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Review of Investment/Financing Interaction Models

Traditionally, financial management theory has emphasized the separation of the capital investment and financing decisions [2, pp. 81 and 176]. This separation assumes that the firm's financing decision is taken as given when the investment decision is made or that the two decisions are independent of each other. In reality, these decisions are seldom independent. Mergers and acquisitions are typical examples of capital investments that make the investment/financing separation inappropriate.

The tone of research on the interaction between investment and financing decisions was set by Myers [16]. Myers advanced the concept of Adjusted Present Value (APV), which permits an examination and evaluation of the consequences of interactions between the firm's financing and investment decisions. A new project's APV is defined as the sum of the present value of its net operating income assuming all-equity financing plus the value of any additional debt capacity to the firm contributed by the project. The definition of the new project's APV presented above represents a fairly simplified version. However, a closer look at the concept of APV brings up other potential issues relating to stock purchase decisions, dividend policy, and transaction costs associated with new sources of funding, etc. These are important financial management variables that have been considered in other research efforts [1, 3, 7, 17].

The second phase of the investment/financing interaction process was developed by Bower and Jenks (BJ) [1]. They used the simplified concept of APV in their effort to estimate divisional screening rates for decentralized investment decisions. After assuming that each investment project had its implicit optimal debt ratio, BJ used this implicit ratio to estimate the project's cut-off rate in the framework of the capital asset pricing model. While BJ's study does provide an important application of the APV concept, it does not go far enough: (a) It does not provide any theoretical basis for assessment of the implicit debt ratio of each project. Their analysis instead relies on average debt ratios of different industries observed on an *ex post* basis. (b) It assumes that the firm's debt capacity is increased by an amount equivalent to the project's debt capacity. Although the additivity of the firm's debt capacity and the new project's debt capacity was originally suggested by Solomon [22], this assumption is not always valid.

Martin and Scott (MS) [13] addressed these two unresolved issues. First, MS provided a conceptual basis for determining debt capacity. They defined it in terms of the target probability of insolvency a firm wants to maintain. Second, MS contended that the diversification effect reduces the target probability of insolvency and at the same time increases the firm's debt capacity. The analytical framework provided by MS implicitly assumes that corporate insolvency is costless. However, costless insolvency makes the probability of insolvency a doubtful criterion for capital budgeting decisions. More recently, Hong and Rappaport (HR) [7] extended the MS analysis by introducing costly insolvency into the firm's valuation model to determine its optimal level of debt. HR suggest that insolvency costs should be considered in order to avoid overstating the additional debt capacity to the firm contributed by the new project. HR's analysis has three limitations. First, their valuation equation still overstates the present value of any tax subsidy and, subsequently, the optimal level of debt because they assume that the cost of debt remains constant. Second, HR (as well as MS) consider the diversification effect as "good news" for the post-investment debt capacity level, which is not necessarily true. Third, they overlook the important issue of potential wealth transfers between stockholders and bondholders.

Given their limitations, past analyses could lead financial managers to make sub-optimal decisions about their firm's capital structure. We will illustrate how an optimal debt ratio can be determined in the presence of costly insolvency using an alternative valuation model that does not require that the cost of debt be fixed. The model allows an evaluation of the consequences of interactions between financing and investment decisions of the firm.

Search for Optimal Level of Debt

The alternative valuation model is developed in the appendix. We assume risk neutrality considering the position of Gonzales, Litzenberger, and Rolfo [5], who correctly point out that potential problems of misspecification will arise when the capital asset pricing model is used to determine the optimal debt ratio of a levered firm. Scott [21] had earlier discussed the theory of optimal capital structure under the assumption of risk neutrality.

For a simple version of a valuation model in a multiperiod setting, we assume that the term structure of interest rates is flat and both operating income ($\tilde{X}$) and corporate debt are treated as perpetuities. The use of the perpetuity assumption is a matter of convenience and does not affect the results in any significant way. We further assume that the firm's operating income is normally distributed and that insolvency is defined as a state in which the firm's operating income does not cover its debt obligation, *i.e.*, $\tilde{X} < R$ where R is interest expense due at the end of each period. For further simplicity, we ignore the possibility of the firm using cash reserves to meet debt service requirements. Insolvency costs are assigned to the risk of insolvency as was formalized by Kraus and Litzenberger [11] and Kim [9].

Our alternative valuation model in the presence of costly insolvency is stated as:

$$V_\ell = V_u + tV_d^* - V_k, \qquad (1)$$

where V_ℓ = the market value of the levered firm,
V_u = the market value of the identical but unlevered firm,
V_d^* = the market value of debt in the presence of *costless* insolvency,
V_k = the present value of insolvency costs, and
t = the corporate income tax rate.

From Equation (1), two valuation equations are derived, representing special cases as some of the basic assumptions are relaxed. When no insolvency risk is assumed in a perfect capital market, Equation (1) is reduced to Modigliani and Miller's (MM) [14] tax-correction model as shown by Equation (2).

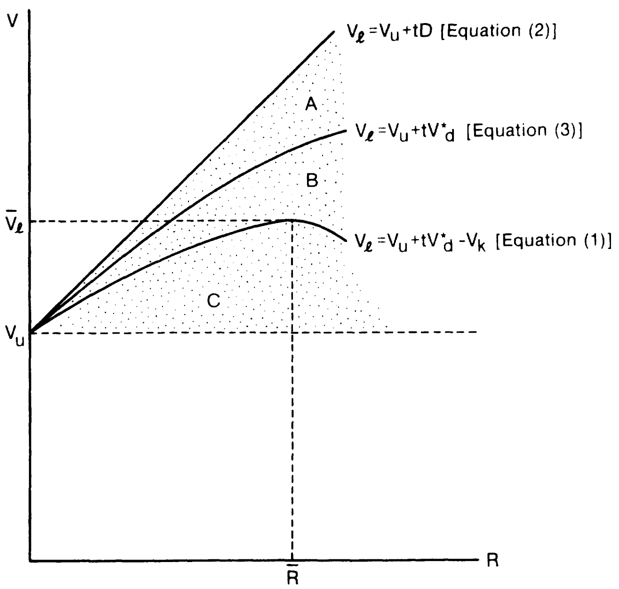
$$V_\ell = V_u + tD. \qquad (2)$$

When a costless insolvency risk is introduced, Equation (1) is reduced to another variation of MM's tax-correction model:

$$V_\ell = V_u + tV_d^*. \qquad (3)$$

Exhibit 1 provides a graphical illustration of how the valuation models defined by Equations (1), (2), and (3) differ. It shows the V_ℓ curve when total interest expense (R) increases under the different sets of assumptions. All three equations have a common intercept, V_u, which represents the value of the unlevered firm. The tax subsidies on debt financing increase as indicated by the different shaded areas (A, B, and C). Under the MM tax-correction model [Equation (2)], the V_ℓ curve is a linear function of R with a constant slope of t/R_I. The shaded area (= A + B + C) represents the present value of the tax subsidy (= $tD = V_\ell - V_u$) which increases proportionally as the amount of debt increases. Once we introduce insolvency risk (but with

Exhibit 1. Valuation Models



no attendant costs) [Equation (3)], the present value of any tax subsidies decreases by the shaded area A because of the reduced market value of debt in the presence of a *costless* insolvency. The slope of the V_ℓ curve in this case increases at a diminishing rate. To prove this, we need to show that the first and second derivatives of Equation (3) are positive and negative, respectively. That is, $dV_\ell/dR = t(dV^*_d/dR) = t[1-F(R)]/R_f > 0$ and $d^2V_\ell/dR^2 = -tf(R)/R_f < 0$.

Observe that the V_ℓ curve never slopes downward in the presence of costless insolvency. This implies that 100% debt financing would remain optimal as in MM's tax-correction model which assumes no insolvency. Therefore, it is not appropriate, theoretically, to consider the value of any additional debt capacity generated by reducing insolvency risk in a world of costless insolvency, as business firms would try to carry as much debt as possible (or equivalently, try to accommodate the highest probability of insolvency). As a result, Martin and Scott [13] and Gahlon and Stover [3] lose much of the theoretical merit of their analyses as they assume that insolvency is costless. However, when costly insolvency risk is introduced, the V_ℓ curve eventually slopes downward as soon as the marginal insolvency cost of debt financing outweighs the marginal tax benefit of debt financing [Equation (1)].

Note that the present value of tax subsidies [represented by the shaded area C] is smallest in the presence of costly insolvency. As presented in Exhibit 1, the market value of the firm under costly insolvency reaches its maximum when the slope of the V_ℓ curve is zero. The corresponding values on the vertical and horizontal axes where the slope is zero indicate the maximum value of the firm ($\bar{V}_\ell$) and the optimal amount of interest expense ($\bar{R}$).

Thus:

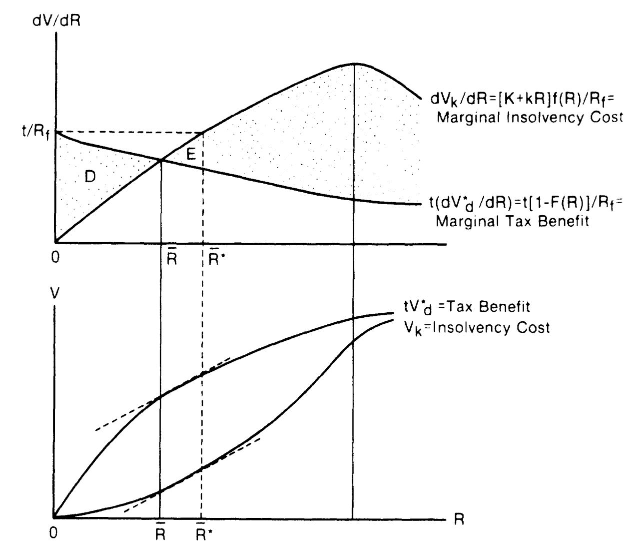
$$dV_\ell/dR = t(dV^*_d/dR) - dV_k/dR = 0, \quad (4)$$

where $dV^*_d/dR = [1-F(R)]/R_f$,
$dV_k/dR = (K + kR)f(R)/R_f$,
$F(\bullet)$ = the cumulative probability distribution function of a normally distributed random variable,
$f(\bullet)$ = the normal probability density function,
K = a fixed component of insolvency costs, and
k = a variable component scale factor of insolvency costs.

The first term on the right-hand side of Equation (4) represents the marginal benefit of debt financing and the second term the marginal cost. Exhibit 2 illustrates the marginal relationship between tax benefits and insolvency costs, and the optimal amount of interest expense.

As Exhibit 2 shows, the optimal amount of interest expense ($\bar{R}$) is determined where the marginal tax

Exhibit 2. Marginal Cost and Benefit of Debt Financing



benefit of debt financing is equal to its marginal insolvency costs; that is, the slope of the tax benefit curve equals that of the insolvency cost curve. The shaded area D in the Exhibit therefore represents the net benefit of debt financing when the firm carries the optimal amount of debt. The shaded area E represents the net decrease in the value of the firm as the amount of debt surpasses this optimal level. The marginal tax benefit curve is monotonically decreasing as the amount of debt financing increases with a corresponding increase in the probability of insolvency, [F(R)]. The valuation model employed by HR [7] explicitly assumes that the marginal tax benefit is constant regardless of the size of the debt financing. This assumption produces an overstatement of the value of tax subsidies as well as of the optimal level of debt. As indicated by $\bar{R}^*$ in Exhibit 2, the constant cost of debt assumption of HR yields a much larger optimal amount of interest expense than it should. Oversimplification of this nature can yield the embarrassing result of an optimal debt ratio greater than unity. Given an extremely small variance of cash flows (and, therefore, small probability of insolvency), HR's model may yield a large optimal level of debt which can be greater than the total value of the firm. For example, if we use $\sigma(\tilde{X}_A) = \$15{,}000$ instead of \$80,000 for firm A in HR's numerical example, we then obtain the optimal amount of debt (\$1,029,375) and the value of the firm A (\$983,806.25).

Numerical Example

Assume the following data for firm A which plans to acquire firm B.

	Firm A	*Firm B*
$E(\tilde{X})$	\$120,000	\$20,000
$\sigma(\tilde{X})$	40,000	7,000
K	40,000	8,000
k	.2	.2
t	.5	.5

$R_f = .05$
$\rho(\tilde{X}_A, \tilde{X}_B) = .5$

Equation (4) is rewritten for the convenience of using tables for the normal density function and cumulative normal distribution. As $F(\bar{R}) = F(\bar{Z})$ and $f(\bar{R}) = f(\bar{Z})/\sigma(\tilde{X})$ where $\bar{Z} = [\bar{R} - E(\tilde{X})]/\sigma(\tilde{X})$, Equation (5) represents an operational form of Equation (4):

$$t[1 - F(\bar{Z})] = (K + kR)f(\bar{Z})/\sigma(\tilde{X}). \quad (5)$$

Given the equality condition specified by Equation (5), we need to find $\bar{R}$ for firm A by trial and error. As shown in Exhibit 3, we illustrated the search process beginning with a Z-value of −.85.

When Z = −.85, we find that F(Z) = .1977 and f(Z) = .2780 from tables for the cumulative normal distribution and ordinates of the normal density function [15, pp. 551–552]. Because $E(\tilde{X}) = \$120{,}000$ and $\sigma(\tilde{X}) = \$40{,}000$, we know R = \$86,000 given Z = −.85 for firm A. Therefore, the LHS of Equation (5) is .4012, the RHS .3975, and their difference is .0037 as shown in the last column of Exhibit 3. When we use Z = −.84 (or R = \$86,400), we find that the difference between the LHS and the RHS is only −.0016, while it is −.0070 when Z = −.83 (or R = \$86,800). Because the Z-value of −.84 gives us the smallest difference between both sides of Equation (5), we can conclude that R = \$86,400 is the optimal amount of interest expense. This is the Z-value that equates the marginal tax benefit to the marginal insolvency cost of debt financing and maximizes the value of firm A.

Given the optimal amount of interest expense, $\bar{R}_A = \$86{,}400$, the next step is to estimate market values of the firm ($\bar{V}_\ell$), debt ($\bar{V}_d$), and equity ($= \bar{V}_s = \bar{V}_\ell - \bar{V}_d$) (where the bar denotes the values when the firm carries the optimal amount of debt).

Since $V_\ell = V_u + tV_d^* - V_k$, we need to estimate the value of the unlevered firm (V_u), the value of debt (V_d^*) when insolvency is costless, and the present value of insolvency costs (V_k). Using Equations (A-1), (A-6), and (A-5) in the Appendix, we have:

$$\begin{aligned}
V_u &= (1-t)\{\sigma^2(\tilde{X})f(0) + E(\tilde{X})[1-F(0)]\}/R_f \\
&= (1-.5)[40{,}000(.0044) \\
&\quad + 120{,}000(1-.0013)]/.05 \\
&= \$1{,}200{,}200, \\
V_d^* &= \{R[1-F(R)] + \sigma^2(\tilde{X})[f(0)-f(R)] \\
&\quad + E(\tilde{X})[F(R)-F(0)]\}/R_f \\
&= \{86{,}400[1-.2005] + 40{,}000[.0044-.2803] \\
&\quad + 120{,}000[.2005-.0013]\}/.05 \\
&= \$1{,}638{,}896, \text{ and} \\
V_k &= (K[F(R)-F(\theta)] + k\{\sigma^2(\tilde{X})[f(\theta)-f(R)] \\
&\quad + E(\tilde{X})[F(R)-F(\theta)]\} + \{\sigma^2(\tilde{X})[f(0)-f(\theta)] \\
&\quad + E(\tilde{X})[F(\theta)-F(0)]\})/R_f \\
&= (40{,}000[.2005-.0401] + .2\{40{,}000 \\
&\quad [.0863-.2803] \\
&\quad + 120{,}000[.2005-.0401]\} + \\
&\quad \{40{,}000[.0044-.0863] \\
&\quad + 120{,}000[.0401-.0013]\})/.05 \\
&= \$201{,}872.
\end{aligned}$$

Exhibit 3. Search for Optimal Interest Expenses for Firm A.

(1) Z	(2) F(Z)	(3) f(Z)	(4) R	(5) LHS{= t[1 − F(Z)]}	(6) RHS[= (K + kR) f(Z)/σ(X̄)]	(7) Difference [= (5) − (6)]
−.85	.1977	.2780	86,000	.4012	.3875	.0037
−.84	.2005	.2803	86,400	.3998	.4014	−.0016
−.83	.2033	.2827	86,800	.3984	.4054	−.0070

Therefore, the market value of firm A is:

$$\bar{V}_f = V_u + tV_d^* - V_k = 1,200,200 + (.5)(1,638,896) - 201,872 = \$1,817,776.$$

Further, using Equation (A-7) in the Appendix, the market value of debt in the presence of costly insolvency is:

$$\bar{V}_d = V_d^* - V_k = 1,638,896 - 201,872 = \$1,437,024.$$

Now the market value of equity ($\bar{V}_s$) of firm A and its optimal debt ratio ($\bar{L}$) are easily obtained:

$$\bar{V}_s = \bar{V}_f - \bar{V}_d = 1,817,776 - 1,437,024 = \$380,752, \text{ and}$$
$$\bar{L} = \bar{V}_d/\bar{V}_f = 1,437,024/1,817,776 = .79.$$

As a small standard deviation of operating income relative to its expected value is used, the optimal debt ratio tends to be larger than normally observed. In order to eliminate the potential impact of the financial leverage effect, we made the optimal debt ratios of firms A and B as close as possible. A recent paper by Rhee and Tadikamalla [20] examines the financial leverage effect as well as the diversification effect of corporate mergers.

The same procedures are used for the estimation of $\bar{V}_f$, $\bar{V}_d$, and $\bar{V}_s$ for firm B as well as firm A after it acquires firm B. For the post-merger valuation of firm A, we assume that no operating synergistic effects are generated by the merger, *i.e.*, $\bar{X}_{A'} = \bar{X}_A + \bar{X}_B = \$140,000$ where prime indicates the post-merger value. Given the coefficient of correlation between the pre-merger income streams $[\rho(\bar{X}_A,\bar{X}_B) = .5]$, the standard deviation of the post-merger income stream is:

$$\sigma(\bar{X}_{A'}) = [\sigma^2(\bar{X}_A) + \sigma^2(\bar{X}_B) + 2\rho(\bar{X}_A,\bar{X}_B)\sigma(\bar{X}_A)\sigma(\bar{X}_B)]^{1/2} = \$43,920.$$

Exhibit 4 presents summary results. The first and third columns show the estimated values for firm A before and after the new investment, respectively. The second column provides the relevant information on firm B. Net gains after the acquisition are presented in the last column, where a few striking results are observed. First, the post-investment optimal interest expense ($\bar{R}$) is less than the sum of the optimal interest expenses of the pre-merger firms. Second, the post-investment market value of equity ($\bar{V}_s$) is less than the sum of pre-investment equity values of firms A and B. Third, bondholders ($\bar{V}_d$) of pre-merger firms A and B have windfall gains at the cost of stockholders' wealth.

Diversification Effect

Following the contention of Lewellen [12], Hong and Rappaport [7] suggest that diversification generated by the less-than-perfectly correlated income streams increases the optimal level of debt for the post-merger firm. Our example, though, clearly shows that the post-investment optimal interest expense is less than the sum of the pre-investment optimal interest expenses of firms A and B evaluated separately. A caveat is in order. Diversification is not always "good" news for the post-investment optimal level of debt in the presence of positive insolvency cost. The sensitivity analysis in Exhibit 5 shows how the optimal amount of interest expense varies when the size of insolvency cost and the correlation coefficient of the pre-merger income streams change.

Exhibit 4. Summary of Pre- vs. Post-Investment Values

	A	B	A'*	A' − (A + B)
$\bar{Z}$	−.84	−.90	−.91	
F($\bar{Z}$)	.2005	.1841	.1814	
f($\bar{Z}$)	.2803	.2661	.2637	
θ	50,000	10,000	60,000	
$\bar{R}$	\$86,400	\$13,700	\$100,032.80	−\$67.20
$\bar{V}_d$	1,437,024	227,057.40	1,693,381.58	29,300.18
$\bar{V}_s$	380,752	70,028.70	442,988.54	−7,792.16
$\bar{V}_f$	1,817,776	297,086.10	2,136,370.12	21,508.02
$\bar{L}$**	.79	.76	.79	

*Prime indicates the post-investment value.
**$\bar{L}$ = Optimal debt ratio = $\bar{V}_d/\bar{V}_f$.

Exhibit 5 shows what happens to the post-investment optimal amount of interest expense as we increase the size of insolvency cost (by increasing the variable cost scale factor), holding the correlation coefficient constant. The larger the insolvency cost, the smaller the optimal amount of interest expense. This result is intuitively obvious. Exhibit 5 also shows what happens to the post-investment optimal interest expense amount as we decrease the coefficient of correlation. The last two columns show that the smaller the correlation, the larger the optimal interest expense for the post-investment firm. Quite a surprising fact, however, is indicated by the first two columns. As we reduce the correlation, the optimal amount of interest expense does not increase despite the diversification effect. In general, with an increasing insolvency cost, diversification helps the firm increase its borrowing capacity after the merger but the reverse is found to be true when the size of insolvency cost is small. Rhee [19] provides various conditions under which the post-merger optimal level of debt is greater than, equal to, or less than the sum of pre-merger firms' optimal levels of debt in a framework of capital asset pricing model.

Wealth Transfer

Given the information in the last column of Exhibit 4, it is obvious that undertaking the new investment would result in a critical violation of the so-called "me-first" rule [2, p. 179]. This violation would make the project unacceptable for stockholders. The wealth transfer from stockholders to bondholders in this numerical example has an important implication for potential agency costs, in a narrow sense, based upon the firm-creditor relationship. (Recall that Jensen and Meckling [8] define agency costs in a broad context to include information costs, costs of financial distress, issues costs, or costs of managing the firm.) The owner's losses created by the investment decision constitute a substantial portion of agency costs simply because the firm provides more benefits to creditors than required by the debt contract.

One may suggest, as a simple solution, that bondholders can pay back their windfall gains to stockholders in the form of compensating side payments. However, this may not be a complete solution in this particular case as bondholders' windfall gains are not the same as stockholder's losses. As bondholders gain more than stockholders lose, something more should be done to negate whatever bondholders gain. Higgins and Schall [6] and Galai and Masulis [4] suggest that the acquiring firm's financial leverage can be raised to the point where the post-merger default risk is sufficiently large to neutralize the wealth transfer. Kim and McConnell [10], in an empirical examination of this issue, find it to be a realistic solution to the acquiring firm.

Exhibit 5. Sensitivity Analysis for the Post Investment Optimal Interest Expense

ρ \ k	.05	.10	.20	.40
0.5	$109,256	$105,742.40	$100,032.80	$92,566.40
0.0	108,731.84	105,483.20	100,610.24	93,706.88
–0.5	108,550	105,590	101,150	95,230

A difficult situation may arise, however, when bondholder gains are less than stockholder losses or, equivalently, the post-investment market value of the combined firm is smaller than the sum of pre-merger values. It is not difficult to draw up a numerical example which shows this kind of situation by adjusting the standard deviations of income streams and the magnitude of insolvency costs. This is a typical case of a merger that is unacceptable to stockholders. Therefore, it is important for managers to consider carefully the potential wealth transfer from the stockholders' perspective. Of course, a meaningful evaluation of potential wealth transfers can be made only when interactions between financing and investment decisions are taken into account, that is, changes in insolvency costs, optimal debt levels, and tax subsidies on debt financing. Thus, the Adjusted Present Value framework advanced by Myers [16] becomes a more effective concept to be used for large scale capital investments.

Conclusion

This paper extends previous analyses of corporate debt capacity and capital budgeting decisions when the investment and financing decisions of firms interact. Our alternative valuation model in the presence of costly insolvency permits a search for an optimal debt ratio for a firm making a relatively large investment decision. It also permits a more explicit discussion of the impact of the diversification effect on the optimal capital structure decision, the value of the firm, potential wealth transfers between bondholders and stockholders, and related agency cost implications.

We conclude that: 1) Diversification is *not* always "good news" when insolvency is costly. It does not necessarily increase the optimal level of debt after a merger. Only when insolvency costs are large can corporate borrowing capacity increase under the diversification effect. 2) Diversification is *not* always favorable for the post-merger value of a firm, either. While the post-investment value of the firm may increase, the distribution of the gains between bondholders and stockholders may make the project unacceptable unless protective covenants or me-first rules are strictly enforced. When evaluating merger consequences, Myers' Adjusted Present Value is effectively reflected in the framework of our valuation.

References

1. R. S. Bower and J. M. Jenks, "Divisional Screening Rates," *Financial Management* (Autumn 1975), pp. 42–49.
2. E. F. Fama and M. H. Miller, *The Theory of Finance*, New York, Holt, Rinehart, and Winston, Inc., 1972.
3. J. M. Gahlon and R. D. Stover, "Debt Capacity and the Capital Budgeting Decision: A Caveat," *Financial Management* (Winter 1979), pp. 55–59.
4. D. Galai and R. W. Masulis, "The Option Pricing Model and the Risk Factor of Stock," *Journal of Financial Economics* (January–March 1976), pp. 53–81.
5. N. Gonzales, R. Litzenberger, and J. Rolfo, "On Mean Variance Models of Capital Structure and the Absurdity of Their Predictions," *Journal of Financial and Quantitative Analysis* (June 1977), pp. 165–179.
6. R. C. Higgins and L. D. Schall, "Corporate Bankruptcy and Conglomerate Merger," *Journal of Finance* (March 1975), pp. 93–113.
7. H. Hong and A. Rappaport, "Debt Capacity, Optimal Capital Structure, and Capital Budgeting Analysis," *Financial Management* (Autumn 1978), pp. 7–11.
8. M. C. Jensen and W. H. Meckling, "The Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics* (October 1976), pp. 306–359.
9. E. H. Kim, "A Mean Variance Theory of Optimal Capital Structure and Corporate Debt Capacity," *Journal of Finance* (March 1978), pp. 45–63.
10. E. H. Kim and J. J. McConnell, "Corporate Mergers and the Coinsurance of Corporate Debt Hypothesis," *Journal of Finance* (May 1977), pp. 349–363.
11. A. Kraus and R. H. Litzenberger, "A State-Preference Model of Optimal Financial Leverage," *Journal of Finance* (September 1973), pp. 911–922.
12. W. G. Lewellen, "A Pure Financial Rationale for the Conglomerate Mergers," *Journal of Finance* (May 1971), pp. 521–537.
13. J. D. Martin and D. F. Scott, Jr., "Debt Capacity and the Capital Budgeting Decision," *Financial Management* (Summer 1976), pp. 7–14.
14. F. Modigliani and M. H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review* (June 1963), pp. 433–443.
15. A. M. Mood, F. A. Graybill, and D. C. Boes, *Introduction to the Theory of Statistics*, 3rd ed., New York, McGraw-Hill, 1974.
16. S. C. Myers, "Interactions of Corporate Financing and Investment Decisions — Implications for Capital Budgeting," *Journal of Finance* (March 1974), pp. 1–25.
17. S. C. Myers and G. A. Pogue, "A Programming Approach to Corporate Financial Management," *Journal of Finance* (May 1974), pp. 579–599.
18. G. Pye, "Gauging the Default Premium," *Financial Analysts Journal* (January–February 1974), pp. 49–52.
19. S. G. Rhee, "Financial Consequences of Mergers: A Critical Examination of the Diversification Effect," A paper presented at the 14th Annual Conference of the Western Finance Association, San Francisco, 1979.
20. S. G. Rhee and P. Tadikamalla, "A Further Reexamination of Co-Insurance Effect," University of Pittsburgh Working Paper (December 1980).
21. J. H. Scott, "A Theory of Optimal Capital Structure," *The Bell Journal of Economics* (Spring 1976), pp. 33–53.
22. E. Solomon, "Measuring a Company's Cost of Capital," *Journal of Business* (October 1955), pp. 240–252.
23. R. A. Taggart, "Capital Budgeting and the Financing Decision: An Exposition," *Financial Management* (Summer 1977), pp. 59–64.

Appendix: An Alternative Valuation Model with Costly Insolvency

We denote $\tilde{Y}_u$ as random cash flows to an unlevered firm. $\tilde{Y}_u$ is defined as follows:

$$\tilde{Y}_u = \begin{cases} (1-t)\tilde{X} & \text{if } \tilde{X} > 0 \\ 0 & \text{if } \tilde{X} \leq 0 \end{cases}$$

where $\tilde{X}$ = operating income and t = the corporate income tax rate.

In a risk-neutral world, the expected value of $\tilde{Y}_u$ is discounted at the risk-free rate of interest (R_f) to obtain the market value of the unlevered firm (V_u):

$$V_u = E(\tilde{Y}_u)/R_f = (1-t) \int_0^{\infty} \tilde{X}\, f(X)dX/R_f$$

To derive a closed form expression for V_u, we need to define $\int_A^B \tilde{X}f(X)dX$. Since $df(X)/dX = -f(X)\,[\tilde{X} - E(\tilde{X})]/\sigma^2(\tilde{X})$, it follows that $\tilde{X}f(X) = -\sigma^2(\tilde{X})df(X)/dX + E(\tilde{X})f(X)$. Therefore,

$$\int_A^B \tilde{X}f(X)dX = \int_A^B[-\sigma^2(\tilde{X})df(X)/dX + E(\tilde{X})f(X)]dX$$
$$= -\sigma^2(\tilde{X})[f(B) - f(A)] + E(\tilde{X})[F(B) - F(A)].$$

Thus, $$V_u = (1-t)\{\sigma^2(\tilde{X})f(0) + E(\tilde{X})[1 - F(0)]\}/R_f \quad \text{(A-1)}$$

where $F(\bullet)$ = the cumulative distribution function of a normally distributed random variable,
$f(\bullet)$ = the normal probability density function, and
$E(\bullet)$ and $\sigma^2(\bullet)$ denote the expected value and the variance operators, respectively.

We can further define random cash flows to stockholders ($\tilde{Y}_s$), bondholders ($\tilde{Y}_d$), and the levered firm ($\tilde{Y}_l = \tilde{Y}_s + \tilde{Y}_d$) as follows:

$$\tilde{Y}_s = \begin{cases} (1-t)(\tilde{X}-R) & \text{if } \tilde{X} > R \\ 0 & \text{if } R \geq \tilde{X} > \theta \\ 0 & \text{if } \theta \geq \tilde{X} > 0 \\ 0 & \text{if } 0 \geq \tilde{X}. \end{cases}$$

$$\tilde{Y}_d = \begin{cases} R & \text{if } \tilde{X} > R \\ \tilde{X}-\tilde{K} & \text{if } R \geq \tilde{X} > \theta \\ 0 & \text{if } \theta \geq \tilde{X} > 0 \\ 0 & \text{if } 0 \geq \tilde{X}. \end{cases}$$

$$\tilde{Y}_l = \begin{cases} (1-t)\tilde{X} + tR & \text{if } \tilde{X} > R \\ \tilde{X}-\tilde{K} & \text{if } R \geq \tilde{X} > \theta \\ 0 & \text{if } \theta \geq \tilde{X} > 0 \\ 0 & \text{if } 0 \geq \tilde{X}. \end{cases}$$

where $\tilde{K} = K + k\tilde{X}$ = stochastic insolvency costs,
$\theta = K/(1-k)$ = a truncated point representing the limit of integration, and
R = interest expense.

As was suggested by Kim [9], stochastic insolvency costs ($\tilde{K}$) are the sum of a fixed component of K dollars and a variable component equal to a fraction of $k < 1$ of $\tilde{X}$. Denoting $\tilde{Y}_k$ as random cash flows to claim holders of insolvency costs, we can define it as follows:

$$\tilde{Y}_k = \begin{cases} 0 & \text{if } \tilde{X} > R \\ K+k\tilde{X} & \text{if } R \geq \tilde{X} > \theta \\ \tilde{X} & \text{if } \theta \geq \tilde{X} > 0 \\ 0 & \text{if } 0 \geq \tilde{X} \end{cases}$$

Again invoking the assumption of risk-neutrality and the perpetuity assumption, we derive the market values of equity (V_s), debt (V_d), the levered firm (V_l), and the present value of insolvency costs (V_k):

$$\begin{aligned} V_s &= E(\tilde{Y}_s)/R_f = \textstyle\int_R^\infty (1-t)(\tilde{X}-R)\, f(X)dX/R_f \\ &= (1-t)\{[E(\tilde{X})-R]\,[1-F(R)] \\ &\quad + \sigma^2(X)\, f(R)\}/R_f \end{aligned} \tag{A-2}$$

$$\begin{aligned} V_d &= E(\tilde{Y}_d)/R_f \\ &= \{\textstyle\int_R^\infty R\, f(X)dX + \int_\theta^R [(1-k)\tilde{X}-K]\, f(X)dX\}/R_f \\ &= \big[R[1-F(R)] + (1-k)\{\sigma^2(X)\,[f(\theta)-f(R)] \\ &\quad + E(\tilde{X})\,[F(R)-F(\theta)]\} - K[F(R)-F(\theta)]\big]/R_f \end{aligned} \tag{A-3}$$

$$\begin{aligned} V_l &= E(\tilde{Y}_l)/R_f \\ &= \{\textstyle\int_R^\infty [(1-t)\tilde{X}+tR]f(X)dX \\ &\quad + \textstyle\int_\theta^R [(1-k)\tilde{X}-K]f(X)dX\}/R_f \\ &= \big[(1-t)\{\sigma^2(\tilde{X})f(R) + E(\tilde{X})\,[1-F(R)]\} \\ &\quad + tR[1-F(R)] + (1-k)\{\sigma^2(\tilde{X})[f(\theta)-f(R)] \\ &\quad + E(\tilde{X})[F(R)-F(\theta)]\} - K[F(R)-F(\theta)]\big]/R_f \end{aligned} \tag{A-4}$$

$$\begin{aligned} V_k &= E(\tilde{Y}_k)/R_f \\ &= [\textstyle\int_\theta^R (K+k\tilde{X})\, f(X)dX + \int_0^\theta \tilde{X}f(X)dX]/R_f \\ &= \big[K[F(R)-F(\theta)] + k\{\sigma^2(X)\,[f(\theta)-f(R)] \\ &\quad + E(\tilde{X})[F(R)-F(\theta)]\} + \{\sigma^2(\tilde{X})[f(0)-f(\theta)] \\ &\quad + E(\tilde{X})[F(\theta)-F(0)]\}\big]/R_f \end{aligned} \tag{A-5}$$

If we assume that insolvency is costless, *i.e.*, $K = 0$, $k = 0$, and therefore, $\theta = 0$, then the market value of debt is reexpressed from Equation (A-3) as follows: The value of debt with costless insolvency is denoted by V_d^* to distinguish it from the market value of debt when insolvency is costly (V_d).

$$\begin{aligned} V_d^* &= [\textstyle\int_R^\infty Rf(X)dX + \int_0^R \tilde{X}f(X)dX]/R_f \\ &= \{R[1-F(R)] + \sigma^2(\tilde{X})[f(0)-f(R)] \\ &\quad + E(\tilde{X})[F(R)-F(0)]\}/R_f, \end{aligned} \tag{A-6}$$

where V_d^* = the market value of debt when insolvency is costless.

It is noted that the difference between V_d^* and V_d is exactly equal to the present value of insolvency costs (V_k). Without proof, we have:

$$V_d^* - V_d = V_k. \tag{A-7}$$

Equation (A-7) provides us with an interesting starting point for estimation of the fixed component (K) and the variable component (k) of insolvency costs as discussed later.

We can transform the complicated expression for the market value of the levered firm [Equation (A-4)] into a simplified one which is comparable to the original tax-correction model of Modigliani and Miller [14]. We modify the right-hand side expression of Equation (A-4) as shown below:

$$\begin{aligned} E(\tilde{Y}_l) &= (1-t)\textstyle\int_0^\infty \tilde{X}f(X)dX - (1-t)\int_0^R \tilde{X}f(X)dX \\ &\quad + \textstyle\int_R^\infty tRf(X)dX + \int_\theta^R (\tilde{X}-\tilde{K})f(X)dX \\ &= (1-t)\textstyle\int_0^\infty Xf(X)dX + t[\int_R^\infty Rf(X)dX \\ &\quad + \textstyle\int_0^R \tilde{X}f(X)dX] - [\int_0^R \tilde{X}f(X)dX \\ &\quad - \textstyle\int_\theta^R \tilde{X}f(X)dX + \int_\theta^R \tilde{K}f(X)dX] \\ &= (1-t)\textstyle\int_0^\infty \tilde{X}f(X)dX + t[\int_R^\infty Rf(X)dX \\ &\quad + \textstyle\int_0^R \tilde{X}f(X)dX] \\ &\quad - [\textstyle\int_\theta^R \tilde{K}f(X)dX + \int_0^\theta \tilde{X}f(X)dX] \end{aligned}$$

Using Equations (A-1), (A-5), and (A-6), the above

Equation is simplified into

$$E(\tilde{Y}_{\ell}) = E(\tilde{Y}_u) + tE(\tilde{Y}^*_d) - E(\tilde{Y}_k).$$

Therefore,

$$V_{\ell} = V_u + tV^*_d - V_k. \qquad (A\text{-}8)$$

It is obvious from the valuation equations for V_u, V_{ℓ}, V^*_d, and V_k that market values of the levered firm (V_{ℓ}), debt [$= V_d = V^*_d - V_k$], and equity [$V_s = V_{\ell} - V_d$] can be estimated when we have information on $E(\tilde{X})$, $\sigma^2(\tilde{X})$, R, t, K, k, and R_f.

One critical problem associated with the practical use of our model concerns the size of the fixed charge of K dollars and the variable portion (k) of insolvency costs. They should vary from one firm to another and from one industry to another. Given the scanty empirical research on this topic, the determination of both K and k must remain the subject of future research. For the present, a rough approximation is suggested based upon Equation (A-7) for the present value of insolvency costs. We can rewrite $V_k = V^*_d - V_d$ using the perpetuity assumption as follows:

$$V_k = \frac{R}{r^*} - \frac{R}{r}, \qquad (A\text{-}9)$$

where r^* = cost of debt when insolvency is costless and

r = cost of debt when insolvency is costly.

For any firm, we can observe the annual interest expense (R). We may use the annual yield of the long-term Treasury Bond yield or the AAA-rated corporate bond yield as an admittedly crude proxy for r^* and the annual yield of the A-rated long-term corporate bond as the proxy for r, assuming that the firm's bond is rated A. Thus, V_k can be estimated each year. By comparing the annual operating income ($\tilde{X}$) of the firm, we may obtain some idea about the magnitude of K and k. In this regard, the pioneering research by Pye [18] can provide further insight. Once again, we do not claim that this is the only recourse for the financial analyst. Further research in this direction is necessary.