that of ask-to-ask, bid-to-bid, or the mid-point return series. The second part is to test the bid–ask bounce effect and the spread size effect. The only requirement for these tests is that the transaction prices must bounce between bid and ask prices. We first estimate the return autocorrelation coefficients adjusted for small sample bias (up to ten lags) at the transaction, ask-to-ask, bid-to-bid, and the mid-point of bid–ask prices levels for each firm and report the mean of the return autocorrelations of the thirty firms. Among for four return series autocorrelation measures, we expect to see the strongest negative autocorrelation (especially the first-order autocorrelation) in the transaction-to-transaction returns series and the weakest negative autocorrelation in the mid-point of bid–ask return series. The main reason is that bid–ask spread induces two types of measurement error, the bid–ask bounce effect and the spread size effect. Both effects are contained in transaction returns, but only the spread size effect is

contained in the ask-to-ask or the bid-to-bid returns. Due to the averaging process, the mid-point of bid–ask return series should have a lesser degree [1] of the spread size effect than that of either the ask-to-ask or bid-to-bid return series. Also, to examine the relative magnitude of bid–ask errors in transaction returns in longer measurement intervals, we employ the variance ratio (VR) approach. That is,

$$VR(k) = (1/N) \cdot [(var(R^k)/(var(R)],$$

where $R$ represents the five-minute return (the base), $R^k$ stands for $k$-minute return, $k = 10, 20, 30, 60, 90$ and $180$ (market open to close) min and $N = k/5$ (French and Roll, 1986). Since under the call market trading mechanism there exist no bid and ask (disclosure) price before the market open, to compute a $k$-minute return series from transaction, ask-to-ask, bid-to-bid and the mid-point of bid–ask prices, the first available price set (i.e. transaction, ask and bid prices) are generated by the second transaction. The last pair of ask and bid prices of a trading day is generated by the last transaction of the trading day. Since there is no trading after this final transaction, it is proper not to use this last pair of ask and bid prices in the variance (or return autocorrelation) estimation procedure.

Due to the price reversal caused by bid–ask errors, we expect that the variance ratios, VR($k$), of all four return series will be less than unity or insignificantly different from one. Among the four return series, the variance ratios of the transaction return sequence should be the smallest, due to the influence of both the bid–ask bounce and the spread size effects. The variance ratios of the mid-point bid–ask return series should be the largest among the four return series. The reason is, as stated earlier, the mid-point return series suffers least from the spread size effect.

## 5. Empirical results

### 5.1. Return variance

In Table 1, we show the average return variances of the stocks in our sample, for all the 30 stocks and for the stocks in the subgroups. Each subgroup contains 15 stocks, according to firm's market capitalization or trading frequency (the average daily number of transaction). The average 30-stock transaction return variance is significantly higher than that of bid-to-bid, ask-to-ask, or the mid-point return variance. This is consistent with the hypothesis shown in Appendix A,

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[1] Again, if spreads are symmetric, then the mid-point of the bid–ask return series is the 'true' return series. In this case, we shouldn't expect any autocorrelation in the mid-point return series under the market efficiency condition.

Table 1
Average variances for 30 'blue-chip' stocks in the Taiwan stock exchange from January to April, 1994

| Average | Average transaction return variance | Average ask-to-ask return variance | Average bid-to-bid return variance | Average mid-point return variance |
|---|---|---|---|---|
| 30 stocks | 13.685 (8.113) | 4.437 (1.870) | 4.185 (1.725) | 2.816 (1.093) |
| low market cap. | 13.728 (6.260) | 5.339 (2.287) | 5.077 (1.933) | 3.182 (1.150) |
| high market cap. | 13.642 (10.206) | 3.535 (1.676) | 3.294 (1.433) | 2.450 (0.989) |
| low trading freq. | 13.596 (5.220) | 5.616 (2.296) | 5.334 (1.936) | 3.373 (1.293) |
| high trading freq. | 13.774 (10.206) | 3.259 (1.712) | 3.037 (1.449) | 2.260 (0.994) |

Variances are based on returns of transaction, ask-to-ask, bid-to-bid and the average of bid and ask price of 30 'blue-chip' stocks in the Taiwan stock exchange. We present the means and medians (in the parentheses) of those return variances of the thirty firms and by the rank of the average value of market capitalization and trading frequency. All estimates have been multiplied by $10^6$.

In all groups, among the four return series, the mean transaction return variance is the largest (Wilcoxon two sample test), followed by the mean of ask-to-ask return variance and the mean of bid-to-bid return variance. The mean min-point return variance is the smallest.

which indicates that even in a call market trading environment, such as the TSE, transaction prices still bounce between bid and ask prices, same as that in a continuous trading environment. As expected, the mid-point return series has the smallest variance is consistent with the notion of averaging effect. That is, for instance, with information (Fig. 4), if spread is symmetric, the variance of the mid-point returns is a measure of the 'true' return innovation. However, if the spread is asymmetric, the return variance of the mid-point returns is still likely to be smaller than that of ask-to-ask, or bid-to-bid. The main reason is that the ask spread is offset (at least, partially) by the bid spread. Hence, the mid-point return series does not have the full impact from either the ask or the bid spread. This result also prevails in all subgroups.

Based upon the return series derived from actual transaction, ask-to-ask, bid-to-bid, and the mid-point of bid–ask price sequences, Table 2 reports autocorrelation estimates from 1 to 10 lags. All individual stock autocorrelation estimates are corrected for small sample bias (Moran, 1948). To facilitate the examination of return behavior, we rank our thirty sample stocks according to their mean value of market capitalization and daily trading frequencies. Within each category, we evenly divide the thirty stocks into two subgroups. We average the individual autocorrelation estimates across all thirty stocks and the four subgroups in the two categories.

The mean return autocorrelation estimates of actual transactions are presented in panel A (Table 2). In general, the 30-stock transaction mean return autocorrelations are negative and are most prevalent in the first four lags. The strongest negative autocorrelation is at lag one. The magnitude of the first order autocorrelation is $-0.271$, which is more than three times as large as the second order

Table 2
Average autocorrelations of 30 'blue-chip' stocks in the Taiwan stock exchange using transaction, ask-to-ask, bid-to-bid and the mid-point returns, from January to April, 1994

| Average | Lag 1 | Lag 2 | Lag 3 | Lag 4 | Lag 5 | Lag 6 | Lag 7 | Lag 8 | Lag 9 | Lag 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: Transaction returns** | | | | | | | | | | |
| 30 stocks | −0.271 (0.015) | −0.077 (0.005) | −0.017 (0.004) | −0.008 (0.004) | 0.006 (0.002) | 0.003 (0.002) | 0.003 (0.003) | 0.008 (0.002) | 0.001 (0.002) | −0.001 (0.002) |
| Low market cap. | −0.255 (0.021) | −0.075 (0.008) | −0.023 (0.005) | −0.001 (0.004) | 0.002 (0.003) | 0.003 (0.003) | 0.001 (0.004) | 0.003 (0.003) | −0.000 (0.003) | 0.004 (0.003) |
| High market cap. | −0.288 (0.023) | −0.080 (0.007) | −0.012 (0.005) | −0.016 (0.006) | 0.009 (0.003) | 0.004 (0.002) | 0.005 (0.003) | 0.014 (0.003) | 0.003 (0.002) | −0.006 (0.003) |
| Low trading frequency | −0.255 (0.022) | −0.067 (0.006) | −0.020 (0.005) | 0.000 (0.004) | −0.001 (0.003) | 0.000 (0.003) | 0.004 (0.004) | 0.002 (0.003) | −0.001 (0.003) | −0.001 (0.003) |
| High trading frequency | −0.288 (0.021) | −0.087 (0.007) | −0.015 (0.006) | −0.017 (0.006) | 0.013 (0.002) | 0.006 (0.002) | 0.002 (0.003) | 0.014 (0.003) | 0.004 (0.002) | −0.001 (0.003) |
| **Panel B: Ask-to-ask returns** | | | | | | | | | | |
| 30 stocks | −0.097 (0.009) | −0.014 (0.007) | −0.014 (0.005) | 0.019 (0.004) | 0.006 (0.004) | 0.025 (0.004) | 0.020 (0.003) | 0.022 (0.004) | 0.005 (0.003) | 0.015 (0.002) |
| Low market cap. | −0.117 (0.014) | −0.006 (0.010) | −0.005 (0.006) | 0.024 (0.008) | 0.007 (0.008) | 0.026 (0.004) | 0.025 (0.004) | 0.013 (0.002) | 0.005 (0.004) | 0.014 (0.003) |
| High market cap. | −0.078 (0.008) | −0.021 (0.011) | −0.024 (0.006) | 0.014 (0.004) | 0.004 (0.003) | 0.023 (0.006) | 0.015 (0.004) | 0.031 (0.007) | 0.006 (0.004) | 0.015 (0.004) |
| Low trading frequency | −0.123 (0.012) | −0.004 (0.010) | 0.002 (0.005) | 0.027 (0.007) | 0.004 (0.008) | 0.027 (0.004) | 0.026 (0.004) | 0.015 (0.004) | 0.005 (0.005) | 0.013 (0.003) |
| High trading frequency | −0.072 (0.008) | −0.023 (0.011) | −0.031 (0.005) | 0.012 (0.004) | 0.007 (0.003) | 0.023 (0.006) | 0.013 (0.003) | 0.029 (0.006) | 0.006 (0.003) | 0.016 (0.004) |
| **Panel C: Bid-to-bid returns** | | | | | | | | | | |
| 30 stocks | −0.071 (0.007) | −0.001 (0.006) | −0.008 (0.006) | 0.027 (0.005) | 0.009 (0.003) | 0.027 (0.004) | 0.019 (0.003) | 0.023 (0.004) | 0.011 (0.003) | 0.021 (0.003) |
| Low market cap. | −0.087 (0.011) | 0.001 (0.008) | 0.004 (0.009) | 0.029 (0.009) | 0.008 (0.005) | 0.026 (0.006) | 0.021 (0.004) | 0.019 (0.006) | 0.010 (0.006) | 0.017 (0.004) |
| High market cap. | −0.056 (0.007) | −0.003 (0.010) | −0.020 (0.007) | 0.024 (0.006) | 0.010 (0.004) | 0.027 (0.005) | 0.018 (0.004) | 0.027 (0.006) | 0.012 (0.002) | 0.024 (0.005) |
| Low trading frequency | −0.094 (0.010) | −0.000 (0.009) | 0.010 (0.008) | 0.029 (0.009) | 0.009 (0.006) | 0.029 (0.006) | 0.018 (0.005) | 0.015 (0.006) | 0.010 (0.006) | 0.018 (0.004) |
| High trading frequency | −0.049 (0.005) | −0.003 (0.010) | −0.027 (0.006) | 0.024 (0.006) | 0.009 (0.004) | 0.025 (0.005) | 0.021 (0.003) | 0.031 (0.005) | 0.012 (0.002) | 0.024 (0.005) |
| **Panel D: Mid-point returns** | | | | | | | | | | |
| 30 stocks | −0.057 (0.006) | −0.056 (0.003) | −0.025 (0.004) | −0.010 (0.003) | −0.009 (0.003) | −0.003 (0.003) | 0.001 (0.003) | −0.001 (0.002) | −0.006 (0.002) | −0.001 (0.002) |
| Low market cap. | −0.070 (0.009) | −0.053 (0.006) | −0.019 (0.005) | −0.009 (0.005) | −0.010 (0.005) | −0.002 (0.005) | 0.004 (0.005) | 0.000 (0.003) | −0.007 (0.004) | −0.002 (0.003) |
| High market cap. | −0.044 (0.005) | −0.059 (0.003) | −0.031 (0.005) | −0.011 (0.003) | −0.006 (0.004) | −0.004 (0.003) | −0.002 (0.003) | −0.003 (0.003) | −0.006 (0.003) | −0.000 (0.003) |
| Low trading frequency | −0.075 (0.008) | −0.052 (0.007) | −0.017 (0.005) | −0.011 (0.006) | −0.011 (0.006) | −0.000 (0.005) | 0.002 (0.005) | 0.000 (0.003) | −0.008 (0.004) | −0.000 (0.004) |
| High trading frequency | −0.039 (0.004) | −0.060 (0.002) | −0.033 (0.004) | −0.010 (0.002) | −0.005 (0.002) | −0.006 (0.003) | 0.000 (0.003) | −0.003 (0.003) | −0.005 (0.003) | −0.001 (0.002) |

Autocorrelation estimates are based on transaction, ask-to-ask, bid-to-bid and the mid-point returns of 30 'blue-chip' stocks in the Taiwan Stock Exchange. We present up to 10 lagged average statistics across all firms and firms which are evenly divided into two groups according to their ranks of the average value of market capitalization and trading frequency. All the autocorrelation statistics have been corrected for small-sample bias (Moran, 1948). The numbers in the parentheses are the standard errors of the average estimates.

autocorrelation. At lag four, it sharply decreases to $-0.008$. This phenomenon also exists within the subgroups. This result supports the notion of the bid–ask bounce effect in the literature. Our findings are consistent with the transaction return autocorrelation in Hasbrouck and Ho (1987) and the *weekly* return autocorrelation patterns (all quintiles) reported in Lo and MacKinlay (1990), but contrast the *daily* return autocorrelation patterns documented in French and Roll (1986) and Kaul and Nimalendran (1990). That is, large firms in the U.S. stock markets, on average, tend to have positive daily first order return autocorrelation, [2] while smaller firms, on average, have negative daily first order return autocorrelations.

Panel B in Table 2 presents the mean ask-to-ask return autocorrelation estimates for all 30 stocks as well as for the ten subgroups in the five categories. In comparison to the positive [3] bid-to-bid autocorrelations documented in Kaul and Nimalendran (1990), our results show that the average bid-to-bid return autocorrelation is negative up to lag 3 and the higher order autocorrelations are positive but almost trivial in magnitude. The large negative first-order ask-to-ask return autocorrelation $(-0.097)$ is direct evidence in support of the existence of the spread size effect. Furthermore, the magnitude of the first order ask-to-ask return autocorrelations is significantly smaller than that of transaction return autocorrelations $(-0.097$ versus $-0.271)$, which implies the existence of the bid–ask bounce effect. It is worthwhile to note that the bid–ask bounce effect has never been independently examined before. Perhaps, this is due to the fact that it is impossible to separate the bid–ask bounce from the magnitude of the spread size (Roll (1984) assumes the size of spreads is constant over time). In other words, without the size of the spread, there would be no bid–ask bounce effect or it would be undetectable. The size of spread is a necessary condition to make the measure of the bid–ask bounce effect meaningful. As shown in panel C of Table 2, similar results are also found for the bid-to-bid return autocorrelations.

The autocorrelations for the mid-point of bid–ask returns are negative (panel D in Table 2) in almost all lags and this result is common to all four subgroups. The mean magnitude of the first order autocorrelation of the 30 stocks is $-0.057$ which is significantly smaller than that of ask-to-ask. This finding shows that the bid–ask averaging process reduces the force of the spread size effect. Moreover, it

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[2] Kaul and Nimalendran (1990) find that in their NASDAQ sample the bid–ask error component of transaction returns can explain over 50% of daily return variances. They further point out that the impact in return variance from bid–ask error is likely to be even more severe in intraday data. The bid–ask spread model in Roll (1984) indicates that if the bid–ask errors are independently and identically distributed over time, then the negative return autocorrelation due to the bid–ask errors can only last up to lag 1. Hence, it is reasonable to see that the trade-to-trade return autocorrelation is negative for both large and small stocks (e.g. in terms of market capitalization).

[3] Kaul and Nimalendran (1990) use over-lapping observations to get more efficient estimates. We are concerned that when autocorrelation estimation is performed, part of its covariance is actually variance which is non-negative. The proportion of the variance in the covariance measure is positive related to the portion of the over-lapping.

also shows that bid–ask spread is not symmetric over time. This is consistent with the hypothesis that if spreads are symmetric, then the mid-point return autocorrelation should be insignificantly different from zero. However, if the spread is asymmetric (i.e. ask spreads are not equal to bid spreads), then the spread size effect will still exist, but possibly to a lesser degree, in the mid-point return series. Our finding is consistent with the result in Hasbrouck and Ho (1987) who also found a negative mid-point return first-order autocorrelation whose magnitude is much smaller than that of transaction return first-order autocorrelation. We discover an interesting phenomenon that low market capitalization firms tend to have larger ask-to-ask, bid-to-bid or the mid-point first order return autocorrelation. This phenomenon also prevails in low trading frequency subgroup. However, for transaction returns, in each category. all two high subgroups have larger first order return autocorrelations than those in the low subgroups. This phenomenon implies that a firm's characteristics (e.g. firm size and trading frequency) affect the bid–ask bounce and spread size effects.

## 5.2. Intraday pattern of the first-order autocorrelation

Since the first order autocorrelation plays a dominant role in the return autocorrelations, we further examine whether there is an intraday pattern within the first order return autocorrelation and how it relates to the bid–ask bounce and the spread size effect. Niederhoffer and Osborne (1966) and Roll (1984) show that the bid–ask spread causes price reversals and these reversals are most likely to induce first order autocorrelation (assuming a transaction is equally likely occurs at the bid or the ask price) rather than higher order autocorrelations, thus, we only examine the first order return autocorrelation. The theory of intraday autocorrelation analysis is that, with information asymmetry, if an informed trader prefers to only trade once at a given price on a particular trading day, conditional on her/his private information on true value of the underlying asset, then it is unlikely the return correlation will have an intraday pattern. However, if an informed trader attempts to camouflage her/his trading activity by making several smaller (volume) trades (i.e. strategic trading), rather than one large trade (Kyle, 1985; Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990), then there may exist an intraday pattern in return behavior. More specifically, if an informed trader likes to trade when the market is deep and trade in several smaller transactions, then, holding everything else constant, the return autocorrelation in the transaction, ask-to-ask, bid-to-bid, or the mid-point of bid–ask return series will be more positively (or less negatively) correlated during certain time periods of the trading day. Note that the autocorrelation is more positive (less negative) due to positive autocorrelation generated from the information (due to the trading strategy suggested above) which is *gradually* incorporated into prices through trading. However, transaction returns still contain the effect of the negative autocorrelation generated by the

bid–ask errors. Hence, if trading is heavier around market open and close [4] and informed traders split their trading into several smaller trades, then we should see a return autocorrelation that is less negatively correlated during these periods. We employ the model used in Keim and Stambaugh (1984) and Bessembinder and Hertzel (1993). That is,

$$r_{i,t} = a_0 + \sum_{j=2}^{6} b_j + \sum_{j=2}^{6} c_i \cdot D_{j,t} \cdot r_{i,t-1} + d \cdot r_{i,t-1} + \epsilon_{i,t},$$

where $r_{i,t}$ = a return measure in interest of stock $i$ in period $t$ i.e. trade-to-trade returns, ask-to-ask, or bid-to-bid returns. $D_{j,t}$ = indicator variables for five 30-minute intervals during the day, They are 1st, 2nd, 3rd, 5th and 6th 30-minute trading intervals. $\epsilon_{i,t}$ = random error terms.

Table 3 reports the results from an expanded version of the dummy regression model in Keim and Stambaugh (1984) and Bessembinder and Hertzel (1993). In this expanded model, we consider the interactive effects between the sub-trading intervals and the first lagged returns. For the 4th 30-minute (about in the middle of a trading day) interval in a trading day, we find that the first order transaction return autocorrelation is significantly negative (as reported before). The interaction terms of the 2nd, 3rd and 5th 30 min trading intervals is generally not significantly different from that of 4th 30 min interval (there are only 4, 6 and 6 stocks significant out of total 30 stocks, respectively). However, for all 30 stocks, the first 30 min trading interval has a positive interaction term which implies that its negative first-order autocorrelation is less than that of the 4th 30 min trading interval. A similar results (i.e. positive interactive effects) is also found for the last 30 min trading interval, although there are two stocks which have insignificant interactive effects.

After controlling for the subperiod interaction effect, the mean first order return autocorrelations of the ask-to-ask, bid-to-bid and mid-point returns are all negative. Most of their subperiod interactive terms are not significant. This implies that the intraday patterns in these return autocorrelations, due to the spread size effect and/or spread asymmetry, are not as prominent as those found in the transaction returns. The less negative first order transaction return autocorrelation that exists in the first and last 30 min trading interval is possibly due to the mitigated effect of the bid–ask bounce (suggested in (Roll, 1984) and (Kaul and Nimalendran, 1990)) due to a series of small-sized informed trades (Kyle, 1985; Admati and Pfleiderer, 1988; Foster and Viswanathan, 1990) around market open and close. The spread size effect does not seem to be influential in this case because there is an insignificant intraday pattern in the ask-to-ask, bid-to-bid and mid-point returns,

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[4] The concentrated trading around market open and close can be due to over night information/liquidity accumulation (Admati and Pfleiderer, 1988; Brock and Kleidon, 1992) and market close delays the resolution of uncertainty and redistributing risk across time and agents. Thus, there is additional trades based on risk-sharing motives occurs (Slezak, 1994).

Table 3

The interaction between 30-minute trading intervals and the first order return autocorrelation for 30 'blue-chip' stocks in the Taiwan stock exchange using transaction, ask-to-ask, bid-to-bid and the mid-point of bid–ask returns, from January to April, 1994

| 30-stock average | 1st 30 min (c2) | 2nd 30 min (c3) | 3rd 30 min (c4) | 5th 30 min (c5) | 6th 30 min (c6) | Lagged dependent, d |
|---|---|---|---|---|---|---|
| Transaction returns | 0.107 (0.005) [30/30] | 0.011 (0.006) [4/30] | 0.008 (0.010) [6/30] | 0.016 (0.005) [6/30] | 0.121 (0.007) [28/30] | −0.322 (0.015) [30/30] |
| Ask-to-ask returns | 0.045 (0.013) [9/30] | 0.010 (0.009) [8/30] | 0.024 (0.015) [11/30] | 0.005 (0.010) [7/30] | 0.031 (0.010) [7/30] | −0.121 (0.014) [27/30] |
| Bid-to-bid returns | 0.022 (0.014) [9/30] | 0.001 (0.010) [6/30] | 0.003 (0.010) [8/30] | 0.007 (0.011) [6/30] | 0.028 (0.012) [11/30] | −0.087 (0.010) [26/30] |
| Mid-point of bid–ask returns | 0.009 (0.006) [4/30] | 0.004 (0.007) [5/30] | −0.002 (0.007) [5/30] | 0.008 (0.008) [5/30] | 0.016 (0.006) [5/30] | −0.063 (0.008) [18/30] |

We regress transaction returns of 30 'blue-chip' stocks in the Taiwan stock exchange on its first lagged return, the 30 min trading interval dummies and the interactions. That is, $r_t = a + \sum_{j=2}^{6} b_j \cdot D_j + \sum_{j=2}^{6} c_j \cdot D_j \cdot r_{t-1} + d \cdot r_{t-1} + \epsilon_t$, where $r_t$ represents the variable in interest i.e. trade-to-trade, ask-to-ask, bid-to-bid returns, or the mid-point of bid–ask returns, $D_j$ corresponds to the 1st, 2nd, 3rd, 5th and 6th 30 min trading intervals in a day. Then the trading interval interaction estimates are averaged across all thirty firms. The numbers in the parentheses are the standard errors of the averages, and the ratios in bracket are the number of significant (at least 10% level) coefficients over the total number of firms.

in contrast to the intraday pattern found in the transaction returns. We will present additional evidence in the next section.

### 5.3. Return variance ratios

Table 4 reports variance ratio estimates based on transaction, ask-to-ask, bid-to-bid and the mid-point returns, for six measurement trading intervals (non-overlapping) of $k$-minutes ($k = 10$, 20, 30, 60, 90 and 180 (i.e. open-to-close minutes). The return variance ratios compare $k$-minute return variances to the base case 5 min return variance (French and Roll, 1986). All reported variance ratios are adjusted for small sample bias (Moran, 1948). Panel A in Table 4 shows the transaction based return variance ratio estimates. These return variance ratios are all less than unity and typically decrease as the measurement interval increases, which indicates that price reversals are a function of the length of the measurement interval. The only exception to this pattern is for the open-to-close to 5 min variance ratio which is greater than the 90 min to 5 min variance ratio. This suggests that the open to close transaction return variance is still subject to price reversals (the bid–ask bounce and/or the spread size effects), but only to a lesser degree. This is consistent with the finding in Table 3 that the transaction return autocorrelation around the market open and close is less negative than during the rest of a trading day. This phenomenon can be found in all ten subgroups. Our finding that as the measurement interval increases, the mean transaction variance ratio decreases, up to $k = 90$, but that between $k = 90$ and $k = 180$ the ratio increases, put our finding between those of French and Roll (1986) and Kaul and Nimalendran (1990). In the former study, the variance ratios (their Table 4, all NYSE and AMEX firms) decrease as the measurement interval increases, but in the latter study, the variance ratios increase (their Table 2, NASDAQ firms). Their results are produced from daily data.

Panel B and C in Table 4 show the ask-to-ask and the bid-to-bid return variance ratios. In each table, the variance ratio's intraday pattern is initially decreasing and then finally increasing, which is similar to that of the transaction return variance

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Note to Table 4:

Variance ratios are based on transaction, ask-to-ask, bid-to-bid, and mid-point returns of 30 'blue-chip' stocks in the Taiwan stock exchange. We present average statistics across thirty firms and by the rank of the average value of market capitalization and trading frequency. The variance ratio statistic is defined as $VR(k) = (1/N)[var(R^k)/var(R)]$, where $R$ is the 5-minute return (the base), $R^k$ is the $k$-minute return, $k = 10$, 20, 30, 60, 90 and 180 (open to close) min and $N = k/5$. All the variance ratios are corrected for small-sample bias. All statistics are estimated for each firm, using non-overlapping data to calculate the $k$-minute (including 5-minute) return variance from January to April, 1994. The individual-firm statistics are averaged across firms within each group. The numbers in the parentheses are the standard errors of the average ratios.

Table 4

Variance ratios for 30 'blue-chip' stocks in the Taiwan stock exchange using transaction, ask-to-ask, bid-to-bid and mid-point returns from January to April, 1994

| Average | 10 min | 20 min | 30 min | 60 min | 90 min | 180 min |
|---|---|---|---|---|---|---|
| Panel A: Transaction returns | | | | | | |
| 30 stocks | 0.756 (0.015) | 0.561 (0.017) | 0.530 (0.018) | 0.467 (0.020) | 0.444 (0.021) | 0.510 (0.029) |
| Low market cap. | 0.756 (0.013) | 0.556 (0.021) | 0.520 (0.024) | 0.461 (0.025) | 0.437 (0.028) | 0.482 (0.035) |
| High market cap. | 0.757 (0.028) | 0.566 (0.027) | 0.539 (0.028) | 0.474 (0.032) | 0.452 (0.030) | 0.539 (0.045) |
| Low trading frequency | 0.742 (0.015) | 0.535 (0.019) | 0.508 (0.022) | 0.441 (0.022) | 0.430 (0.027) | 0.493 (0.040) |
| High trading frequency | 0.770 (0.027) | 0.587 (0.027) | 0.552 (0.029) | 0.494 (0.033) | 0.459 (0.031) | 0.528 (0.041) |
| Panel B: Ask-to-ask returns | | | | | | |
| 30 stocks | 0.925 (0.017) | 0.772 (0.016) | 0.779 (0.023) | 0.734 (0.025) | 0.721 (0.028) | 0.797 (0.034) |
| Low market cap. | 0.895 (0.029) | 0.753 (0.027) | 0.720 (0.035) | 0.691 (0.039) | 0.663 (0.046) | 0.729 (0.046) |
| High market cap. | 0.956 (0.014) | 0.791 (0.017) | 0.837 (0.021) | 0.776 (0.029) | 0.778 (0.027) | 0.866 (0.044) |
| Low trading frequency | 0.894 (0.027) | 0.738 (0.022) | 0.719 (0.029) | 0.680 (0.032) | 0.671 (0.041) | 0.772 (0.057) |
| High trading frequency | 0.957 (0.017) | 0.806 (0.020) | 0.838 (0.029) | 0.787 (0.034) | 0.771 (0.036) | 0.823 (0.038) |
| Panel C: Bid-to-bid return | | | | | | |
| 30 stocks | 0.948 (0.013) | 0.807 (0.016) | 0.813 (0.018) | 0.765 (0.020) | 0.762 (0.026) | 0.858 (0.037) |
| Low market cap. | 0.941 (0.017) | 0.804 (0.027) | 0.794 (0.030) | 0.766 (0.027) | 0.751 (0.043) | 0.807 (0.048) |
| High market cap. | 0.955 (0.020) | 0.810 (0.018) | 0.832 (0.020) | 0.765 (0.029) | 0.774 (0.032) | 0.909 (0.055) |
| Low trading frequency | 0.916 (0.015) | 0.780 (0.022) | 0.785 (0.028) | 0.754 (0.030) | 0.759 (0.044) | 0.834 (0.059) |
| High trading frequency | 0.980 (0.018) | 0.834 (0.021) | 0.841 (0.021) | 0.777 (0.026) | 0.766 (0.031) | 0.882 (0.046) |
| Panel D: Mid-point returns | | | | | | |
| 30 stocks | 1.013 (0.017) | 0.900 (0.019) | 0.918 (0.026) | 0.882 (0.029) | 0.882 (0.040) | 0.914 (0.042) |
| Low market cap. | 1.025 (0.027) | 0.939 (0.025) | 0.911 (0.040) | 0.912 (0.040) | 0.890 (0.064) | 0.908 (0.061) |
| High market cap. | 1.000 (0.021) | 0.861 (0.024) | 0.925 (0.036) | 0.852 (0.043) | 0.874 (0.049) | 0.920 (0.060) |
| Low trading frequency | 1.022 (0.028) | 0.929 (0.027) | 0.939 (0.040) | 0.918 (0.040) | 0.927 (0.062) | 0.966 (0.061) |
| High trading frequency | 1.004 (0.021) | 0.871 (0.024) | 0.898 (0.035) | 0.847 (0.041) | 0.837 (0.048) | 0.862 (0.056) |

ratios for 30 stock average and the ten subgroups. Though these variance ratios are consistently less than unity, they are greater than those transaction return variance ratios, which further supports the hypothesis of the spread size effect and is indirectly consistent with the bid–ask bounce effect. Our findings contrast the findings in Kaul and Nimalendran (1990). They show that the bid-to-bid return variance ratios tend to be greater than unity as the measurement interval increases (their Table 3). Panel D in Table 4 shows the variance ratios of the mid-point bid–ask returns have the same pattern as those of the transaction, ask-to-ask and bid-to-bid return variance ratios. However, the mid-point return variance ratios are greater than those of the ask-to-ask or bid-to-bid return variance ratios. This is consistent with the notion of the price averaging effect and at the same time supports the spread asymmetry hypothesis. That is, if spreads are symmetric, then the variance ratios of the mid-point return should be insignificantly different from unity.

## 6. Conclusion

In this paper, we study the nature of the bid–ask spread on the Taiwan stock exchange. We first show that, like stock prices under a continuous market trading mechanism, the stock prices on the TSE, on which stocks are traded under the call trading mechanism, jump between bid and ask prices. We further examine the significance of bid–ask errors in return behavior which has been examined by Roll (1984) and Kaul and Nimalendran (1990). We propose that there are two major forces, the bid–ask bounce and the spread size errors, in bid–ask errors which may cause price reversals in a market in which the trading mechanism causes prices to bounce between bid and ask prices, such as the Taiwan stock market. Previous studies emphasize the bid–ask bounce effect which is viewed as the major source of price reversals. We show analytically that bid–ask errors induce both the bid–ask bounce and the spread size effect and they each will bias the observed transaction return behavior. We also show that the spread symmetry issue further compounds the effect of bid–ask errors in the observed return behavior. Following the approach in French and Roll (1986) and Kaul and Nimalendran (1990), we conduct return autocorrelation estimation and variance ratios tests. We find that the transaction return autocorrelations are negative with significant intraday patterns. That is, the returns around the market open and close are less negative than those of the rest of a trading day. This phenomenon probably is due to a series of small-size information trades around market open and close. The ask-to-ask and bid-to-bid return autocorrelations are also negative but to a lesser degree than those of the transaction returns. This finding serves as direct evidence of the existence of the spread size effect in both the ask-to-ask (and the bid-to-bid) and the observed transaction returns and indirectly supports

the existence of the bid–ask bounce effect in the observed transaction returns. This finding is important, because it implies that the spread size is a necessary condition for the existence of a significant bid–ask bounce effect, whether the size of spread is constant or stochastic over time (i.e. the bid–ask bounce effect can exist but may not be significant if the size of spread is trivial). Moreover, in our analysis, both the bid–ask bounce and the spread size effect can cause negative return autocorrelation. However, empirically, the bid–ask bounce effect cannot be separated from the size of spreads, since we consider the spread size a random variable (but we can isolate the spread size effect without involving the bounce effect). Hence, without justifying the existence of the spread size effect, it would be unclear what effect contributes the negative autocorrelation to the observed transaction returns. We also examine the mid-point of bid–ask return autocorrelations. We find that the mid-point return series has negative autocorrelations which support the hypothesis of spread asymmetry.

To expand the study to longer intervals, we employ variance ratio tests. The results are consistent with those found in the return autocorrelations. The variance ratios from various measurement intervals are less than unity for all observed transaction, ask-to-ask, and bid-to-bid returns. The ask-to-ask (bid-to-bid) return variance ratios are greater than those of observed transaction returns which is consistent with our findings in return autocorrelations and support the bid–ask bounce and spread size effects. The return variance ratios of the mid-point returns demonstrate a similar pattern as the ask-to-ask and bid-to-bid returns, but with a greater magnitude. This is consistent with the hypothesis of spread asymmetry.

The conclusion of our finding is that bid–ask errors exists not only in a market governed by a continuous market trading mechanism, but also in a market governed by a call market trading mechanism. The bid–ask spreads may contain two types of measurement error in the observed returns. Using ask-to-ask or bid-to-bid returns does not eliminate bid–ask errors in returns, because of the existence of the spread size effect. The mid-point of the bid–ask prices may be a better candidate if the spread asymmetry issue is insignificant. When conducting an empirical study, we must be aware of the characteristics of the observed return series and choose one with minimum measurement errors to avoid data mining as indicated in (Black, 1993).

# Acknowledgements

## Appendix A

In this appendix, we want to show that if transaction prices bounce randomly between bid and ask prices in order to maximize trading volumes (trading rule (iv) mentioned in Section 2), then the transaction return volatility should be greater than that of ask-to-ask returns or bid-to-bid returns.

Let $P_t$ be the (unobservable) price which is free of the bid–ask errors at time $t$. Define the continuous compounded rate of return on $P_t$ as

$$R_t = \ln P_t - \ln P_{t-1} = \mu + \eta_t, \tag{A.1}$$

where $\mu$ = unconditional mean of $R_t$ in period $t$, $\eta_t$ = idiosyncratic white noise, where $\eta_t \sim N(0, \sigma_\eta^2)$.

Let $s_t^A$ be the upper bound of the observable transaction price $P_t^*$, where $s_t^A$ is expressed as a percentage of the (unobservable) transaction price at time $t$. Similarly, $s_t^B$ is the lower bound of the transaction price $P_t^*$, where $s_t^B$ is expressed as a percentage of the (unobservable) transaction price at time $t$. We have

$$P_t^A = P_t + P_t \cdot s_t^A = P_t \cdot \left(1 + s_t^A\right).$$

Thus,

$$\ln P_t^A = \ln P_t + \ln\left(1 + s_t^A\right),$$

$$P_t^B = P_t - P_t \cdot s_t^B = P_t \cdot \left(1 - s_t^B\right). \tag{A.2}$$

Thus,

$$\ln P_t^B = \ln P_t + \ln\left(1 - s_t^B\right). \tag{A.3}$$

We define the transaction price $P_t^*$ at time $t$ as

$$P_t^* = P_t \cdot \left(1 + s_t^A\right)/\left(1 + \lambda_t\right),$$

$$\ln P_t^* = \ln P_t + \ln\left(1 + s_t^A\right) - \ln\left(1 + \lambda_t\right), \tag{A.4}$$

where $(1 + \lambda_t)$ is a random variable which sets $P_t^*$ to maximize the transaction volume in period $t$ and $0 \leqslant \ln(1 + \lambda_t) \leqslant (\ln(1 + s_t^A) - \ln(1 - s_t^B))$. Note that $\ln(1 + s_t^A) - \ln(1 - s_t^B)$ is the full spread at time $t$.

According to the trading rules on the TSE, the upper bound price for the transaction price $P_t^*$ is the ask (disclosure) price, $P_t^A$. Similarly, the lower bound price for the transaction price $P_t^*$ is the bid (disclosure) price. Hence,

$$P_t^A = P_t \cdot \left(1 + s_t^A\right) \quad \text{and} \quad P_t^B = P_t \cdot \left(1 - s_t^B\right).$$

The return and its variance of $P_t^A$ are

$$R_t^A = \ln P_t^A - \ln P_{t-1}^A$$

$$= \left[\ln P_t + \ln\left(1 + s_t^A\right)\right] - \left[\ln P_{t-1} + \ln\left(1 + s_{t-1}^A\right)\right]$$

$$= R_t + \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right]$$

$$= \mu + \eta_t + \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right],$$

where $[\ln(1 + s_t^A) - \ln(1 + s_{t-1}^A)]$ is the change in the ask spreads from time $t - 1$ to time $t$.

$$\operatorname{Var}\left(R_t^A\right) = \operatorname{Var}(\eta_t) + \operatorname{Var}\left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right]$$

$$+ 2 \cdot \operatorname{Cov}\left\{\eta_t, \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right]\right\}. \tag{A.5}$$

The (observable) transaction return $R_t^*$ and its variance

$$R_t^* = \ln P_t^* - \ln P_{t-1}^*$$

$$= \left[\ln P_t + \ln\left(1 + s_t^A\right) - \ln\left(1 + \lambda_t\right)\right]$$

$$- \left[\ln P_{t-1} + \ln\left(1 + s_{t-1}^A\right) - \ln\left(1 + \lambda_{t-1}\right)\right]$$

$$= R_t + \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right] - \left[\ln\left(1 + \lambda_t\right) - \ln\left(1 + \lambda_{t-1}\right)\right]$$

$$= \mu + \eta_t + \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right] - \left[\ln\left(1 + \lambda_t\right) - \ln\left(1 + \lambda_{t-1}\right)\right].$$

$$\operatorname{Var}\left(R_t^*\right) = \operatorname{Var}(\eta_t) + \operatorname{Var}\left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right]$$

$$+ \operatorname{Var}\left[\ln\left(1 + \lambda_t\right) - \ln\left(1 + \lambda_{t-1}\right)\right]$$

$$+ 2 \cdot \operatorname{Cov}\left\{\eta_t, \left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right]\right\}$$

$$+ 2 \cdot \operatorname{Cov}\left\{\eta_t, \left[\ln\left(1 + \lambda_t\right) - \ln\left(1 + \lambda_{t-1}\right)\right]\right\}$$

$$+ 2 \cdot \operatorname{Cov}\left\{\left[\ln\left(1 + s_t^A\right) - \ln\left(1 + s_{t-1}^A\right)\right],\right.$$

$$\left.\left[\ln\left(1 + \lambda_t\right) - \ln\left(1 + \lambda_{t-1}\right)\right]\right\}. \tag{A.6}$$

If $\ln(1 + \lambda_t) = c$, $c \in \Re^+$ (a finite positive constant), $\forall t$, i.e. a transaction price always occurs at or in a constant distance (in percentage) from the ask price at time $t$, $\forall t$, then $\operatorname{Var}(R_t^*)$ is equal to $\operatorname{Var}(R_t^A)$.

If $\ln(1 + \lambda_t)$ is random, $\forall t$, and $\eta_t$ is uncorrelated with $[\ln(1 + s_t^A) - \ln(1 + s_{t-1}^A)]$ and $[\ln(1 + \lambda_t) - \ln(1 + \lambda_{t-1})]$, and $[\ln(1 + s_t^A) - \ln(1 + s_{t-1}^A)]$ is uncorrelated with $[\ln(1 + \lambda_t) - \ln(1 + \lambda_{t-1})]$, then it is straight forward to see that Eq. (A.6) is greater than Eq. (A.5) because Eq. (A.6) has an extra term, $\operatorname{Var}[\ln(1 + \lambda_t) - \ln(1 + \lambda_{t-1})]$.

Thus, we have shown that the variance of transaction returns can be greater than that of the ask-to-ask or the bid-to-bid returns. The reason of a higher transaction return variance is due to the randomness of price movement between the bid and ask price to maximize the trading volume.

## Appendix B

In this section, we show that the return behavior in a market in which the trading mechanism causes transaction prices jump between ask and bid prices. We intend to explain that the bid–ask errors potentially introduce two kinds of randomness into transaction prices. Both of them potentially introduce additional variation in the return volatility. If one of them is likely to cause price reversals (Roll, 1984; Kaul and Nimalendran, 1990), then, by controlling it, it is interesting to see how the other one affects the price behavior, assuming the market is efficient.

Let $P_t^*$ be the transaction prices from a market in which trading prices jump between ask and bid prices due to trading mechanism. We set

$$P_t^* = P_t + P_t \cdot s_t \cdot Q_t = P_t \cdot [1 + s_t \cdot Q_t], \tag{B.1}$$

where $P_t$ = unobservable transaction prices without bid–ask errors, $s_t$ = random error component induced by the ask or bid quotation and it is expressed as a fraction of $P_t$ (i.e. $0 \leqslant s_t < 1$), $Q_t$ = unobservable random indicator for the bid–ask classification of the transaction price ($Q_t = 1$, if transaction $t$ is at the ask and $Q_t = -1$, if it is at the bid.

Let $R_{i,t}^*$ be the continuously compounded rate of return of security $i$ based on observable transaction prices at time $t - 1$ and $t$. From Eq. (B.1), it follows that

$$
\begin{aligned}
R_{i,t}^* &= \ln P_{i,t}^* - \ln P_{i,t-1}^* \\
&= \ln\left[ P_{i,t} \cdot (1 + s_{i,t} \cdot Q_{i,t}) \right] - \ln\left[ P_{i,t-1} \cdot (1 + s_{i,t-1} \cdot Q_{i,t-1}) \right] \\
&= \ln P_{i,t} - \ln P_{i,t-1} + \ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1}) \\
&= \mu + \eta_{i,t} + \left[ \ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1}) \right], \tag{B.2}
\end{aligned}
$$

where $\mu$ is the unconditional mean of $R_{i,t}$ which is the return derived from unobservable transaction prices, $\eta_{i,t}$ is the innovation of $R_{i,t}$ in period $t$ (thus, $R_{i,t} = \mu + \eta_t$), and the third term in the bracket is the change in the bid–ask errors. Note that this bid–ask errors contain both bounce ($Q_{i,t}$ and $Q_{i,t-1}$) and size ($s_{i,t}$ and $s_{i,t-1}$) effects which will be discussed later.

By taking variance measure on Eq. (B.2), we have

$$
\begin{aligned}
\mathrm{Var}(R_{i,t}^*) = {}&\mathrm{Var}(\eta_{i,t}) + \mathrm{Var}\left[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})\right] \\
&+ 2 \cdot \mathrm{Cov}\left[\eta_{i,t}, \ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})\right]. \tag{B.3}
\end{aligned}
$$

Now we can have the following analysis on Eq. (B.3):

(1) If both $s_{i,t}$ and $Q_{i,t}$ are constant i.e. $s_{i,t}$ is time invariant and $Q_{i,t}$ always takes value 1 or always takes value $-1$ (in other words transaction prices are

always occurred at bid prices or always occurred at ask prices), then the volatility behavior of $R_{i,t}^{*}$ will be the same as that of $R_{i,t}$. That is,

$$\text{Var}(R_{i,t}^{*}) = \text{Var}(\eta_{i,t}) \tag{B.4}$$

(2) If both $s_{i,t}$ and $Q_{i,t}$ are stochastic i.e. $s_{i,t}$ is a random variable ($0 \leqslant s_{i,t} < 1$) and $Q_{i,t}$ is an independent indicator random variable and $\ln[1 + s_{i,t} \cdot Q_{i,t}]$ is uncorrelated to $\eta_{i,t}$, $\forall t$, then Eq. (B.3) degenerates into

$$\text{Var}(R_{i,t}^{*}) = \text{Var}(\eta_{i,t}) + \text{Var}[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})]. \tag{B.5}$$

(3) If $Q_{i,t}$ is constant (i.e. $Q_{i,t} = 1$, $\forall t$, or $Q_{i,t} = -1$, $\forall t$) and the spread $\ln[1 + s_{i,t} \cdot Q_{i,t}]$ is an independent random variable, then, from Eq. (B.4), we have the return variances from the ask-to-ask and the bid-to-bid price series, respectively,

$$\text{Var}(R_{i,t}^{A}) = \text{Var}(\eta_{i,t}) + \text{Var}[\ln(1 + s_{i,t}^{A}) - \ln(1 + s_{i,t-1}^{A})], \tag{B.6}$$

$$\text{Var}(R_{i,t}^{B}) = \text{Var}(\eta_{i,t}) + \text{Var}[\ln(1 - s_{i,t}^{B}) - \ln(1 - s_{i,t-1}^{B})], \tag{B.7}$$

where $\ln(1 + s_{i,t} \cdot Q_{i,t}^{A}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1}^{A})$ and $\ln(1 - s_{i,t} \cdot Q_{i,t}^{B}) - \ln(1 - s_{i,t-1} \cdot Q_{i,t-1}^{B})$ are the change in ask spread and bid spread, respectively.

(4) By the same analogy, if $s_{i,t} = s$ is constant over time and $Q_{i,t}$ is an indicator random variable, then, from Eq. (B.4), we have

$$\text{Var}(R_{i,t}^{*}) = \text{Var}(\eta_{i,t}) + \text{Var}[\ln(1 + s \cdot Q_{i,t}) - \ln(1 + s \cdot Q_{i,t-1})], \tag{B.8}$$

where $\ln(1 + s \cdot Q_{i,t}) - \ln(1 + s \cdot Q_{i,t-1})$ is the change in spread due to the random bounce between ask and bid prices.

In all above cases, the return sequences derived from the price series in a market which trading prices jump between ask and bid prices will have higher volatility than that of the returns free of the bid–ask errors (Eq. (B.4)).

By taking autocovariance measure on Eq. (B.2), assuming

(1) $\eta_{i,t}$ is an independent random variable (market efficiency).

(2) $s_{i,t} \cdot Q_{i,t}$, $\forall t$, are independent random variables, then $\ln(1 + s_{i,t} \cdot Q_{i,t})$, $\forall t$ are uncorrelated random variables (p. 39, Breiman, 1968).

(3) $\ln[1 + s_{i,t} \cdot Q_{i,t}]$ is uncorrelated to $\eta_{i,t}$, $\forall t$.

Thus, we have

(i) $\quad \text{Cov}(\eta_{i,t}, \eta_{i,t+1}) = 0$,

(ii) $\quad \text{Cov}[\ln(1 + s_{i,t} \cdot Q_{i,t}), \quad \ln(1 + s_{i,t+j} \cdot Q_{i,t+j})] = 0$, $i \neq j$,

(iii) $\text{Cov}\{\eta_{i,t}, [\ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})]\} = 0$.

The first order autocovariance measure on the observed one-period returns is

$$\text{Cov}(R^*_{i,t}, R^*_{i,t+1})$$

$$= \text{Cov}\{\mu + \eta_{i,t} + [\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})],$$

$$\mu + \eta_{i,t+1} + [\ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})]\}$$

$$= \text{Cov}(\eta_{i,t}, \eta_{i,t+1})$$

$$+ \text{Cov}\{\eta_{i,t}, [\ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})]\}$$

$$+ \text{Cov}\{[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})], \eta_{i,t+1}\}$$

$$+ \text{Cov}\{[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})],$$

$$[\ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})]\}$$

$$= \text{Cov}[\ln(1 + s_{i,t} \cdot Q_{i,t}), \ln(1 + s_{i,t+1} \cdot Q_{i,t+1})] - \text{Var}[\ln(1 + s_{i,t} \cdot Q_{i,t})]$$

$$- \text{Cov}[\ln(1 + s_{i,t-1} \cdot Q_{i,t-1}), \ln(1 + s_{i,t+1} \cdot Q_{i,t+1})]$$

$$+ \text{Cov}[\ln(1 + s_{i,t-1} \cdot Q_{i,t-1}), \ln(1 + s_{i,t} \cdot Q_{i,t})]$$

$$= -\text{Var}[\ln(1 + s_{i,t} \cdot Q_{i,t})]. \tag{B.9}$$

If the assumptions hold, the expression Eq. (B.9) implies that the first order observed return autocorrelation is negative which contains the randomness from both the price bounce and spread size. Clearly, if trading prices do not jump between bid and ask prices (i.e. $Q_{i,t}$ is constant) and the size of spread is non-stochastic, then $\text{Cov}(R^*_{i,t}, R^*_{i,t+1})$ in Eq. (B.9) reduces to zero. This means that, consistent with the notion of market efficiency, the transaction returns uncorrelated at lag one if trading prices do not jump between bid and ask prices and spreads are constant over time.

In order to examine the separate impact from $Q_{i,t}$ and $s_{i,t}$ on the return autocorrelation, we first assume that the size of the spread $s_{i,t}$ is constant and we have the impact of $Q_{i,t}$, which is called the bid–ask bounce effect:

$$\text{Cov}(R_{i,t}, R_{i,t+1}) = -\text{Var}[\ln(1 + s \cdot Q_{i,t})]. \tag{B.10}$$

Consistent with the analysis in Roll (1984), the given constant in the size of spreads, the observed first-order return autocorrelation is negative in Eq. (B.10). It is worthwhile to note that the size of spreads, even it is constant over time, can magnify the observed first order return autocorrelation. That is, if $s \to 0$, then $s \cdot Q_{i,t} \to 0$. We have $\lim_{s \to \infty} \text{Var}[\ln(1 + s \cdot Q_{i,t})] = 0$. Thus, in order to have the bid–ask bounce effect, $\text{Var}[\ln(1 + s \cdot Q_{i,t})] > 0$, the necessary condition is that the size of spreads is non-trivial.

Now, assuming there is no bounce effect in prices, in other words, the price

movement is only determined from one side of the market ($Q_{i,t} = 1$, $\forall t$, or $Q_{i,t} = -1$, $\forall t$), similarly, we have the bid–ask spread size effect:

$$\text{Cov}\left(R_{i,t}^{A}, R_{i,t+1}^{A}\right) = -\text{Var}\left[\ln\left(1 + s_{i,t}^{A}\right)\right],$$

or

$$\text{Cov}\left(R_{i,t}^{B}, R_{i,t+1}^{B}\right) = -\text{Var}\left[\ln\left(1 - s_{i,t}^{B}\right)\right]. \tag{B.11}$$

Given the assumptions, the expression Eq. (B.11) implies that the ask-to-ask or the bid-to-bid first order return autocorrelation can still be negative. It is important to recognize that the size of spreads plays an important role in both the bounce and size effects. If $s_{i,t}$ is random but the magnitude is trivial, then Eqs. (B.10) and (B.11) will both approach zero. This suggests that a less degree of variation in spreads can make Eq. (B.11) insignificant, but a trivial amount of spread can also make Eq. (B.10) unnoticeable, even the prices still jump between the bid and ask prices.

Similarly, we derive the return from the mid point of the bid and ask price and the return autocovariance as follows:

$$R_t^{AVG} = \ln\left[\left(P_{i,t}^{A} + P_{i,t}^{B}\right)/2\right] - \ln\left[\left(P_{i,t-1}^{A} + P_{i,t-1}^{B}\right)/2\right]$$

$$= \ln\left\{1/2 \cdot \left[P_{i,t} \cdot \left(1 + s_{i,t}^{A}\right) + P_{i,t} \cdot \left(1 - s_{i,t}^{B}\right)\right]\right\}$$

$$\quad - \ln\left\{1/2 \cdot \left[P_{i,t-1} \cdot \left(1 + s_{i,t-1}^{A}\right) + P_{i,t-1} \cdot \left(1 - s_{i,t-1}^{B}\right)\right]\right\}$$

$$= \mu + \eta_{i,t} + \ln\left(2 + s_{i,t}^{A} - s_{i,t}^{B}\right) - \ln\left(2 + s_{i,t-1}^{A} - s_{i,t-1}^{B}\right),$$

$$\text{Cov}\left(R_{i,t}^{AVG}, R_{i,t+1}^{AVG}\right)$$

$$= \text{Cov}\left[\mu + \eta_{i,t} + \ln\left(2 + s_{i,t}^{A} - s_{i,t}^{B}\right) - \ln\left(2 + s_{i,t-1}^{A} - s_{i,t-1}^{B}\right),\right.$$

$$\left. \mu + \eta_{i,t+1} + \ln\left(2 + s_{i,t+1}^{A} - s_{i,t+1}^{B}\right) - \ln\left(2 + s_{i,t}^{A} - s_{i,t}^{B}\right)\right]$$

$$= -\text{Var}\left[\ln\left(2 + s_{i,t}^{A} - s_{i,t}^{B}\right)\right]. \tag{B.12}$$

From the expression Eq. (B.12), we have that if the spread is symmetric or the magnitude of the asymmetry in spread is constant i.e. $s_{i,t}^{A} = s_{i,t}^{B}$, or $s_{i,t}^{A} - s_{i,t}^{B} = k$, $k \in \Re$ (a finite constant), $\forall t$, then $\text{Cov}(R_{i,t}^{AVG}, R_{i,t+1}^{AVG}) = 0$. However, if the spread is asymmetric and this asymmetry is non-constant over time, then $\text{Cov}(R_{i,t}^{AVG}, R_{i,t+1}^{AVG}) < 0$.

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