Theorem. The number $\sqrt{2}$ is irrational.

Proof. Assume, to the contrary, that $\sqrt{2}$ is __________
$\Rightarrow \sqrt{2} = \frac{a}{b}$, such that $a$ and $b$ are __________ and $b \neq ____$.
Moreover, we may assume that the fraction $\frac{a}{b}$ is in __________ terms.
$\Rightarrow 2 = (\sqrt{2})^2 = (\frac{a}{b})^2 = \frac{a^2}{b^2}$
$\Rightarrow 2b^2 = ____$
$\Rightarrow a^2$ is even, since ____ is an integer
$\Rightarrow a$ is __________
$\Rightarrow a = 2c$, for some integer $c$
$\Rightarrow a^2 = (2c)^2 = 4 ____; but, a^2 = 2b^2$
$\Rightarrow 4c^2 = 2b^2$
$\Rightarrow b^2 = 2 ____$
$\Rightarrow b^2$ is even, since $b^2$ is an __________ $\Rightarrow$ ____ is even.
However, this is absurd, because $a$ and $b$ cannot both be even as $\frac{a}{b}$ is in lowest terms. Therefore, we have reached a contradic-
tion. \qed