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STOCHASTIC DEMAND AND A DECOMPOSITION OF SYSTEMATIC RISK

S. Ghon Rhee

I. INTRODUCTION

The relationship between the systematic risk of common stock and financial leverage has received considerable attention in the past (e.g., Beaver, Kettler, and Scholes, 1970; Hamada, 1972; Hill and Stone, 1980; and Mandelker and Rhee, 1984). The role of operating leverage, however, in explaining the systematic risk has yet to be fully explored. Conflicting theories remain unresolved on the relationship between beta and operating leverage. Rubinstein (1973) and Lev (1974) demonstrated a positive relationship between the two, while Subrahmanyam and Thomadakis (1980) proved that beta is a decreasing function of operating leverage. This apparent conflict needs to be settled.

Looking at the balance sheet of a firm, one may easily ascertain that both asset structure and financial structure represent two important determinants of the relative riskiness of common stock. Nevertheless, there seems to be no satisfactory answer to the question of what portion of systematic risk is explained by operating leverage. In fact, no one can tell the exact magnitude of the operating risk premium (solely attributable to operating leverage) in

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the equilibrium rate of return on a firm's common stock. For that matter, the proper variable to serve as a proxy for operating leverage is not well defined either. Many proxy variables may be recommended, but in the absence of a theoretical model, it is not a simple matter to identify the variable.

The reason for the lack of acceptable answers to these questions can be traced back to the manner in which firm valuation has been carried out since the publication of Modigliani and Miller's pathbreaking papers (1958, 1963). A firm's valuation starts with the estimation of net operating income (NOI) assuming all-equity financing. This NOI is determined by the given investment decision. The firm's asset structure or production cost structure are implicitly built into the investment decisions. The next step involves the evaluation of the financing mix and its impact on the firm's value. On the analogy of this valuation approach, the traditional wisdom recommends that the systematic risk of common stock be decomposed into two components: business (or operating) risk and financial risk (e.g., Hamada, 1972; and Rubinstein, 1973). Business risk represents the systematic risk of the financially unlevered firm, while financial risk measures the additional risk assumed by common stockholders with the firm's use of debt. In this traditional dichotomous scheme, operating leverage is already built into business risk. Hence, there is no need to isolate the impact of operating leverage from business risk. It would even be futile to attempt to do so in the traditional framework, leaving the questions unanswered.

A series of recent studies by Ferri and Jones (1979), Mandelker and Rhee (1984), and Dotan and Ravid (1985) has revived interest in the relationship between operating leverage and financial leverage. Ferri and Jones and Mandelker and Rhee reported empirical evidence on the negative relationship between the two types of leverage. Dotan and Ravid provided a rigorous proof that optimal operating leverage is a decreasing function of financial leverage. The common focus of these studies is on the interaction of investment and financing decisions, which was formalized by a classic paper by Myers (1974) in his evaluation of capital budgeting decisions. Myers recognized an interaction of financing and investment decisions in the Modigliani-Miller Proposition 1 in the presence of corporate income taxes, and he developed the adjusted present value (APV) framework, thereby allowing various interaction effects in addition to the present value of interest tax shields. The interaction between the two types of leverage as recently reported by these studies brings up an interesting challenge: How does this interaction fit into Myers' APV framework?

In order to resolve various issues addressed so far, this paper proposes a trichotomous decomposition of common equity beta as opposed to the traditional dichotomous decomposition. The proposed trichotomization of beta isolates the impact of operating leverage from business (or operating)

risk, thereby identifying the proper variable that measures operating leverage and estimating the magnitude of the operating risk premium solely responsible for the firm's use of operating leverage. By so doing, the controversy between Subrahmanyam-Thomadakis (1980) and Rubinstein (1973) and Lev (1974) is resolved. This paper demonstrates that the theoretical conflicts arise from the differing assumption underlying the firm's decision-making behavioral mode when uncertainty is introduced: quantity-setting or price-setting. The Subrahmanyam-Thomadakis analytical results rest upon the quantity-setting firms in line with past studies that examined either competitive or monopolistic firms under price uncertainty (e.g., Sandmo, 1971; Leland, 1972; Long and Racette, 1974; Thomadakis, 1976; Hite, 1977; and Booth, 1981). In contrast, the Rubinstein-Lev results are built upon the price-setting firms in the tradition of past literature that studied monopolistic firms under demand uncertainty (e.g., Baron, 1971; McCall, 1971; Leland, 1972; and Chen, 1975). As the microeconomic structure of the firm's optimal output or pricing decision is recognized, this paper concludes that the theoretical conflicts are not as real as they may have originally seemed. This paper discusses the implications of the proposed risk decomposition for Myers' APV framework. This should place the interaction between the two types of leverage in a proper perspective within the APV capital budgeting framework.

II. THE DECOMPOSITION OF BETA

In this section the systematic risk (or beta) of common stock is decomposed into three components; business risk, operating risk, and financial risk. What is needed for the planned trichotomization of beta is a unique sort of income that is independent not only of financial structure but also of asset structure, whereas the traditionally defined NOI is independent of financial structure only. Introduced in this paper is a firm that is both operationally and financially unlevered, and such a firm's earnings should be what is needed. [The notion of a completely unlevered firm was originally introduced by Mandelker and Rhee (1984, p. 50).] This completely unlevered firm will incur zero interest expenses and zero fixed costs, and as a result, its variable cost per unit will be much higher than ordinarily observed. Business risk is measured by the beta of this completely unlevered firm. As the firm levers itself operationally and financially, both operating risk and financial risk are introduced to magnify its business risk. Operating risk captures the added systematic risk solely due to the firm's use of operating leverage, and financial risk is attributable to the use of financial leverage.

Several assumptions underlying this paper's approach to the risk decomposition need to be discussed: The firm issues only two types of

securities, common equity and risky debt. Depreciation expenses are not explicitly considered in the model, and the total amount of fixed obligations (including principal and interest) is assumed to be tax-deductible.¹ Although these are not realistic assumptions, the analysis is simplified without loss of generality. The number of units produced (and sold), Q ; total fixed costs, F ; and the total amount of fixed obligations, Y , are all treated as random variables.² The price per unit, p , is determined at the beginning of the period before demand uncertainty is resolved by the market value rule. The variable cost per unit, v , is assumed to be known to simplify the analysis.

By introducing demand uncertainty and treating the price per unit as an *ex ante* control variable, the firm is regarded as a price setter rather than as a quantity setter. The main reason why the price-setting behavioral mode is chosen for this study is twofold. First, the analytical results remain compatible with traditional risk decomposition, as was demonstrated by Hamada (1972) and Rubinstein (1973). Second, on the basis of quantity-setting firms, the relationship between the financially unlevered firm's beta and operating leverage has been examined by other studies (e.g., Subrahmanyam and Thomadakis, 1980; and Goldenberg and Chiang, 1983). Being a price setter, the firm is allowed to have some degrees of monopolistic power with respect to its output, and the usual assumption that financial assets are traded in perfectly competitive markets is retained. In general, the imperfection in output markets can be introduced for price-setting firms as well as for quantity-setting firms. Thomadakis (1976) and Subrahmanyam and Thomadakis (1980) introduced the imperfection by allowing the demand curve to be downward-sloping for the quantity-setting firms, whereas the price-setting behavioral mode is sufficient to introduce the imperfection for the price-setting firms.³ An analysis built upon the imperfect output market structure seems more realistic and useful than the perfectly competitive output market (applicable only to quantity setters) for the following reasons: (1) The identification of perfectly competitive firms (or industries) is an extremely difficult task in reality, if not impossible and (2) the perfect competition in output markets produces rather trivial results, which suggest that the project's net present value has to be zero and further that the firm's market value equals its book value (or asset cost).

The decomposition of beta begins with the derivation of the market value of common stock for an ordinary firm (which is both operationally and financially levered) in the single-period capital asset pricing model.⁴ The market value of common equity is the risk-adjusted present value of the end-of-period earnings to stockholders:

$$S = \frac{E(\tilde{I}) - \lambda \text{cov}(\tilde{I}, \tilde{R}_m)}{1 + r} \quad (1)$$

where

$\tilde{\Pi}$ = end-of-period earnings to stockholders;
 $= (1 - t)[(p - v)\tilde{Q} - \tilde{F} - \tilde{Y}]$;

t = corporate income tax rate;

\tilde{R}_m = rate of return on the market portfolio;

λ = market price of risk;

$$= \frac{E(\tilde{R}_m) - r}{\sigma^2(\tilde{R}_m)};$$

$E(\cdot), \sigma^2(\cdot), \text{cov}(\cdot)$ = expected value, variance, and covariance operators, respectively.

Substitution of $\tilde{\Pi} = (1 - t)[(p - v)\tilde{Q} - \tilde{F} - \tilde{Y}]$ into Eq. (1) yields

$$S = \frac{(1 - t)[(p - v)CE(\tilde{Q}) - CE(\tilde{F}) - CE(\tilde{Y})]}{1 + r} \quad (2)$$

where $CE(\cdot)$ denotes the certainty equivalent. Thus,

$$CE(\tilde{Q}) = E(\tilde{Q}) - \lambda \text{cov}(\tilde{Q}, \tilde{R}_m)$$

$$CE(\tilde{F}) = E(\tilde{F}) - \lambda \text{cov}(\tilde{F}, \tilde{R}_m)$$

$$CE(\tilde{Y}) = E(\tilde{Y}) - \lambda \text{cov}(\tilde{Y}, \tilde{R}_m)$$

Similarly, the market value of the identical but completely unlevered firm can be expressed as:

$$S^o = \frac{(1 - t)(p - v^o)CE(\tilde{Q})}{1 + r} \quad (3)$$

where the superscript denotes variables unique to the unlevered firm. Note that both $CE(\tilde{F})$ and $CE(\tilde{Y})$ are not shown in Eq. (3) because this firm is operationally as well as financially unlevered. As a result, this firm's variable cost per unit $v^o > v$ to compensate for zero fixed costs in its production processes.

The beta of the levered firm's common stock is defined as

$$\beta_s = \frac{\text{cov}(\tilde{R}_s, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} \quad (4)$$

where \tilde{R}_s is the rate of return on common stock. In the single-period setting, $\tilde{R}_s = \tilde{\Pi}/S - 1$. Substitution of \tilde{R}_s into Eq. (4) yields

$$\beta_s = \frac{(1 - t)[(p - v) \text{cov}(\tilde{Q}, \tilde{R}_m) - \text{cov}(\tilde{F}, \tilde{R}_m) - \text{cov}(\tilde{Y}, \tilde{R}_m)]}{S\sigma^2(\tilde{R}_m)} \quad (5)$$

The beta of the unlevered firm can be easily inferred from Eq. (5) after dropping $\text{cov}(\tilde{F}, \tilde{R}_m)$ and $\text{cov}(\tilde{Y}, \tilde{R}_m)$ and changing v to v^0 and S to S^0 , respectively:

$$\beta^0 = \frac{(1-t)(p-v^0)\text{cov}(\tilde{Q}, \tilde{R}_m)}{S^0\sigma^2(\tilde{R}_m)} \quad (6)$$

Common equity beta can be expressed as a function of β^0 after dividing Eq. (5) by Eq. (6):

$$\beta_s = \frac{S^0}{S} \left[\frac{p-v}{p-v^0} - \frac{\text{cov}(\tilde{F}, \tilde{R}_m)}{(p-v^0)\text{cov}(\tilde{Q}, \tilde{R}_m)} - \frac{\text{cov}(\tilde{Y}, \tilde{R}_m)}{(p-v^0)\text{cov}(\tilde{Q}, \tilde{R}_m)} \right] \beta^0 \quad (7)$$

Unfortunately, it is not easy to interpret the meaning of Eq. (7). In order to define the relationship between β_s and β^0 in the form appropriate for readily comprehensible interpretation, further modifications are necessary. As shown in Appendix 1, Eq. (7) may be reduced to

$$\beta_s = \beta^0 + (1-t)(\beta^0 - \beta_F) \frac{C}{S} + (1-t)(\beta^0 - \beta_d) \frac{D}{S} \quad (8)$$

where

C = risk-adjusted present value of total fixed costs;

$$= \frac{CE(\tilde{F})}{1+r};$$

D = market value of risky debt;

$$= \frac{CE(\tilde{Y})}{1+r};$$

β_F = beta of fixed costs;

β_d = beta of risky debt.

Equation (8) effectively decomposes the beta of common stock into three distinct components; business risk, operating risk, and financial risk. The unlevered firm's beta, β^0 , measures business risk of common stock before the firm levers itself operationally and financially. As indicated by Eq. (6), business risk is determined by the systematic, or market-related, portion of demand uncertainty (or sales variability). This means that cyclical firms whose sales depend on the state of the economy tend to have high business risk. Business risk as measured by β^0 has an important implication for empirical studies that estimate the cost of capital. The identification of homogeneous risk classes of firms (or common stocks) is the first step to be completed for the estimation of the cost of capital, as was demonstrated by Miller and Modigliani (1966). Because β^0 is independent of both operating leverage and financial leverage, it can serve as a proper measure for

identifying homogeneous risk class; whereas the traditional business (or operating) risk as measured by the financially unlevered firm's beta (usually denoted by β_u in the past literature) may not be an appropriate benchmark to use for the same purpose, since operating leverage may vary from one firm to another or from one industry to another.

The second term on the right-hand side of Eq. (8) captures operating risk. Note that operating leverage is measured by the ratio of the risk-adjusted present value of fixed costs to the market value of common equity. In past studies various proxies for operating leverage were used. For example, the contribution margin (e.g., Rubinstein, 1973; and Lev, 1974), the percentage change in earnings before interest and taxes to the percentage change in sales (e.g., Mandelker and Rhee, 1984; and Ferri and Jones, 1979), and the ratio of fixed assets to total assets measured in book values (e.g., Ferri and Jones) are the usual accounting variables frequently used. While the use of these variables is justified in light of the specific hypotheses examined, Eq. (8) indicates that the ratio C/S measured in market values is the proper variable to serve as a proxy for operating leverage. As the counterpart of the debt-equity ratio from the right-hand side of the balance sheet, the ratio C/S emerges from the left-hand side of the balance sheet to gauge the asset structure of the firm.

A. The Relationship Between Beta and Operating Leverage

Rubinstein (1973) and Lev (1974) demonstrated that common equity beta is an increasing function of operating leverage, while Subrahmanyam and Thomadakis (1980) established a negative relationship between the two. They demonstrated that the negative relationship holds regardless of the output market structure considered: perfect competition or imperfect competition. In reconciling the apparent contradiction, one should note that the Subrahmanyam-Thomadakis analysis is based on quantity-setting firms facing price uncertainty, while the Rubinstein-Lev analysis is based on price-setting firms facing demand uncertainty. One may further argue that the Rubinstein-Lev analysis does not recognize the optimizing behavior of the firm for its pricing/output decisions. In Appendix 2 the common equity beta of a price-setting firm is derived in parallel with the Subrahmanyam-Thomadakis approach, recognizing the firm's optimizing effort. The main conclusion that emerges from the analysis in Appendix 2 is that the price-setting firm's common stock beta does not necessarily respond to the change in operating leverage in the same manner as the quantity-setting firm's counterpart. Another important conclusion indicates that the analytical results, with and without recognizing the firm's optimizing behavior, are the same in terms of predicting the response of beta to the change in operating leverage.

Note from Eq. (8) that the common equity beta is a linear function of operating leverage and further that their relationship may vary depending upon the sign of $\beta^o - \beta_F$. If demand uncertainty is greater than fixed cost uncertainty, $\beta^o > \beta_F$, then an increase in operating leverage raises the beta, as was observed by Rubinstein and Lev. Otherwise, the Subrahmanyam and Thomadakis conclusion holds even for the price-setting firms. More recently, Goldenberg and Chiang (1983) generalized the Subrahmanyam-Thomadakis model by admitting the covariance structure between price uncertainty and wage uncertainty. As a result, they found that the impact of the change in labor-capital ratio on the common equity beta is positive [negative] depending upon whether the price uncertainty is greater [smaller] than the wage uncertainty. While their analysis remains in the quantity-setting decision model with perfect (output) markets, their conclusion is of immediate interest to the analytical results in Appendix 2 because of the parallel logic. For the price-setting firm, which is the subject of this study, the dependence of beta on operating leverage is determined by the sign of the difference between quantity uncertainty and fixed costs uncertainty. The comparison between demand uncertainty and fixed cost uncertainty is an empirical proposition to be investigated in the future.

The last term represents financial risk when corporate debt is risky. Note that the debt-equity ratio remains as the proper variable to measure financial leverage of the firm as in the traditional decomposition model. With the introduction of risky debt, its beta enters into the model to adjust the magnitude of financial risk. Previous studies by Bierman and Oldfield (1979), Conine (1980), and Mandelker and Rhee (1984) demonstrated the same adjustment in financial risk as risky corporate debt is recognized. One may surmise that the condition $\beta^o > \beta_d$ holds under the normal circumstances.

If fixed costs are not random and corporate debt is risk-free, then Eq. (8) is reduced to the traditional decomposition of systematic risk advanced by Hamada (1972) and Rubinstein (1973). Given $\beta_F = 0$ and $\beta_d = 0$, it follows that

$$\beta = \beta^o + (1 - t)\beta^o \frac{C}{S} + (1 - t)\beta^o \frac{D}{S} \quad (9)$$

in which C now denotes the present value of fixed costs (discounted at the risk-free interest rate) and D is the market value of risk-free debt. The first two terms combined together measure the traditional version of business (or operating) risk and the last captures financial risk.

B. An Illustration

To illustrate the risk decomposition just described, common stocks issued by Amerada Hess and Raytheon are used. It must be emphasized that this illustration is not intended to address all empirical issues associated with

the proposed risk decomposition, which should be the focus of subsequent empirical studies. This is why Eq. (9) is used rather than Eq. (8) for the illustration. The summary results are presented in Table 1.

Note that business risk is higher for Raytheon than for Amerada Hess, but Raytheon's operating risk and financial risk are lower than those of Amerada Hess because of lower operating leverage and financial leverage. As a result, the beta of Amerada Hess's common stock is higher than that of Raytheon's common stock.

The equilibrium rate of return on common stock or the cost of equity capital can be easily derived from Eq. (8) along with the respective premium for business risk, operating risk, and financial risk. After multiplying both sides of Eq. (8) by $[E(\tilde{R}_m) - r]$, Eq. (8) is re-expressed using $E(\tilde{R}_s) - r = [E(\tilde{R}_m) - r]\beta_s$:

$$E(\tilde{R}_s) = r + [E(\tilde{R}_m) - r]\beta^o + (1 - t)[E(\tilde{R}_m) - r](\beta^o - \beta_F) \frac{C}{S} \\ + (1 - t)[E(\tilde{R}_m) - r](\beta^o - \beta_d) \frac{D}{S} \quad (10)$$

Table 1. Decomposition of Systematic Risk

Company	β_s	C/S	D/S	Business Risk	Operating Risk	Financial Risk
Amerada Hess	1.40	1.09	1.58	.60	.33	.47
Raytheon	1.15	.60	.46	.75	.23	.17

Notes.

- β_s was obtained from the *Value Line Investment Survey* (1985).
- S = the number of shares outstanding $\times [(High\ price + Low\ price)/2]$ as of 1984.
- As a surrogate for D , the book value of total debt as of 1984 is used.
- The corporate income tax rate of 50 percent is assumed.
- The estimation of fixed costs is more troublesome than estimating any other variables. Two estimation methods may be feasible (1) Annual 10-K reports filed with the Securities and Exchange Commission isolate at least several different items of such fixed costs as depreciation, rents, property taxes and nonemployment taxes, pension expenses, maintenance and repair, R&D expenses, etc. [See Pakkala (1979) for the analysis of these fixed-cost items] (2) An alternative approach to isolating fixed costs from total costs is to use the following time-series regressions to estimate the slope coefficient, which indicates the variable cost per dollar of net sales [See Lev (1974) for a similar approach.] This alternative approach is employed in this analysis:

$$(COGS)_t = a + b(NS)_t + u_t, \quad t = 1975-1984$$

where COGS denotes the cost of goods sold and NS stands for net sales. The estimated b is .76 for Amerada Hess and .77 for Raytheon, and both estimates are significant at $\alpha = 1\%$. After \hat{b} is estimated, the following equation is used to estimate total fixed costs:

$$C = (1 - \hat{b}) \times \text{Net sales for 1984} + \text{Administrative and selling expenses for 1984} \\ + \text{R\&D expenses for 1984}$$

which is intended to add obvious fixed-cost items available from the firm's Annual Report (1984) to the estimated fixed portion of COGS. For Amerada Hess, oil exploration expenses are also added.

where $E(\tilde{R}_s)$ denotes the equilibrium rate of return on common stock. The last three terms on the right-hand side of Eq. (10) are the business risk premium, the operating risk premium, and the financial risk premium, respectively. Assuming that the fixed costs are not random and corporate debt is risk-free, Eq. (10) is reduced to

$$E(\tilde{R}_s) = r + [E(\tilde{R}_m) - r]\beta^o + (1 - t)[E(\tilde{R}_m) - r]\beta^o \frac{C}{S} + (1 - t)[E(\tilde{R}_m) - r]\beta^o \frac{D}{S} \quad (11)$$

Continuing with the illustration in Table 1, the respective risk premiums can be estimated using Eq. (11) as presented in Table 2 under the assumption that $E(\tilde{R}_m) = .17$ and $r = .08$. The business risk premium for Amerada Hess accounts for 43 percent of its total systematic risk premium, while the comparable figure for Raytheon is 65 percent. The difference in the operating risk premiums of the two firms is relatively small, but the financial risk premiums are substantially different because of the large difference in the debt-equity ratios.

Further substitution of $E(\tilde{R}^o) - r = [E(\tilde{R}_m) - r]\beta^o$, where $E(\tilde{R}^o)$ is the equilibrium rate of return on the unlevered firm's common stock, into Eq. (11) yields

$$E(\tilde{R}_s) = E(\tilde{R}^o) + (1 - t)[E(\tilde{R}^o) - r]\frac{C}{S} + (1 - t)[E(\tilde{R}^o) - r]\frac{D}{S} \quad (12)$$

which is analogous to Proposition 2 of Modigliani and Miller (1963). Equation (12) indicates that the cost of equity capital is always an increasing function of both operating leverage and financial leverage. However, with the introduction of uncertainty into fixed costs and bondholders' claim, this relationship is not as obvious as that discussed in relation to Eq. (8).

Table 2. Estimation of the Respective Premiums for Business Risk, Operating Risk, and Financial Risk

Company	Equilibrium Return on Common Stock	Risk- Free Return	Total Systematic Risk Premium	Business Risk Premium	Operating Risk Premium	Financial Risk Premium
Amerada Hess	.206	.08	.126 (100%)	.054 (43%)	.029 (23%)	.043 (34%)
Raytheon	.184	.08	.104 (100%)	.068 (65%)	.020 (20%)	.016 (15%)

Note: $E(\tilde{R}_m) = .17$ and $r = .08$ are assumed.

III. IMPLICATIONS OF THE RISK DECOMPOSITION FOR MYERS' ADJUSTED PRESENT VALUE

The discussion in this section is directly motivated by the recent studies by Ferri and Jones (1979), Mandelker and Rhee (1984), and Dotan and Ravid (1985). These studies established the negative relationship between the two types of leverage either empirically or theoretically, which indicates the interaction of the firm's investment and financing decisions. The proposed risk decomposition is readily applicable to Myers' (1974) adjusted present value (APV) framework for capital budgeting decision, thus accommodating the previous interaction.

The traditional approach to firm (or project) valuation is to discount expected net operating income (assuming that the firm were financed solely by equity) by a weighted average cost of capital (WACC):

$$V = \sum_{i=1}^n \frac{(1-t)E(\tilde{X}_i)}{(1+\rho^*)^i} \quad (13)$$

where V is the firm's value, \tilde{X} is operating income, and ρ^* denotes the traditional WACC. The computation of ρ^* relies on either the definition of the traditional WACC,

$$\rho^* = E(\tilde{R}_e) \frac{S}{V} + (1-t)E(\tilde{r}) \frac{D}{V} \quad (14)$$

or on an alternative formula developed by Modigliani and Miller (1958),

$$\rho^* = \rho(1-tL) \quad (15)$$

where \tilde{r} is the cost of risky debt, ρ is the discount rate appropriate for the firm under all-equity financing, and L is the firm's target debt ratio. An important advantage of Eq. (15) over Eq. (14) is its underlying valuation approach, which rests on the concept of value additivity. Equations (14) and (15) both require that the firm (or project) generate a level perpetual income stream and support permanent debt to make their estimation of ρ^* exact. Unlike the traditional WACC, as noted by Myers (1974), the alternative Miller-Modigliani formula does not require the assumption that the project is identical to the firm in terms of business risk (in the traditional decomposition) and financial leverage. The Miller-Modigliani formula, however, does not allow various interaction effects other than the present value of interest tax shields for the computation of ρ^* . Myers, therefore, suggested that the estimation of ρ^* be avoided in light of these restrictive assumptions. He further recommended that the valuation approach (based upon the value additivity principle) underlying the *right-hand* side of Eq. (15) (*not* the left-hand side) be borrowed for firm (or project) evaluation,

thereby allowing other interaction effects to be introduced:

$$V = \sum_{i=1}^n \frac{(1-t)E(\tilde{X}_i)}{(1+\rho)^i} + \sum_{i=1}^n \frac{tE(\tilde{Y}_i)}{[1+E(\tilde{r})]^i} + \text{Present values of other interaction effects} \quad (16)$$

which indicates that the firm's value is the sum of its value if it were all-equity financed, the present value of the tax savings on its debt, and present values of other interaction effects.

In the proposed risk decomposition described in the previous section, the firm's value can be expressed as

$$V = \frac{(1-t)E(\tilde{X}^o)}{1+E(\tilde{R}^o)} - (1-t)C + tD \quad (17)$$

where $\tilde{X}^o = (p - v^o)\tilde{Q}$. Equation (17) represents the counterpart of Eq. (16) when both operating leverage and financial leverage are recognized. The valuation approach as suggested by Eq. (17) fits into Myers' adjusted present value in the sense that $(1-t)C$ can be treated as one of the many interaction effects to be admitted. For project evaluation, Eq. (17) suggests the following: (1) Break down the total cash flows to the project into three components: the cash flows assuming zero leverage, the after-tax fixed costs, and the tax savings on debt financing. (2) Estimate the respective value of each component using the discount rate appropriate for each component. (3) Sum up the values of three components.

The project's hurdle rate can be derived after differentiating Eq. (17) with respect to I , where I denotes the level of new investment. Recalling that the project acceptance rule requires $dV/dI \geq 1$, the cutoff point for investment is defined as

$$(1-t) \frac{dE(\tilde{X}^o)}{dI} \geq [1 + E(\tilde{R}^o)] \left[1 + (1-t) \frac{dC}{dI} - t \frac{dD}{dI} \right] \quad (18)$$

where dC/dI and dD/dI are the project's marginal contributions to the firm's operating capacity and debt capacity, respectively. As the firm invests more, it can incur additional fixed costs and interest expenses; hence, the new investment adds to the firm's operating as well as debt capacity. The quantities dC/dI and dD/dI for individual projects may be higher or lower than the firm's overall target operating leverage or target debt ratio. This interpretation is consistent with Myers' generalized perception of the Miller-Modigliani formula as defined by Eq. (15). Eq. (18) suggests that the cutoff point for investment is raised by the project's dC/dI but lowered by the project's dD/dI . If operating leverage is impounded into business risk and the perpetuity assumption is restored, then Eq. (18) is reduced to the familiar Miller-Modigliani Proposition 3:

$$(1-t) \frac{dE(\tilde{X})}{dI} \geq \rho \left[1 - t \frac{dD}{dI} \right] \quad (19)$$

IV. CONCLUSIONS

This paper represents the first attempt at a trichotomous decomposition of systematic risk of common stock as opposed to the traditional dichotomous approach. From the analytical results of the paper, several important contributions have been made to enhance our understanding of the common stock beta: First, the joint impact of operating and financial leverage on the beta has been clearly defined. Second, the real determinants of beta are now easily identified at the firm level; the three main determinants so identified are the systematic portion of demand (or sales) uncertainty and the two types of leverage.⁵ Third, three risk premiums can now be estimated using the model: business risk premium, operating risk premium, and financial risk premium. Fourth, the decomposition model demonstrates that the proper variable to serve as a proxy for operating leverage is the ratio of the risk-adjusted present value of fixed costs to the market value of common equity, while the debt-equity ratio measured in market values remains the valid surrogate of financial leverage.

Most importantly, in connection with the unresolved controversy surrounding the relationship between beta and operating leverage, this paper demonstrated that conflicting theories arise from the differing assumptions underlying the firm's decision-making behavioral modes: quantity-setting or price-setting. The conclusion reached by Subrahmanyam and Thomadakis is valid for the quantity-setting firm, whereas Rubinstein and Lev are correct for the price-setting firm as long as demand uncertainty is greater than fixed-cost uncertainty. If demand uncertainty is less than fixed-cost uncertainty, then the Subrahmanyam-Thomadakis conclusion will be supported even for the price-setting firm. The comparison between the two types of uncertainty introduced in this paper should be the focus of subsequent empirical studies.

In light of the interaction between financing and investment decisions as indicated by the negative relationship between operating and financial leverage, this paper has placed the two types of leverage in a proper perspective within Myers' (1974) adjusted present value framework for capital budgeting decisions.

APPENDIX 1

The main purpose of Appendix 1 is to demonstrate the derivation of Eq. (8) from Eq. (7) in the main text. Eq. (7) is rewritten as

$$\beta_s = \frac{S^o}{S} \left[\frac{p - v}{p - v^o} - \frac{\text{cov}(\tilde{F}, \tilde{R}_m)}{(p - v^o) \text{cov}(\tilde{Q}, \tilde{R}_m)} - \frac{\text{cov}(\tilde{Y}, \tilde{R}_m)}{(p - v^o) \text{cov}(\tilde{Q}, \tilde{R}_m)} \right] \beta^o \quad (7)$$

In order to simplify Eq. (7), three terms on its right-hand side are evaluated in order. First, solving for $(p - v)/(p - v^o)$ using Eqs. (2) and (3) in the

text yields

$$\frac{p-v}{p-v^0} = \frac{S}{S^0} + \frac{(1-t)[CE(\tilde{F}) + CE(\tilde{Y})]}{S^0(1+r)} = \frac{S + (1-t)C + (1-t)D}{S^0} \quad (A1.1)$$

where $C = CE(\tilde{F})/(1+r)$ = the risk-adjusted present value of fixed costs, and $D = CE(\tilde{Y})/(1+r)$ = the market value of risky debt. Multiplying both sides of Eq. (A1.1) by S^0/S yields

$$\frac{S^0}{S} \cdot \frac{p-v}{p-v^0} = 1 + (1-t) \frac{C}{S} + (1-t) \frac{D}{S} \quad (A1.2)$$

Next, the second term on the right-hand side of Eq. (7) is evaluated:

$$\frac{S^0}{S} \cdot \frac{\text{cov}(\tilde{F}, \tilde{R}_m)}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)} = \frac{\text{cov}(\tilde{F}, \tilde{R}_m)/S}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)/S^0} \quad (A1.3)$$

Multiplying the numerator of Eq. (A1.3) by

$$\frac{1-t}{\sigma^2(\tilde{R}_m)} \cdot \frac{C}{C}$$

and the denominator by

$$\frac{1-t}{\sigma^2(\tilde{R}_m)}$$

reduces Eq. (A1.3) to

$$\frac{S^0}{S} \cdot \frac{\text{cov}(\tilde{F}, \tilde{R}_m)}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)} = \frac{(1-t)\beta_F C/S}{\beta^0} \quad (A1.4)$$

where

$$\beta_F = \frac{\text{cov}(\tilde{F}, \tilde{R}_m)}{C\sigma^2(\tilde{R}_m)} = \text{beta of fixed costs};$$

$$\beta^0 = \frac{(1-t)(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)}{S^0\sigma^2(\tilde{R}_m)} = \text{beta of the unlevered firm}.$$

Lastly, the third term of Eq. (7) is evaluated:

$$\frac{S^0}{S} \cdot \frac{\text{cov}(\tilde{Y}, \tilde{R}_m)}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)} = \frac{\text{cov}(\tilde{Y}, \tilde{R}_m)/S}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)/S^0} \quad (A1.5)$$

Multiplying the numerator of Eq. (A1.5) by

$$\frac{1-t}{\sigma^2(\tilde{R}_m)} \cdot \frac{D}{D}$$

and the denominator by

$$\frac{1-t}{\sigma^2(\tilde{R}_m)}$$

yields

$$\frac{S^0}{S} \cdot \frac{\text{cov}(\tilde{Y}, \tilde{R}_m)}{(p-v^0) \text{cov}(\tilde{Q}, \tilde{R}_m)} = \frac{(1-t)\beta_d D/S}{\beta^0} \quad (\text{A1.6})$$

where $\beta_d = \text{cov}(\tilde{Y}, \tilde{R}_m)/D\sigma^2(\tilde{R}_m)$ = the beta of risky debt.

Substitution of Eqs. (A1.2), (A1.4), and (A1.6) into Eq. (7) yields

$$\beta = \beta^0 + (1-t)(\beta^0 - \beta_F) \frac{C}{S} + (1-t)(\beta^0 - \beta_d) \frac{D}{S} \quad (8)$$

APPENDIX 2

The primary purpose of Appendix 2 is to demonstrate that operating leverage impacts upon the systematic risk of common stock in the same manner as described in the text when the firm's optimizing behavior is explicitly recognized, à la Subrahmanyam and Thomadakis (1980). The sources of uncertainty are introduced into the model by random demand and random fixed costs. Since Subrahmanyam and Thomadakis investigated the relationship between a financially unlevered firm's beta and operating leverage, another source of uncertainty associated with the firm's debt obligations is not considered in this Appendix. To be compatible with the Subrahmanyam-Thomadakis model, corporate income taxes are not introduced. Both uncertain demand and fixed costs are specified as

$$\tilde{Q} = Q(1 + \tilde{e}) \quad (\text{A2.1})$$

where $E(\tilde{Q}) = Q$ is the expected value of uncertain demand, and \tilde{e} is a random error with zero demand and $\sigma^2(\tilde{e})$ representing demand fluctuations about the mean, and

$$\tilde{F} = F(1 + \tilde{u}) \quad (\text{A2.2})$$

where $E(\tilde{F}) = F$ is the expected value of uncertain fixed costs and \tilde{u} is a random error term with $E(\tilde{u}) = 0$ and $\sigma^2(\tilde{u}) > 0$.

Given Eq. (A2.1) and Eq. (A2.2), the total revenue and the total costs are defined as

$$\widetilde{TR} = p\tilde{Q} \quad (\text{A2.3})$$

$$\widetilde{TC} = v\tilde{Q} + \tilde{F} \quad (\text{A2.4})$$

where p denotes the price per unit to be determined ex ante for the

price-setting firm and v is the variable cost per unit. From Eqs. (A2.3) and (A2.4), marginal revenue and marginal costs are defined as

$$\widetilde{MR} = \frac{\partial \widetilde{TR}}{\partial p} = (1 - \eta)Q(1 + \tilde{e}) \quad (A2.5)$$

$$\widetilde{MC} = \frac{\partial \widetilde{TC}}{\partial p} = -\eta \frac{v}{p} Q(1 + \tilde{e}) \quad (A2.6)$$

where η is the absolute value of the price elasticity of demand and the condition $\eta > 1$ is imposed to satisfy the second-order condition for the firm's value maximization. The firm's value is defined by Eq. (A2.7) in the capital asset pricing model framework:

$$V = \frac{CE(\widetilde{TR}) - CE(\widetilde{TC})}{1 + r} \quad (A2.7)$$

where

$$\begin{aligned} CE(\widetilde{TR}) &= \text{the certainty equivalent of } \widetilde{TR}; \\ &= E(\widetilde{TR}) - \lambda \text{cov}(\widetilde{TR}, \tilde{R}_m) = pQ[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)] \end{aligned}$$

$$\begin{aligned} CE(\widetilde{TC}) &= \text{the certainty equivalent of } \widetilde{TC}; \\ &= E(\widetilde{TC}) - \lambda \text{cov}(\widetilde{TC}, \tilde{R}_m) \\ &= vQ[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)] + F[1 - \lambda \text{cov}(\tilde{u}, \tilde{R}_m)]; \end{aligned}$$

and λ and $\text{cov}(\cdot)$ have been defined in the text.

To determine the firm's value-maximizing p^* , the first derivative of Eq. (A2.7) with respect to p is obtained:

$$\frac{\partial V}{\partial p} = \frac{CE(\widetilde{MR}) - CE(\widetilde{MC})}{1 + r} \quad (A2.8)$$

where

$$CE(\widetilde{MR}) = (1 - \eta)Q[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)]$$

$$CE(\widetilde{MC}) = -\eta \frac{v}{p} Q[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)]$$

Thus, Eq. (A2.9) represents the first-order maximizing condition:

$$Q(1 - \eta)[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)] + \eta \frac{v}{p} Q[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)] = 0 \quad (A2.9)$$

from which the value-maximizing p^* is determined by

$$p^* = -\frac{\eta}{1 - \eta} v \quad (A2.10)$$

Substitution of Eq. (A2.10) into (A2.7) yields the maximum value of the firm, V^* :

$$V^* = \frac{-vQ[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)]/(1 - \eta) - F[1 - \lambda \text{cov}(\tilde{u}, \tilde{R}_m)]}{1 + r} \quad (\text{A2.11})$$

The common stock beta of the firm is defined as

$$\beta = \frac{\text{cov}(\overline{TR} - \overline{TC}, \tilde{R}_m)}{V^* \sigma^2(\tilde{R}_m)} \quad (\text{A2.12})$$

Substitution of

$$\text{cov}(\overline{TR} - \overline{TC}, \tilde{R}_m) = p^*Q \text{cov}(\tilde{e}, \tilde{R}_m) - vQ \text{cov}(\tilde{e}, \tilde{R}_m) - F \text{cov}(\tilde{u}, \tilde{R}_m)$$

and of V^* as defined by Eq. (A2.11) produces, after rearrangement,

$$\beta = \frac{Q \text{cov}(\tilde{e}, \tilde{R}_m)/(1 - \eta) + y \text{cov}(\tilde{u}, \tilde{R}_m)}{Q[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)]/(1 - \eta) + y[1 - \lambda \text{cov}(\tilde{u}, \tilde{R}_m)]} \cdot \frac{1 + r}{\sigma^2(\tilde{R}_m)} \quad (\text{A2.13})$$

where $y = F/v$, which will serve as a surrogate for operating leverage of the firm. To determine the relationship between beta and operating leverage, $\partial\beta/\partial y$ is obtained:

$$\frac{\partial\beta}{\partial y} = \frac{-(1/(1 - \eta))Q[\text{cov}(\tilde{e}, \tilde{R}_m) - \text{cov}(\tilde{u}, \tilde{R}_m)]}{\{Q[1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m)]/(1 - \eta) + y[1 - \lambda \text{cov}(\tilde{u}, \tilde{R}_m)]\}^2} \cdot \frac{1 + r}{\sigma^2(\tilde{R}_m)} \quad (\text{A2.14})$$

It is obvious from Eq. (A2.14) that

$$\frac{\partial\beta}{\partial y} \cong 0 \quad \text{if} \quad \text{cov}(\tilde{e}, \tilde{R}_m) \cong \text{cov}(\tilde{u}, \tilde{R}_m) \quad (\text{A2.15})$$

Thus, as long as demand uncertainty is greater (smaller) than fixed cost uncertainty, an increase in operating leverage will raise (lower) the beta of common stock. This conclusion is effectively the same as that obtained based upon Eq. (8) in the text:

$$\frac{\partial\beta}{\partial(C/S)} \cong 0 \quad \text{if} \quad \beta^o \cong \beta_F \quad (\text{A2.16})$$

While Subrahmanyam and Thomadakis observed that the beta of common stock is negatively related to operating leverage (as measured by the reciprocal of the labor-capital ratio) for the quantity-setting firm, their conclusion does not necessarily hold for the price-setting firm. Both Eqs. (A2.15) and (A2.16) justify the positive relationship between the two, as was noted by Rubinstein (1973) and Lev (1974) if $\beta^o > \beta_F$. Otherwise, the

Subrahmanyam-Thomadakis conclusion is still valid even for the price-setting firm. The comparison between the respective magnitude of β^0 and β_F should be the focus of future empirical studies.

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NOTES

1. The assumption that both principal and interest are tax-deductible is not new in literature. Among others, see Rubinstein (1973), Kim (1978), and Rhee (1984).

2. The number of output units sold should be less than, or equal to, the total demand. Following Leland (1972, p. 280), however, the two quantities are treated as equal, so that the optimization process remains within the standard control problem in p and Q , where the two variables are related by the demand function. See Brocket et al. (1984) for a recent treatment of the case in which demand is not equal to output.

3. Both McCall (1971, p. 418) and Leland (1972, p. 284) demonstrated that price setters and quantity setters behave differently under uncertainty; price setters behave less monopolistically than quantity setters.

4. The analysis may proceed in parallel with Subrahmanyam and Thomadakis (1980) such that (1) the optimal price is determined to maximize the firm's value, (2) the corresponding optimal quantity and the firm's value are estimated, and (3) the beta of common stock is derived. One advantage of this approach is that the optimal price/output can be expressed as a function of the price elasticity of demand, and subsequently it allows us to introduce Lerner's index of monopoly power. The analysis in the text does not follow these procedures for the following reasons: (1) As demonstrated in Appendix 2, the analytical results obtained when recognizing the firm's optimizing behavior in line with Subrahmanyam and Thomadakis do not differ from those presented in the text. (2) The main purpose of this paper is to decompose beta rather than to establish the relationship between systematic risk and monopoly power. (3) The risk decomposition can be carried out in the manner more comparable with the traditional approach.

5. See Myers (1977), Hill and Stone (1980), and Mandelker and Rhee (1984) for discussions of real determinants of beta and references to the literature.

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