# Chapter 15

# Mechanical Waves

PowerPoint® Lectures for
***University Physics, Twelfth Edition***
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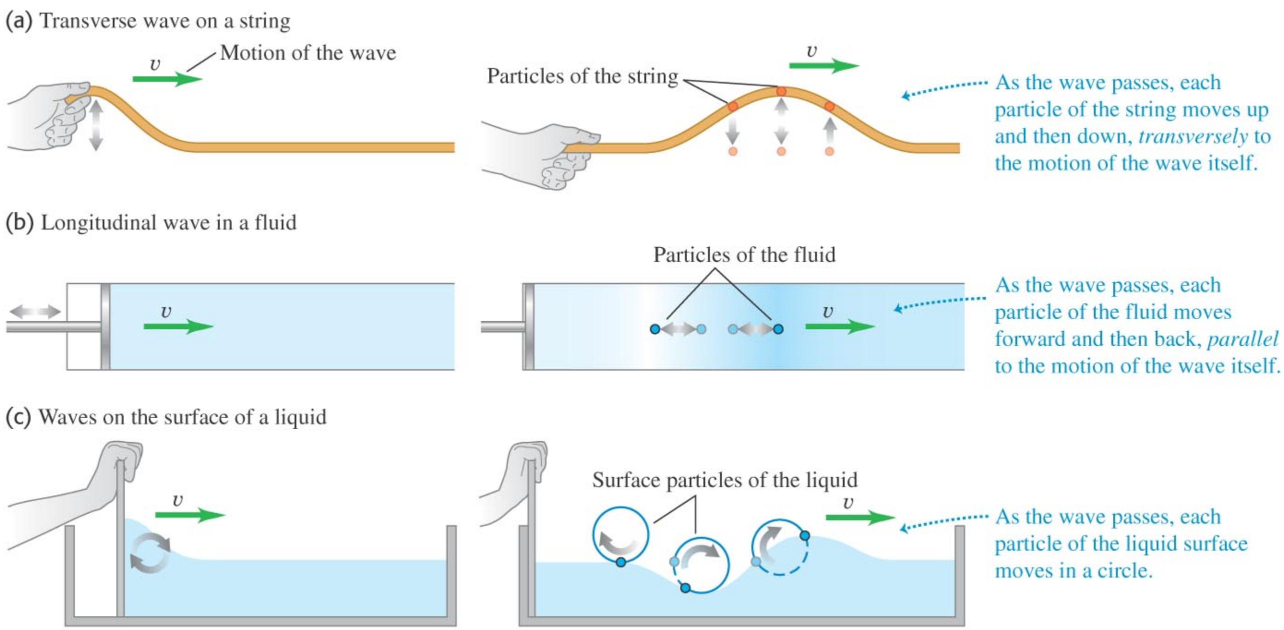
# Topics for Chapter 15

- What is a mechanical wave motion
- Properties of mechanical waves
- Mathematical description of traveling wave
- Energy carried by in traveling wave
- Superposition of waves
- Standing waves

# What is a mechanical wave motion?

- Create a **disturbance** in one region of medium
- The **propagation of this disturbance** to other regions of the medium = **mechanic wave**
- Note: the **medium** must be **elastic** (it has some kind of restoring force) and has **inertia** (mass)
- See examples below (identify the medium and the restoring force(s)):

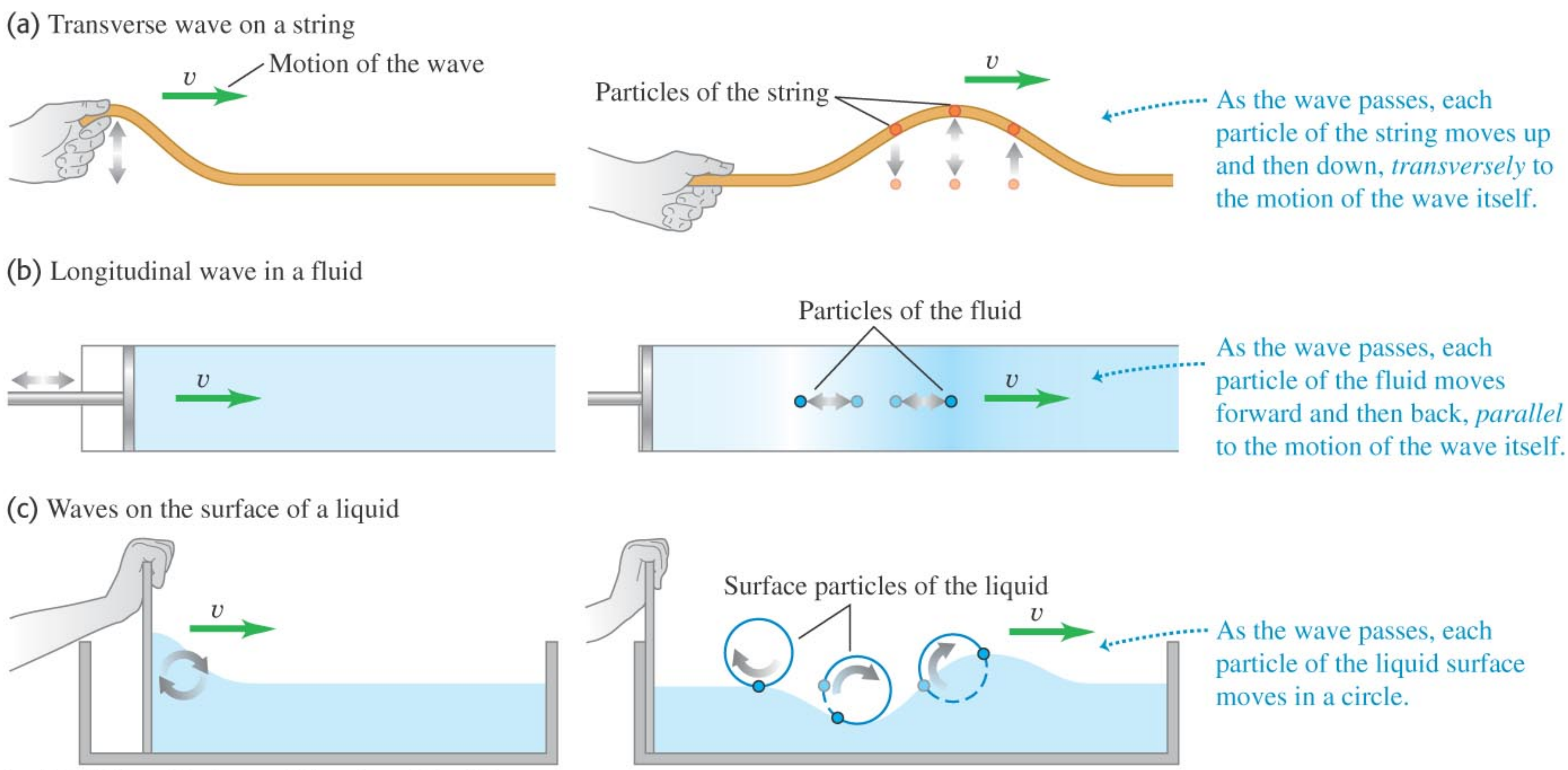


# Do all waves require a medium to travel?

- ALL “mechanical” waves require a medium to travel.
- Exception: Electromagnetic waves (radio wave, microwave, visible light, ultraviolet, X-ray, gamma rays, etc) can travel through empty space (vacuum)

# Types of waves

- **Longitudinal waves** - Waves that have disturbance parallel to the direction of wave propagation are called longitudinal wave
- **Transverse waves** -Waves that have disturbance perpendicular to the direction of propagation. Identify the waves below as longitudinal or transverse.



# Wave speed

- Wave speed depends on the properties of the medium

$$v=\sqrt{\frac{\text{Magnitude of restoring force}}{\text{Inertia of the medium}}}$$

$e.g.$ wave speed of a rope under tension:

$$v=\sqrt{\frac{F}{\mu}};\quad F = tension,\quad \mu = \frac{mass}{length} = \frac{kg}{m}$$

$$\text{check unit: } F=kg\frac{m}{s^2}$$

$$\sqrt{\frac{F}{\mu}} = \sqrt{\frac{kgm}{s^2}\bullet\frac{m}{kg}} = \frac{m}{s}$$

Other examples:
(1) Sound velocity depends on the medium:air vs. solid (both magnitude of restoring force and inertia are different in these two media).
(2) Longitudinal earthquake waves (P-wave) is faster than the transverse earthquake wave (S-wave) - the magnitude of compressional restoring force (P-wave) is greater than the shear restoring force (S-wave).

# Waveform - examples

- Traveling wave pulse - generated by a pulsed driving force
- Traveling periodic wave (harmonic wave) - generated by a continuous driving force

Mathematically (the amplitude of) a travelling is described by:

$\vec{A}(x,t) = f(x - vt)\hat{n}$

$(a)$The function f describes the shape of the wave

(e.g. for periodic wave, it may cosine: $\vec{A}(x,t) = A_{\max}\cos(x - vt)\hat{n}$)

$(b)$The dependence on x and t in this form x-vt signifies that it is

a travelling wave along the x-direction;

v = positive => travelling in positive x-direction

v=negative => travelling in negative x-direction

$(c)$ The direction of the amplitude is denoted by the unit vector $\hat{n}$.

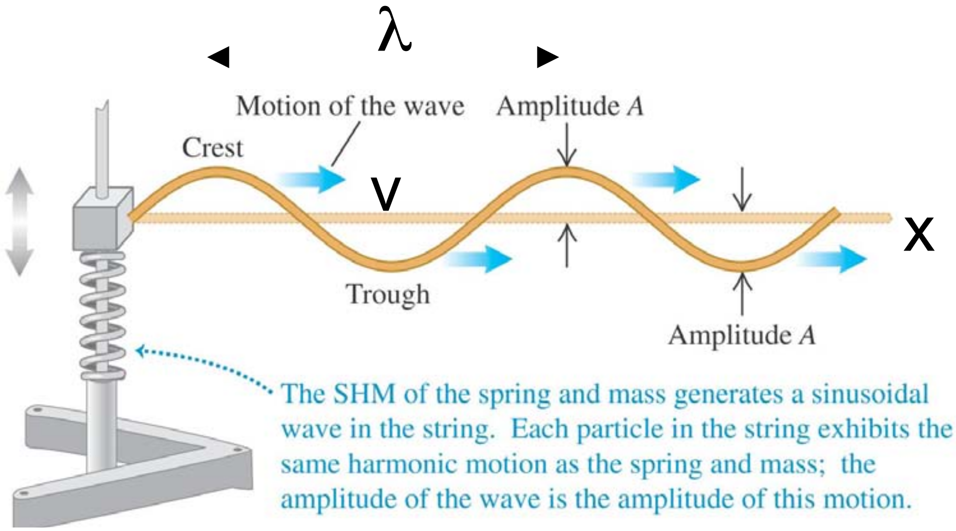
Example: If the wave is longitudinal and it travels along the x-direction,then $\hat{n}=\hat{i}$.

If the wave is transverse and transverse and it travels along x-direction, then $\hat{n}=\hat{j}$ or $\hat{n}=\hat{k}$

Q. Write a general expression for a longitudinal wave travelling in the negative y-direction with a wave speed of 10 m/s.

# Transverse Periodic wave

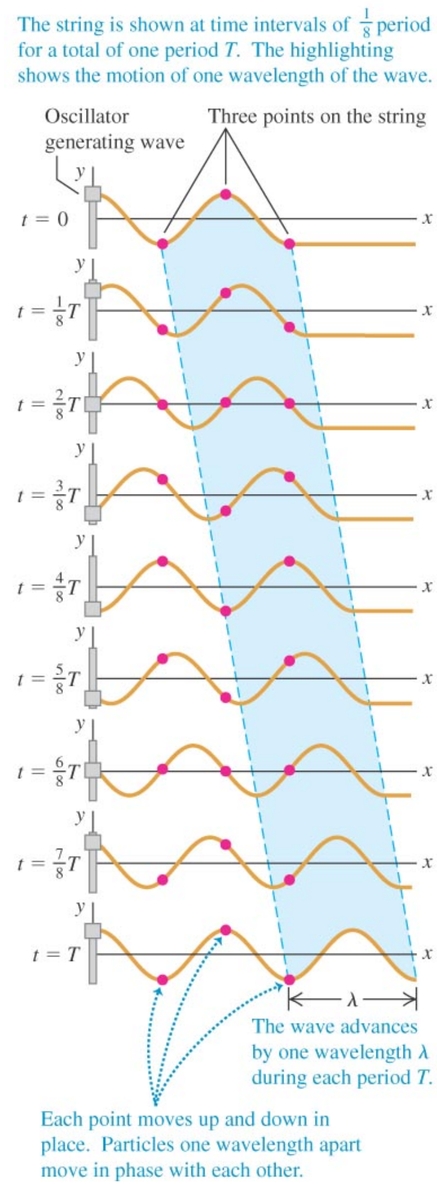
- A detailed look at periodic transverse waves will allow us to extract parameters.




$$\vec{A}(x,t) = A\hat{j}\cos\left[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right]$$

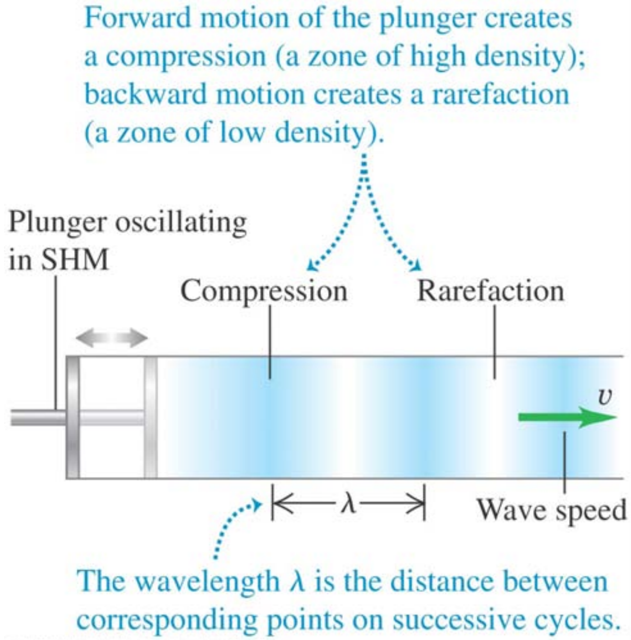
$$= A\hat{j}\cos\left[\frac{2\pi}{\lambda}(x - \frac{\lambda}{T}t)\right]$$

$$= A\hat{j}\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]; \quad v = \frac{\lambda}{T} = \lambda f$$

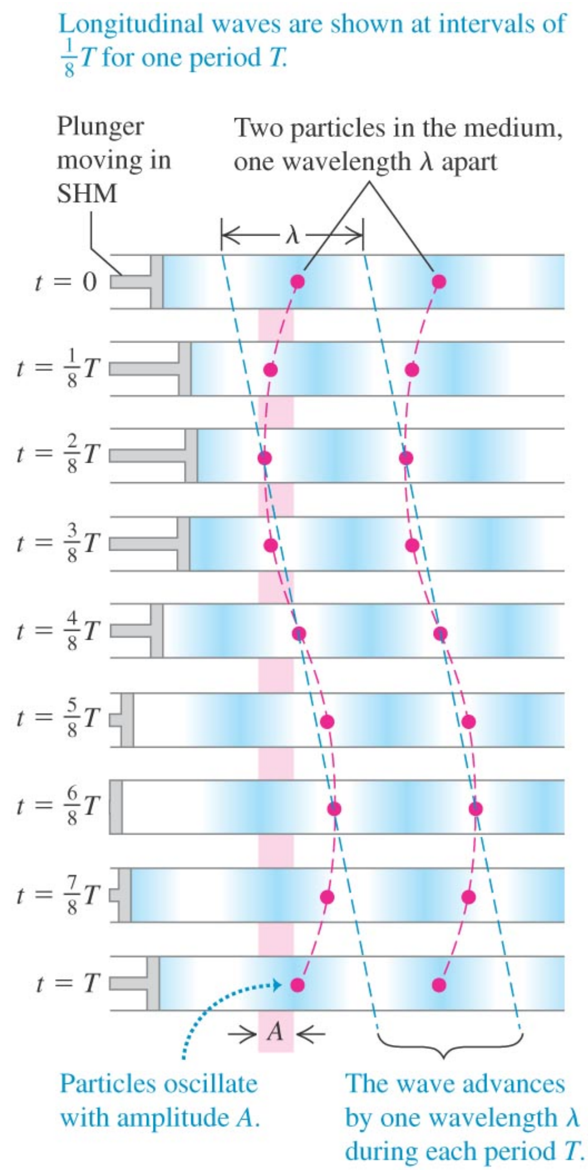


# Longitudinal Periodic waves

Refer to Example 15.1.

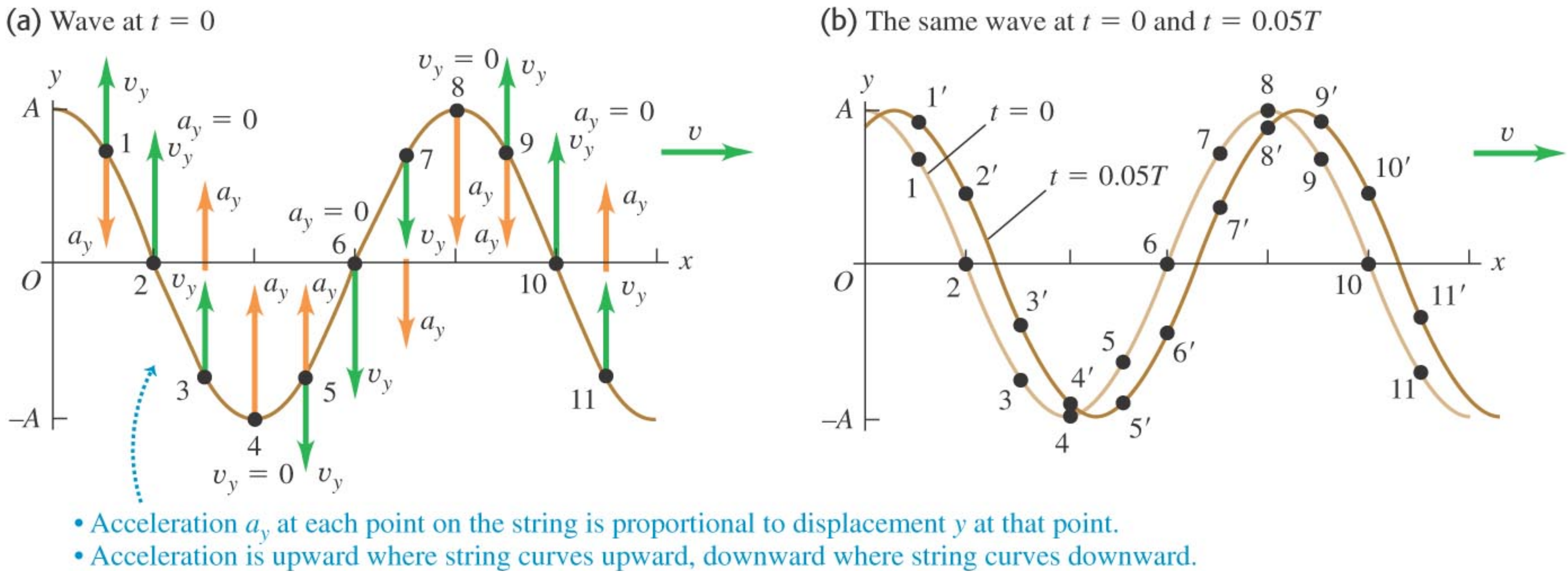



$$\vec{A}(x,t) = A\hat{i}\cos\left[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right]$$



# Particle velocity vs. wave velocity

- Consider a transverse wave on a rope




$$\vec{A}(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x - v)t\right]\hat{j}$$

$$\text{Particle velocity} = \frac{d\vec{A}(x,t)}{dt} = A\frac{2\pi}{\lambda}v\sin\left[\frac{2\pi}{\lambda}(x - v)t\right]\hat{j}$$

$$\Rightarrow \text{Maximum particle speed} = \left| A\frac{2\pi}{\lambda}v \right| = \left| A2\pi f \right| = \left| A\frac{2\pi}{T} \right|$$

# Energy (or Power) carried by a wave

As the wave travels along the medium, it transports the energy that it carries.

$$\frac{\text{transported kinetic energy}}{\text{time}} = \frac{1}{2}\frac{\text{mass}}{\text{length}}(\text{particle velocity})^2 \bullet (\text{ wave velocity})$$

Example : Periodic wave

$$= \frac{1}{2}\mu\left(A\omega \sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2 \bullet v$$

$$= \frac{1}{2}\mu v\omega^2 A^2\left(\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2$$

The $\dfrac{\text{transported potential energy}}{time}$ give exactly the same term

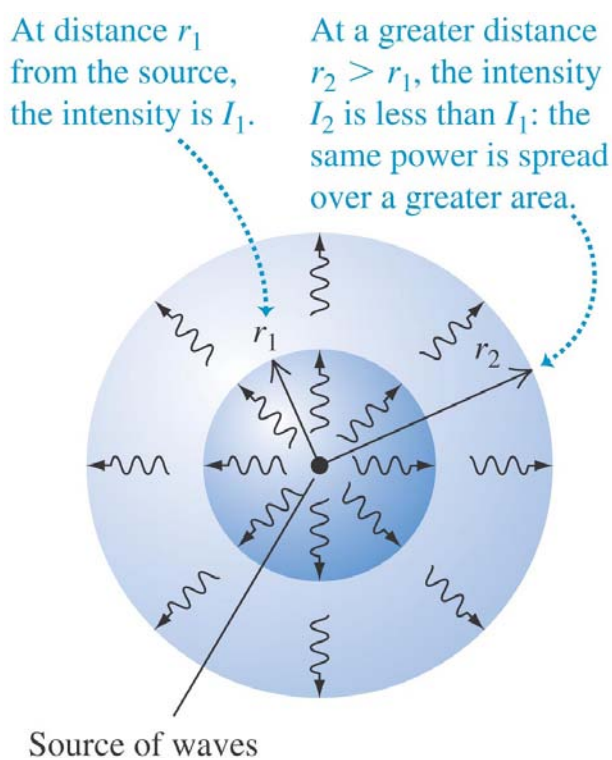
$$\Rightarrow \text{Total instantaneous power} = \text{P} = \mu v\omega^2 A^2\left(\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2$$

$$\Rightarrow \text{Average power} = \frac{1}{2}\mu v\omega^2 A^2 \quad \text{(because sine square } = 1/2)$$

# Wave intensity

- Go beyond the wave on a string and visualize, say … a sound wave spreading from a speaker. That wave has intensity dropping as $1/r^2$ due to conservation of energy.

$$\text{Intensity} = \frac{\text{Energy/time}}{\text{area}} = \frac{Power}{4\pi r^2}$$



# Mathematical connection between wave pulse and periodic waves

A wave pulse can be thought of as a superposition of many periodic waves with various wavelengths and frequencies - Fourier's Theorem.

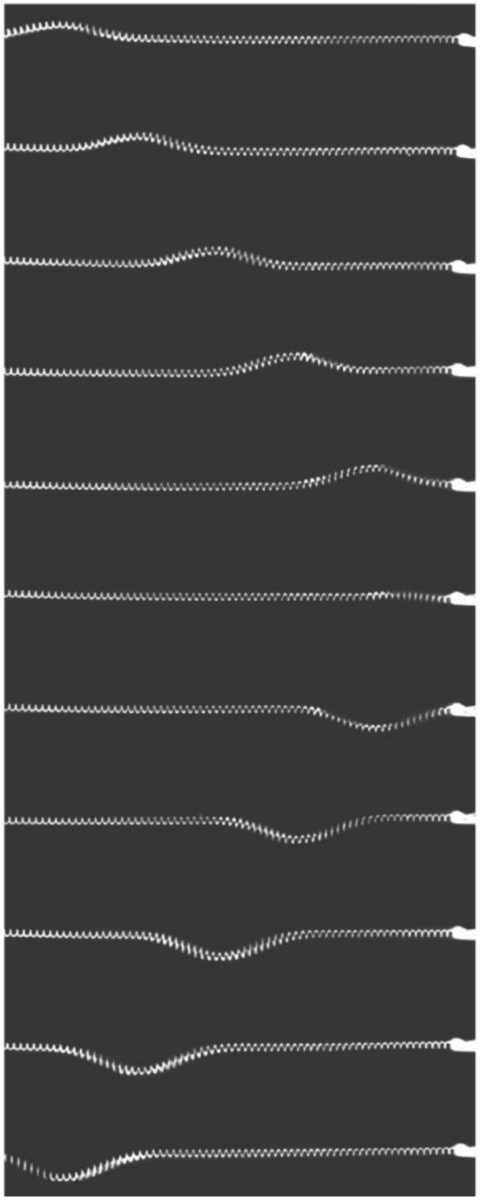
Any fucntion f(x) can be expressed as a sum of cosine and sine fucntions of various wavelengths:

$$f(x) = \sum_{\lambda} A_{\lambda} \cos\left(\frac{2\pi}{\lambda} x\right) + B_{\lambda} \sin\left(\frac{2\pi}{\lambda} x\right)$$

# Non-dispersive vs dispersive medium

- A non-dispersive medium is one where all wavelengths have same wave velocity. For example: ALL electromagnetic waves traveling in vaccuum have the same wave velocity ~ $3\text{x}10^8$ m/s). Another example is wave along a tight rope is is *approximately* non-dispersive,

$$v = \sqrt{\frac{F}{\mu}} \quad \text{independent of } \lambda$$



In a non-dispersive medium, a wave pulse does not disperse (spread out) as it travels, see figure on the right.

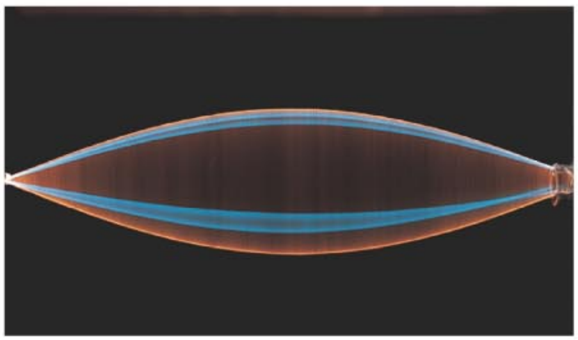
# Non-dispersive vs dispersive medium

- Almost all media are dispersive (the only true non-dispersive medium is the vaccum!)
- For example: Electromagnetic waves (light) travels inside a piece glass have wave velocities depending on the wavelength of light; red and blue light have different wave velocity => white light entering a piece of glass (a prism) will disperse into different colors.
- Dispersion of wave pulse prevents digital signal to travel long distance; eventually the pulse shape will be distorted.

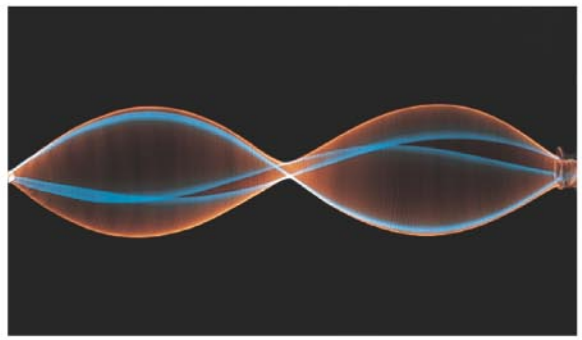
# Standing waves on a string - resonance frequencies

- Although a wave traveling along a string can have any wavelength, a string with both ends fixed have certain "preferred" wavelengths (or resonance frequencies) - (e.g. guitar).

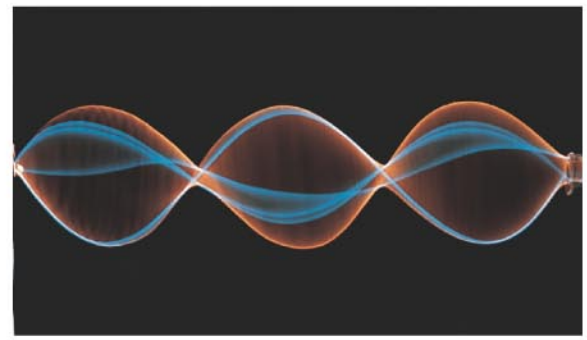
(a) String is one-half wavelength long.



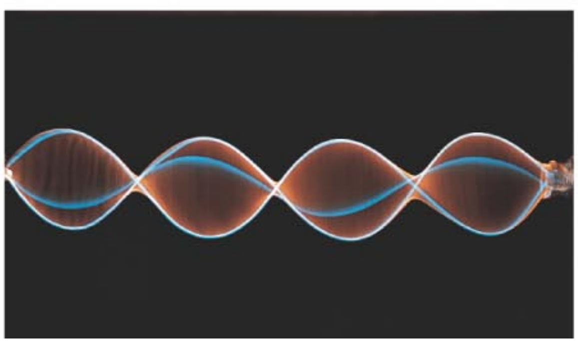
(b) String is one wavelength long.



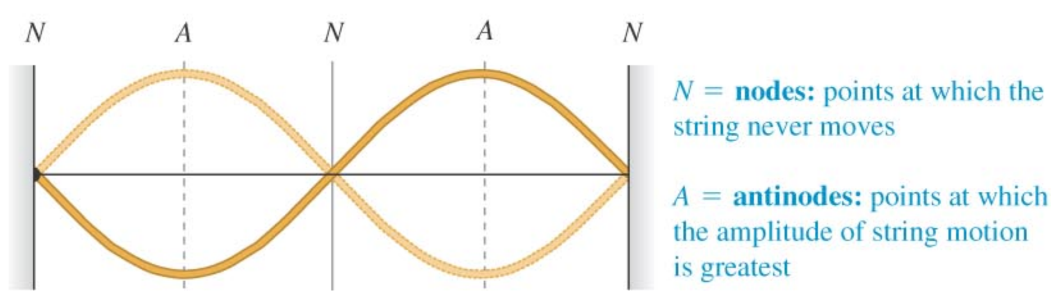
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants

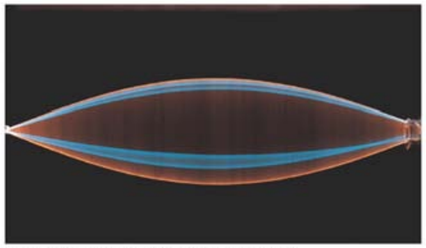


# Calculation of resonance wavelength (or frequencies)

*Given* : The length of a guiter string is 0.3m.

Find the first three "resonance" wavelengths.

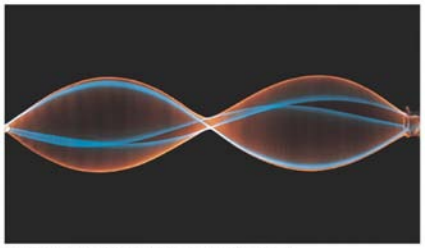
(a) String is one-half wavelength long.





$$\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L = 2(0.3) = 0.6m$$

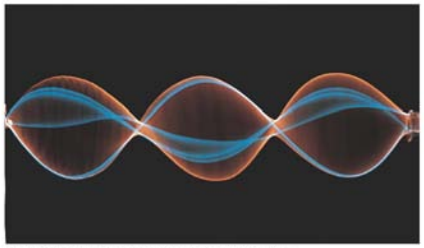
(b) String is one wavelength long.





$$\lambda_2 = L \Rightarrow \lambda_2 = 0.3m$$

(c) String is one and a half wavelengths long.





$$\frac{3\lambda_3}{2} = L \Rightarrow \lambda_3 = \frac{2}{3}L = \frac{2}{3}(0.3) = 0.2m$$

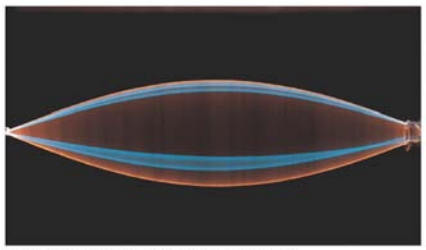
# Calculation of resonance wavelength (normal modes)

*Given* : The length of a guiter string is 0.3m.

Find the first three "resonance" frequencies.

What additional information do we need?

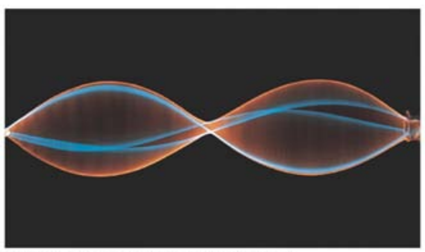
(a) String is one-half wavelength long.





$$\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L = 2(0.3) = 0.6m$$

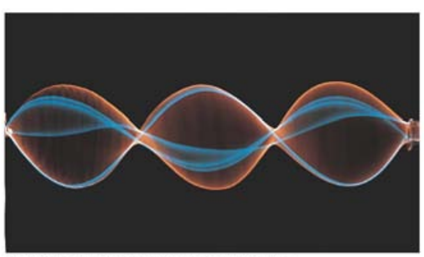
(b) String is one wavelength long.





$$\lambda_2 = L \Rightarrow \lambda_2 = 0.3m$$

(c) String is one and a half wavelengths long.





$$\frac{3\lambda_3}{2} = L \Rightarrow \lambda_3 = \frac{2}{3}L = \frac{2}{3}(0.3) = 0.2m$$