Fill in the blanks.

Theorem. *The number $\sqrt{2}$ is irrational.*

Proof. Assume, to the contrary, that $\sqrt{2}$ is ________________
$\Rightarrow \sqrt{2} = \frac{a}{b}$, such that a and b are ________________ and $b \neq$ _____.
Moreover, we may assume that the fraction $\frac{a}{b}$ is in ________________ terms.
$\Rightarrow 2 = \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$
$\Rightarrow 2b^2 =$ _____
$\Rightarrow a^2$ is even, since _____ is an integer
$\Rightarrow a$ is ________________
$\Rightarrow a = 2c$, for some integer c
$\Rightarrow a^2 = (2c)^2 = 4$ _____; but, $a^2 = 2b^2$
$\Rightarrow 4c^2 = 2b^2$
$\Rightarrow b^2 = 2$ _____
$\Rightarrow b^2$ is even, since b^2 is an ________________ $\Rightarrow$ _____ is even.
However, this is absurd, because a and b cannot both be even as $\frac{a}{b}$ is in lowest terms. Therefore, we have reached a contradiction. $\square$