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Investment in Salvage Equipment

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Manufacturers are faced with three options for disposing of excess finished items: they can (1) continue to mark down the item until it sells; (2) dispose of or scrap it; or (3) salvage it in order to reclaim valuable raw materials and components. In many situations the option of a markdown is not viable, thus the choice is to either scrap or salvage. Although many manufacturers have invested in salvage equipment, they have not deduced the impact of this investment as it affects not only the risk-adjusted value of the project but also the production run size and the selling price. Hence, their decisions frequently turn out to be suboptimal. This paper develops a project-valuation model that adds a new dimension to traditional capital budgeting decisions by incorporating salvage capacity.

INTRODUCTION

Typically, manufacturers must choose one of three possible options to dispose of excess production: the item can be (1) marked down until it is sold, (2) scrapped or otherwise disposed of; or (3) worked or reprocessed so that either valuable raw materials and components are reclaimed or the item is used to make some byproduct. Baked goods and vegetables cannot be sold beyond their freshness dates. High-end fashion jewelry is not sold at markdown for fear that this will cause alarm among current and future customers. Certain greeting-card companies will not mark down their cards in order to maintain a higher status or image for their products. Thus, these manufacturers must choose either the second or third option.

The third option is called salvage. A salvage process is performed by specialized machines or operations in order to lower the manufacturer's cost of carrying unsold items. Many examples may be cited where manufacturers rely on the third option as part of their operations. In the costume jewelry industry, special smelters are being purchased to remove gold and base metals from unsold pieces. Investments in specialized grinders and blenders are being made by wholesale bakers to reprocess unsold bread into bread crumbs and other baked products into ingredients for animal feed. The investment in salvage equipment, however, is not without cost. There is a fixed cost for the equipment itself as well as a variable cost to rework or reprocess the item. Unfortunately, little research has been conducted on the impact of the investment for salvage operations on the firm's market value. Atkinson (1979) is the only author to have examined salvage value. He used this value as an internal transfer price but did not extend his analysis to include investment in salvage processes as an explicit decision variable to maximize the firm's value. As a result, manufacturers may make suboptimal decisions for investments in salvage equipment. They either do not invest when a salvage operation would improve the firm's value or invest excess amounts where a smaller or indeed no operation would be more advantageous.

In this paper we develop a model for an optimal investment decision in salvage equipment and the impact of this decision on the product's selling price and the size of the production run. Our results are applicable to manufacturers of high-end costume jewelry, greeting cards, newspapers, and magazines, wholesale bakeries, or any others who cannot mark down their unsold items. For all these industries, if no investment is made in salvage processes then the unsold finished goods or unused subassemblies are scrapped at little or no value. Yet a significant improvement may be attained on the firm's profitability with an investment in the salvage process. The analysis is conducted using a Capital Asset

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Pricing Model (CAPM) framework to take into account both risk and return of the investment decision under study. Recent applications of the CAPM are found in Anvari (1987) and Kim and Chung (1989) for stochastic inventory valuation.

A FIRM VALUATION WITH INVESTMENT IN SALVAGE EQUIPMENT

Invoking the usual underlying assumptions of the CAPM, the market value of any risky asset (or firm) is expressed as:

$$V = [E(\pi) - \lambda \mathrm{Cov}(\pi, m)]/(1+r) \tag{1}$$

where

π = uncertain cash flows to the firm,
$\lambda = [E(m) - r]/\sigma^2(m)$ = the market price of systematic risk,
m = the rate of return on the market portfolio,
r = the risk-free rate of return, and
$E(.)$, $\sigma^2(.)$, and $\mathrm{Cov}(.,.)$ denote the expected value, variance, and covariance operators, respectively.

A manufacturer must decide how many units to produce and what price to charge. In many (if not most) situations demand is highly stochastic. Hence, the manufacturer faces the risk of over- or underproduction. If more units are produced than are demanded, the manufacturer faces another investment decision in the salvage operation to reclaim or remove valuable components. The most important question is the optimal size of the salvage process to maximize the firm's value. Follow-up questions include the impact of the salvage operation on price, run size, and riskiness.

The capacity of the salvage operation is defined by αR, where R denotes the production run size in units and α is a scale factor that ranges between 0 and 1. In reality, investment in salvage equipment would be for discrete sizes, but for ease of exposition we assume that it is a continuous variable. The salvage operation therefore incurs a fixed cost, $F_s(\alpha)$, that will increase with α while the variable salvage cost per unit, $C_s(\alpha)$, is inversely related to the operation's size. We assume that both $F_s(0)$ and $C_s(0)$ equal zero. P_s is the known selling price of the salvaged unit. Let $D = D(P)(1+e)$ be a stochastic demand function, where $D(P)$ is the expected demand, P the selling price per unit of the item, and e an uncertainty factor with an expected value of zero and a known variance.[1] Additionally, let C be the known variable cost per unit and F be the known fixed cost. Depending upon the magnitude of product demand, the end-of-period cash flows to the firm, π, may be defined as:

$$\pi = \begin{cases} PR - CR - F - F_s(\alpha) & \text{if } D > R \quad \text{(2a)} \\ (PD - CR - F) + \{(R - D)[P_s - C_s(\alpha)] - F_s(\alpha)\} & \text{if } R \geqslant D \geqslant R(1-\alpha) \quad \text{(2b)} \\ (PD - CR - F) + \{\alpha R[P_s - C_s(\alpha)] - F_s(\alpha)\} & \text{if } R(1-\alpha) > D \quad \text{(2c)} \end{cases}$$

When the product demand exceeds the production run size, no salvaging is done, but the fixed cost for the salvage operation is incurred. Equation (2a) denotes the cash flows to the firm under this situation. When the product demand is less than production, the salvage operation will be utilized. If the unsold units are less than the capacity of the salvage operation, Eqn (2b) describes the cash flows to the firm. The first term is income from the sale of the demanded items while the second term enclosed within the braces is the income from the salvage operation. If the unsold units are even greater than the salvage capacity, Eqn (2c) denotes the firm's cash flows. In this case, the excess unsold units over the salvage capacity are assumed to be scrapped. If the manufacturer decides to scrap its excess production, then no investment is made in the salvage process and the end-of-period cash flows described by Eqns (2b) and (2c) will be the same because the expressions enclosed within the braces are defined to be zero.

The optimization process of a typical firm stops after the firm's value-maximizing production run size and the selling price are determined. This typical process, however, neglects the impact of salvage investment on the firm's value. Based upon the end-of-period cash flows defined by Eqns (2a)–(2c), it is possible for the firm to optimize not just price and run size but to simultaneously determine whether a salvage operation should be added and, if it is, what capacity it should have. This investment decision in salvage equipment will have a significant impact on the price and production run size. The conventional wisdom suggests that if an item has a salvage value, the production run size will be greater than if there is no salvage value. This statement, however, assumes that there is no fixed cost to the salvage process and that the variable cost of salvaging is not a function of the capacity of the process. In the presence of the interaction among decisions involving production run size, selling price, and salvage capacity, the end-of-period cash flows as defined by Eqns (2a)–(2c) provide a good starting point for this paper's analysis. There are two boundary points for the firm's cash flows, i.e. when D is equal either to R or to $R(1-\alpha)$, which are based upon the magnitude of stochastic demand for the product as indicated by Eqns (2a)–(2c). Substitution of the full expression for demand, $D=D(P)(1+e)$, into the above boundary points allow us to express the conditions where demand will exceed the upper boundary point as

$$e \geqslant R/D(P)-1=U$$

and the conditions where demand will be below the lower boundary point as:

$$e \leqslant R(1-\alpha)/D(P)-1=L$$

In order to find the expected value of cash flows as defined by Eqns (2a)–(2c) we introduce two expected values of units given the two boundary points. Let $n(.)$ be the probability density function of e and let

$$Q_1=\int_{-\infty}^{U}(U-e)n(e)\mathrm{d}e$$

and

$$Q_2=\int_{-\infty}^{L}(L-e)n(e)\mathrm{d}e$$

Thus, $Q_1D(P)$ is the expected number of unsold units that the firm would have without the salvage operation and would be forced to scrap, $Q_2D(P)$ the expected number of unsold units that the firm would have to scrap despite the investment in the salvage operation, and $(Q_1-Q_2)D(P)$ the expected number of unsold units that would be salvaged given the investment in the salvage operation. The expected value of π as defined by Eqns (2a)–(2c) is now written as:

$$E(\pi)=P[R-D(P)Q_1]-CR-F + [P_s-C_s(\alpha)][D(P)(Q_1-Q_2)]-F_s(\alpha) \quad (3)$$

The term in the first brackets represents the expected satisfied demand while that in the third brackets is the expected units sold at salvage price P_s under the salvage operation. Assuming e and m to have a joint bivariate normal distribution, the following proportionality between $\mathrm{Cov}(\pi, m)$ and $\mathrm{Cov}(e, m)$ can be derived:

$$\mathrm{Cov}(\pi, m)=\mathrm{Cov}(e, m)D(P)\{PN(U) - [P_s-C_s(\alpha)][N(U)-N(L)]\} \quad (4)$$

where $N(.)$ is the cumulative normal probability density function (see Appendix). Substitution of Eqns (3) and (4) into (1) yields:

$$\begin{aligned}V(P,R,\alpha)=&(PD(P)[(R/D(P))-Q_1\\&-\lambda\mathrm{Cov}(e, m)N(U)]-CR-F\\&+\{D(P)[P_s-C_s(\alpha)](Q_1-Q_2\\&+\lambda\mathrm{Cov}(e, m)[N(U)-N(L)])\\&-F_s(\alpha)\})/(1+r)\end{aligned} \quad (5)$$

where the firm's value is denoted by $V(P, R, \alpha)$, showing the three variables of interest: unit price, production run size, and the capacity of the salvage operation. The entire numerator of Eqn (5) is the certainty equivalent of the end-of-period cash flows to the firm. The term in the braces represents the value added by the salvage operation. If this term is less than or equal to zero, then the salvage operation would reduce the firm's value and therefore the firm should choose to scrap rather than salvage its excess inventory.

INVESTMENT IN SALVAGE EQUIPMENT AND ITS STRATEGIC IMPLICATIONS

Equation (5) must now be solved for the optimal values of the three decision variables: the price of the unit, the production run size, and the capacity of the salvage operation. As the decision variable α is restricted by the range $0 \leqslant \alpha \leqslant 1$, the solution to the problem can be obtained by taking the Lagrangian function $\mathscr{L}=V(P, R, \alpha)+\theta(1-\alpha)$, and satisfying the Kuhn–Tucker conditions $\alpha\partial\mathscr{L}/\partial\alpha=0$, $\theta(1-\alpha)=0$, $\theta \geqslant 0$, and

$$\begin{aligned}\partial\mathscr{L}/\partial P=&\{[D(P)+PD'(P)](R/D(P)-Q_1\\&-\lambda\mathrm{Cov}(e, m)N(U))-P(R/D(P))\\&\times(1-N(U)-\lambda\mathrm{Cov}(e, m)n(U))D'(P)\\&+[P_s-C_s(\alpha)][G(U)\\&-G(L)]D'(P)\}/(1+r)=0\end{aligned} \quad (6)$$

$$\partial\mathscr{L}/\partial R = \{P(1-N(U)-\lambda\mathrm{Cov}(e, m)n(U)) + [P_s - C_s(\alpha)]\{N(U)+\lambda\mathrm{Cov}(e, m)n(U) - (1-\alpha)[N(L)+\lambda\mathrm{Cov}(e, m)n(L)]\} - C\}/(1+r) = 0 \quad (7)$$

and

$$\partial\mathscr{L}/\partial\alpha = \{-C_s'(\alpha)D(P)[Q_1 - Q_2 + \lambda\mathrm{Cov}(e, m) \times (N(U)-N(L))] + R[P_s - C_s(\alpha)] [N(L)+\lambda\mathrm{Cov}(e, m)n(L)] - F_s'(\alpha)\}/(1+r) - \theta \leqslant 0 \quad (8)$$

where $G(.) = -N(.) + \sigma^2(e)n(.) + \lambda\mathrm{Cov}(e, m)[N(.) - (1+.)n(.)]$, and θ is a multiplier.[2] The solution will optimize the firm's value.[3]

Numerical Example

For ease of illustration, the following numerical example is employed to derive the optimal price, production run size, and salvage capacity and to demonstrate the interactions of the three decision variables. Let:

$D(P) = 300\,000 - 12\,000P$; $\lambda\mathrm{Cov}(e, m) = 0.125$;
$C = \$10$; $E(m) = 0.21$;
$F = \$250\,000$; $\sigma(m) = 0.08$;
$P_s = \$8$; $\sigma(e) = 0.4$;
$C_s(\alpha) = \$5 - 2.5(\alpha)$; $E(e) = 0$; and
$F_s(\alpha) = \$40\,000(e^{\alpha} - 1)$; $r = 0.08$

Setting α at various intervals between 0 and 1 and then using Newton's method to find P and R yields the maximum value of the firm. From Table 1 the firm's value is a concave function of α. The firm's value increases initially as the investment in the salvage operation increases. The firm's value reaches its maximum of \$121 786.70 at $\alpha = 0.5921$ and then it slopes downward. Its maximum value is obtained at the optimal price of \$18.277 and run size of 77 609, respectively. The α-value of 0.5921 implies that the optimal capacity of the salvage operation is 45 952 units. Further, from Table 1, the optimal price decreases and run size increases as the size of salvage operation increases. When the firm scraps rather than salvages its unsold production, i.e. $\alpha = 0$, the firm's value is only \$103 493 with $P = \$18.456$ and $R = 65\,851$ units. Thus, the adoption of the salvage operation allows the firm to lower the price of the finished product and to increase the production run size, which should, in turn, make it possible to further penetrate the market and increase its market share.

Depending upon the operating environment of the firm, a few variations of the results reported in Table 1 may be discussed. First, consider the firm in a highly competitive market where it must sell at a market-determined price and its physical plant configuration does not allow flexibility in output. Its only decision variable is then the size of the salvage operation. For comparison with the results in Table 1, the same numerical values of $P = \$18.456$ and $R = 65\,851$ units when $\alpha = 0$ are used. As summarized in panel A of Table 2, the firm's value is a concave function of the size of the salvage operation. It is not surprising that this maximum value is approximately 6% smaller than that obtained in the previous example where the firm is allowed to simultaneously search for the three key variables.

Other interesting cases emerge as either price or run size is held constant. Consider the firm that is a price leader but its run size is fixed due to its plant's physical capacity or to labor restrictions. The firm must then decide both price and the size of the salvage operation. As presented in panel B of Table 2, the firm's value is maximized at \$116 410.63 when $\alpha = 0.46851$ and $P = \$18.834$. Unlike the results presented in Table 1, the relationship between the size of salvage operation and the optimal price is positive. The rate of change in the price is larger than that observed in Table 1. This is explained by

Table 1. The Relationship between Investment in Salvage Operation with the Firm's Value, Price, and Run Size

Investment in salvage operation ($=\alpha$)	Price ($=P$) (\$)	Run size ($=R$) (units)	The firm's value ($=V$) (\$)
0.0000	18.456	65 851	103 493.70
0.1000	18.424	67 805	108 148.01
0.2000	18.392	69 861	112 551.86
0.3000	18.361	71 952	116 390.02
0.4000	18.331	74 004	119 361.72
0.5000	18.302	75 951	121 215.46
0.5800	18.280	77 399	121 776.75
0.5900	18.277	77 572	121 786.42
0.5921	18.277	77 609	121 786.70[a]
0.6000	18.274	77 744	121 782.45
0.7000	18.249	79 367	120 993.32
0.8000	18.224	80 843	118 867.15
0.9000	18.201	82 215	115 475.36
1.0000	18.179	83 539	110 900.09

[a] Optimal value.

the firm's price-leader status. Being a price leader, the firm is able to change its price and it does so faster than usual. Its action, however, is not without cost. Too high a price will result in more unsold units, hence a larger salvage operation and a lower value of the firm.

Suppose the firm must accept a market-determined price while it is able to decide the size of both its production run and its salvage operation. This case differs from the case presented in Table 1 in that the price is not a decision variable. This firm's maximum value is \$121 529.81 at $\alpha = 0.58692$ and $R = 75\,861$ units as presented in panel C of Table 2. The same positive relationship between the size of the salvage operation and the run size exists as indicated in Table 1. Because the firm cannot set the price, its value and run size are smaller than those summarized in Table 1. When compared with the results in panel B, the firm has a higher maximal value with the inflexibility in price setting than with the inflexibility in run size. The price setter of panel B has a lower value of \$5119.18 than the market penetrator of panel C. Regardless of the market environment in which the firm operates, an important conclusion is that any over- or under-investment in the salvage operation is suboptimal for the firm's value maximization.

Table 2. Investment in Salvage Operation in Three Different Cases

Investment in salvage operation ($=\alpha$)	Price ($=P$) (\$)	Run size ($=R$) (units)	The firm's value ($=V$) (\$)
A. Price and run size are held constant:			
0.0000	18.456	65 851	103 493.22
0.1000	18.456	65 851	107 828.06
0.2000	18.456	65 851	111 263.23
0.3000	18.456	65 851	113 551.89
0.4000	18.456	65 851	114 546.81
0.4200	18.456	65 851	114 583.95
0.4236	18.456	65 851	114 584.84[a]
0.5000	18.456	65 851	114 187.70
0.6000	18.456	65 851	112 478.70
0.7000	18.456	65 851	109 461.79
0.8000	18.456	65 851	105 191.01
0.9000	18.456	65 851	99 711.73
1.0000	18.456	65 851	93 046.59
B. Run size is held constant:			
0.0000	18.456	65 851	103 493.22
0.1000	18.543	65 851	107 935.17
0.2000	18.630	65 851	111 689.77
0.3000	18.713	65 851	114 472.91
0.4000	18.788	65 851	116 071.20
0.4683	18.834	65 851	116 410.53
0.4685	18.834	65 851	116 410.63[a]
0.5000	18.853	65 851	116 352.59
0.6000	18.908	65 851	115 266.41
0.7000	18.954	65 851	112 826.88
0.8000	18.993	65 851	109 085.78
0.9000	19.027	65 851	104 102.68
1.0000	19.059	65 851	97 920.48
C. Price is held constant:			
0.0000	18.456	65 851	103 493.22
0.1000	18.456	67 546	108 140.45
0.2000	18 456	69 329	112 521.24
0.3000	18.456	71 133	116 320.63
0.4000	18.456	72 893	119 239.09
0.5000	18.456	74 546	121 026.64
0.5860	18.456	75 848	121 529.75
0.5869	18.456	75 861	121 529.81[a]
0.6000	18.456	76 049	121 516.77
0.7000	18.456	77 390	120 642.51
0.8000	18.456	78 591	118 424.80
0.9000	18.456	79 697	114 936.34
1.0000	18.456	80 759	110 259.31

[a] Optimal values.

Strategic Implications of Salvage Investment

Depending upon the functional relationships between the size of the salvage operation on price and production run size, general strategic implications can be considered for manufacturing firms. By letting α be an exogenous parameter, identities (6) and (7) can be solved for the optimum P^* and R^* as functions of α. The derivatives of these functions with respect to α will determine the overall strategic implications. At the optimum, $dP^*/d\alpha = D_1/D_3$ and $dR^*/d\alpha = D_2/D_3$, where D_1, D_2, and D_3 are the determinants of the matrices:

$$D_1 = \begin{vmatrix} -V_{P\alpha} & V_{PR} \\ -V_{R\alpha} & V_{RR} \end{vmatrix} \quad D_2 = \begin{vmatrix} V_{PP} & -V_{P\alpha} \\ V_{PR} & -V_{R\alpha} \end{vmatrix}$$

$$D_3 = \begin{vmatrix} V_{PP} & V_{PR} \\ V_{RP} & V_{RR} \end{vmatrix}$$

where V_{PR}, $V_{R\alpha}$, and $V_{P\alpha}$ are the second-order mixed partial derivatives of V, and V_{PP} and V_{RR} are the second-order partial derivatives of V with respect to P and R. Given that the second-order conditions for an optimum are satisfied, then D_3 is positive. Hence, the respective signs of $dP^*/d\alpha$ and $dR^*/d\alpha$ are determined by D_1 and D_2, respectively. Overall strategic implications may be examined depending upon the signs of D_1 and D_2.

A negative D_1 and a positive D_2 imply that the investment in salvage equipment lowers the selling price and the firm should be able to increase the production run size. The appropriate strategy for the firm is a market-penetration one. The market-penetration policy is particularly effective when product demand is price sensitive, per unit production costs fall with increased run sizes, and the low price discourages competitors from entering the market (Kotler, 1984). Even if the firm must accept a fixed price determined by a price leader in the industry, it should still follow the market-penetration strategy because it gains from the residual value of the salvaged units. The numerical problem provided in this paper is an example of a market-penetration strategy.

If D_1 is positive but D_2 is negative, then the optimal policy for the firm is a skim-pricing strategy. An example of this would be a newly launched high-fashion product. McCarthy and Perrault (1987) suggest a skim-pricing strategy under which the firm sells the product to the top of the market at a high price. Kotler (1984) suggests that skim pricing should be employed when: (1) a sufficient number of buyers have a large current demand; (2) the unit costs of producing a small run size are low enough not to cancel the advantage of skim pricing; (3) the high price does not attract more competitors; and (4) the high price supports the image of a superior product (i.e. prestige pricing). In this case, the higher price will increase marginal returns while the salvage operation and the smaller run size reduce the risk of following the skim-pricing policy.

The situations where D_1 and D_2 are both either positive or negative are not possible given the conditions for concavity. Specifically, as long as the price of the salvaged item, P_s, exceeds the variable cost of salvaging, $C_s(\alpha)$, for all permissible values of α, then D_1 and D_2 must have opposite signs.

Systematic Risk and Salvage Equipment

Investment in a salvage operation will help the firm dispose of excess production at a profit. It is not clear, however, whether such an investment will increase or decrease the systematic risk (or beta) as defined by:

$$\beta = \mathrm{Cov}(\pi, m)/(\sigma^2(m)V(P, R, \alpha)) \qquad (9)$$

The impact of a salvage operation on the firm's systematic risk may be examined by the sign of the derivative of Eqn (9) with respect to α. Unfortunately, the sign is not clearly determined because the change in α may lead to changes in either the price or the run size, or both. For this reason, the numerical example in Table 1 is used to examine the impact of the salvage operation on the firm's beta. Table 3 summarizes the results.

Table 3. The Relationship between Investment in Salvage Operation and Systematic Risk

Investment in salvage operation ($=\alpha$)	The firm's value ($=V$) (\$)	The firm's beta ($=\beta$)
0.0000	103 493.70	4.2800
0.1000	108 148.01	4.1762
0.2000	112 551.86	4.0838
0.3000	116 390.02	4.0092
0.4000	119 361.72	3.9562
0.5000	121 215.46	3.9285
0.5600	121 716.73	3.9262[b]
0.5800	121 776.75	3.9285
0.5900	121 786.42	3.9300
0.5921	121 786.70[a]	3.9300
0.6000	121 782.45	3.9315
0.7000	120 933.32	3.9700
0.8000	118 867.15	4.0492
0.9000	115 475.36	4.1762
1.0000	110 900.09	4.3600

[a] Optimal value.
[b] Minimum beta.

The firm's beta is 4.28 given a zero investment in salvage value. As the size of the salvage operation increases, the firm's beta decreases to reach its minimum of 3.9262 at $\alpha = 0.56$. The firm's beta then slopes upward as the firm increases the amount of salvage operation. The minimum beta is attained before the firm's value is maximized, which implies that the minimization of the firm's cost of capital is not consistent with the maximization of the firm's value.[4] The same trend has been observed from the numerical simulation conducted for the three cases of operating environment reported in Table 2.

SUMMARY AND CONCLUSIONS

Manufacturers in many industries such as wholesale baking, greeting cards, and high-end fashion jewelry are faced with either scrapping or salvaging their unsold production. This paper has examined the impact of the investment in salvage operations. This has been accomplished by modelling the firm's end-of-period cash flows under different semi-elastic demand conditions. In the CAPM framework, it has been demonstrated that the size of the investment in salvage equipment directly affects the price and run size decisions. The investment in salvage operation adds a new dimension to traditional capital budgeting decisions. The findings of this paper indicate that the firm's value is a concave function of the size of the salvage operation, with a maximum reached at a finite point of the operation's size. To ignore this important decision variable will lead the firm to make suboptimal decisions.

APPENDIX

To develop Eqn (4) let

$$\mathrm{Cov}(\pi, m) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\pi - E(\pi)) \times (m - E(m))n(e, m)\mathrm{d}m\mathrm{d}e \qquad \text{(A1)}$$

where $n(e, m)$ is the bivariate normal distribution for (e, m). Then

$$\int_{-\infty}^{+\infty}(m - E(m))n(e, m)\mathrm{d}m = (e - E(e))/\sigma^2(e)n(e)\mathrm{Cov}(e, m)$$

where $n(e)$ is the marginal distribution of e. By assumption, $E(e) = 0$, thus Eqn (A1) reduces to

$$\mathrm{Cov}(\pi, m) = \mathrm{Cov}(e, m)/\sigma^2(e)\left[\int_{-\infty}^{+\infty} e\pi n(e)\mathrm{d}e\right] \qquad \text{(A2)}$$

Let Γ equal the term in the brackets and let

$$\Gamma_1 = \int_{-\infty}^{U} PD(P)(1+e)en(e)\mathrm{d}e \qquad \text{(A3)}$$

$$\Gamma_2 = (P_s - C_s(\alpha))(U - L)D(P)\int_{-\infty}^{L} en(e)\mathrm{d}e \qquad \text{(A4)}$$

$$\Gamma_3 = (P_s - C_s(\alpha))R\left(\int_{L}^{U} en(e)\mathrm{d}e\right) \qquad \text{(A5)}$$

$$\Gamma_4 = -(P_s - C_s(\alpha))\left(\int_{L}^{U} D(P)(1+e)en(e)\mathrm{d}e\right) \qquad \text{(A6)}$$

$$\Gamma_5 = \int_{L}^{U} PRen(e)\mathrm{d}e \qquad \text{(A7)}$$

Thus, Γ also equals the sum of Eqns (A3) through (A7).

For a normal distribution with a mean of zero

$$\int_{a}^{b} en(e)\mathrm{d}e = \sigma^2(e)(n(a) - n(b)) \qquad \text{(A8)}$$

$$\int_{a}^{b} e^2 n(e)\mathrm{d}e = \sigma^2(e)(N(b) - N(a)) + \sigma^2(e)(an(a) - bn(b)) \qquad \text{(A9)}$$

where a and b are such that $-\infty \leqslant a,\ b \leqslant +\infty$.

Using Eqns (A8) and (A9) to re-express Eqns (A3) through (A7), and adding the re-expressed values for $\Gamma_1 - \Gamma_5$ yields (after simplifying)

$$\Gamma = \sigma^2(e)D(P)(PN(U) - (P_s - C_s(\alpha))(N(U) - N(L))) \qquad \text{(A10)}$$

Substituting Eqn (A10) for the term in the brackets in Eqn (A2) yields Eqn (4).

NOTES

1. Kottas and Lau (1979), Constantinides *et al.* (1981) and Lau and Lau (1987) all assumed a stochastic demand function.
2. Having assumed e to be normally distributed with mean of zero, we can use Eqns (A8) and (A9) from the Appendix to obtain $Q_1 = UN(U) + \sigma^2(e)n(U)$ and $Q_2 = LN(L) + \sigma^2(e)n(L)$.
3. There are certain conditions that are necessary to ascertain the concavity of the firm's value curve:
   (a) The demand function, $D(P)$, must be such that $D'(P)$ is strictly negative and that $D''(P)$ is less than or equal to zero which would be the case where expected demand decreases as price increases;
   (b) The variable cost of the salvage operation is such that as the size of the operation increases, the variable cost decreases, i.e. $C_s'(\alpha)$ is strictly negative while $C_s''(\alpha)$ is greater than or equal to zero;
   (c) The fixed salvage cost increases as the operation gets larger or where $F_s'(\alpha)$ is strictly positive while $F_s''(\alpha)$ is greater than or equal to zero;
   (d) The risk discount factor that modifies expected demand, $A = (R/D(P)) - Q_1 - \lambda\mathrm{Cov}(e, m)N(U)$, and the rate of change of A as $U = (R/D(P) - 1)$ changes, $\partial A/\partial U = 1 - N(U) - \lambda\mathrm{Cov}(e, m)n(U)$, are both positive; and
   (e) Mathematically the expression $(1 - \lambda\mathrm{Cov}(e, m)U/\sigma^2(e))$ is positive. This last condition ensures that $(1 - \lambda\mathrm{Cov}(e, m)L/\sigma^2(e))$ is positive and $(G(U) - G(L))$ is negative as $L \leqslant U$ for all permissible values of α.
4. See Arditti (1973) and Haley and Schall (1978).

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