Computing $\pi$ Via New Polynomial Approximations to Arctangent: 

a new contribution to (arguably) the oldest approximation problem

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Rational functions after integration can produce arctangent and therefore can be used to approximate $\pi$. Using rational functions of the form

$$\left\{ \frac{t^{km}(t - \beta)^{lm}}{1 + t^2} \right\}_{m \in \mathbb{N}}$$

for different values of $k$, $l$, and $\beta$, we produce families of efficient polynomial approximations to arctangent, and hence, provide approximations to $\pi$ via known arctangent values. Some of the polynomials produce rational approximations to $\pi$ and others approximations that require only the computation of a single square root; moreover, they are orders of magnitude more accurate than Maclaurin polynomials. We analyze the efficiency of the approximations and provide algebraic and analytic properties of the sequences of polynomials. Finally, we turn the approximations of $\pi$ into series including one that gives about 21 additional decimal digits of accuracy with each successive term.

This talk should be accessible to undergraduates with a good knowledge of integral calculus. i.e., those with a good knowledge of calculus II should be able to follow it.