Chapter 9

Momentum, Impulse, and Collisions

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Learning Goals for Chapter 8

- To understand the concept of momentum and impulse.
- “Impulse-momentum Theorem”
- To know when momentum is conserved and examine the implications of conservation of momentum in a variety of physics problems - e.g. collision
- To understand the concept of center of mass and how to use it to analyze physics problems

Compare kinetic energy and momentum

Kinetic enegy (K) is a scalar (a number)

$$K \equiv \frac{1}{2} m v^2$$

Momentum ($\vec{p}$) is a vector

$$\vec{p} = m\vec{v}$$

Both describe the properties of a mass and its velocity.

Kinetic energy is related to the magnitude of the momentum

$$K = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{|\vec{p}|^2}{2m}$$

Both kinetic energy and momentum are useful concepts in analyzing collisions.

What causes a particle's momentum to change?

Suppose a particle of mass=m has an initial velocity $\vec{v}_i$.

A force, $\vec{F}$, is applied to the mass to cause it to change its velocity to $\vec{v}_f$ in time t seconds.

Q. What are initial and final momentum vectors?

Q. How is the force force related to the change in momentum vector?

(You may assume the force is constant for now)

Numerical example:

Let m=10 kg, $\vec{v}_i = 2\frac{m}{s}\hat{i}$, $\vec{v}_f = 8\frac{m}{s}\hat{i}, t = 2s$.

Find $\vec{p}_i$,$\vec{p}_f$, and $\vec{F}$ (assume $\vec{F}$ is a constant force)

Derive Impulse-momentum Theorem from Newton's Second Law

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

$$\Rightarrow \vec{F} = \frac{d\vec{p}}{dt} \quad \text{(this is another form of Newton's second law)}$$

$$\Rightarrow \int_{t_i}^{t_f} \vec{\mathrm{F}}\,\mathrm{dt} = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt}\mathrm{dt} = \vec{p}_f - \vec{p}_i \quad \text{(Impulse-momentum Theorem)}$$

$$\int_{t_i}^{t_f} \vec{\mathrm{F}}\,\mathrm{dt} \equiv I \text{ is called the "}\textit{implulse}\text{".}$$

If $\vec{\mathrm{F}}$ is a constant force then,

the impulse simplfies to $\mathrm{I}=\vec{\mathrm{F}}\,\Delta t = \vec{p}_f - \vec{p}_i$.

If $\vec{\mathrm{F}}$ is not a constant force then, $\mathrm{I}=\vec{F}_{average}\Delta t = \vec{p}_f - \vec{p}_i$

Compare with Work-kinetic energy Theorem

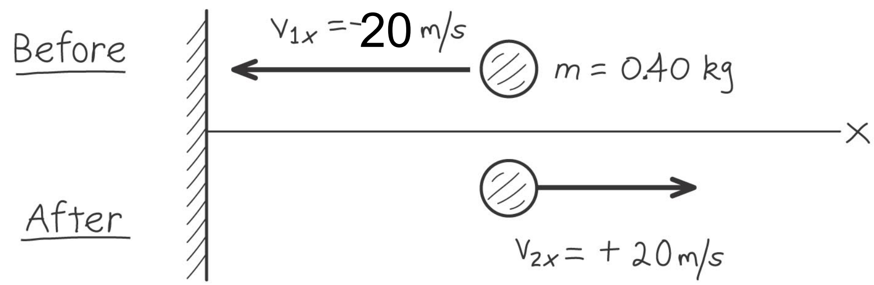
$$\int_i^f \vec{\mathrm{F}} \bullet \mathrm{d}\vec{\ell} = K_f - K_i \quad \text{(Work-kinetic energy Theorem)}$$

Compare momentum and kinetic energy

Example: Batting a baseball

In this case, is there a change in momentum?

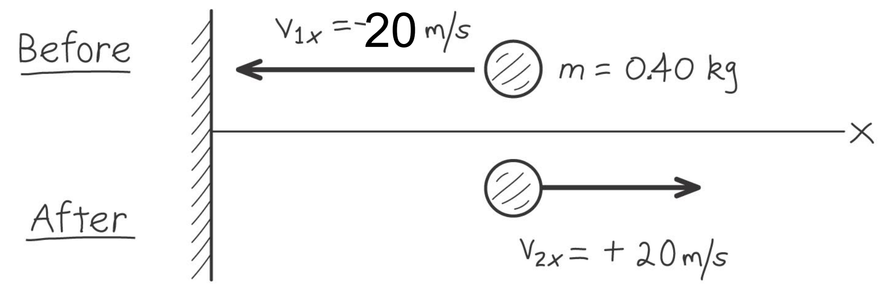
Is there a change in KE?



How do you explain our answers in terms of Impulse-momentum Theorem & Work-K.E. Theorem?

Deduce impulse and force from change of momentum

Example: Batting a baseball



Questions:

A. What is the impulse on the ball provided by the bat?

B. What is the average force if duration of contact is 0.01s?

(Give both magnitude and direction)

Examples of impulse-momentum theorem - quantified

- To change the momentum of an object, *one needs to apply a force on the object.*
- *The amount of force depends on the change in momentum and the time the force acts on the object*
- Impulse-momentum Theorem

$$\vec{p}_f - \vec{p}_i = \vec{F}_{average}\Delta t$$

Dropping a wine glass on a hard floor
vs.
Dropping a wine glass on a soft carpet
Hard floor Δt small => $F_{average}$ is large

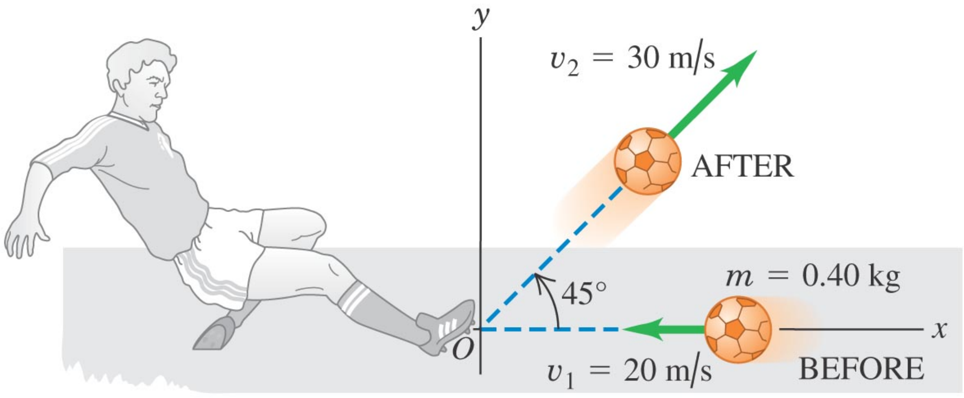
A 0.2kg wine glass is dropped on a hard floor. Let the initial velocity of the glass right before it hits the ground is 5 m/s and it took 0.01 second to stop the glass. Find the average force acting on the glass, both direction and magnitude.

Examples of impulse-momentum theorem - quantified

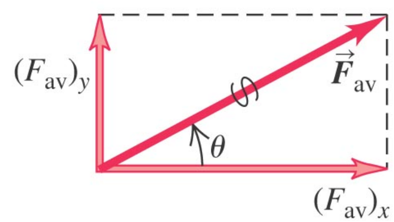
$$\vec{p}_f - \vec{p}_i = \vec{F}_{average}\Delta t$$

Identify the initial and final momentum vectors
and deduce $F_{average}$
(both direction and magnitude)
Given the ball made contact with the player's foot for 0.1s.

(a) Before-and-after diagram

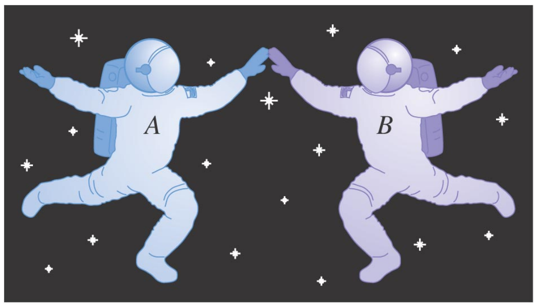


(b) Average force on the ball

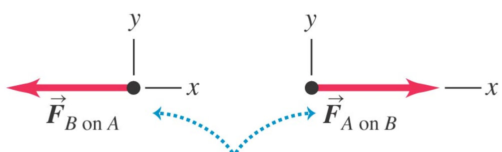


Like energy, momentum also has conservation rules

- When the **net "external" force is zero**, the **total momentum of the system is conserved** $$(m_1\vec{v}_1 + m_2\vec{v}_2)_{initial} = (m_1\vec{v}_1 + m_2\vec{v}_2)_{final}$$

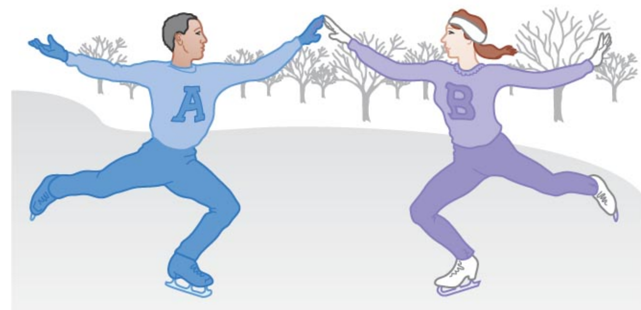


No external forces act on the two-astronaut system, so its total momentum is conserved.

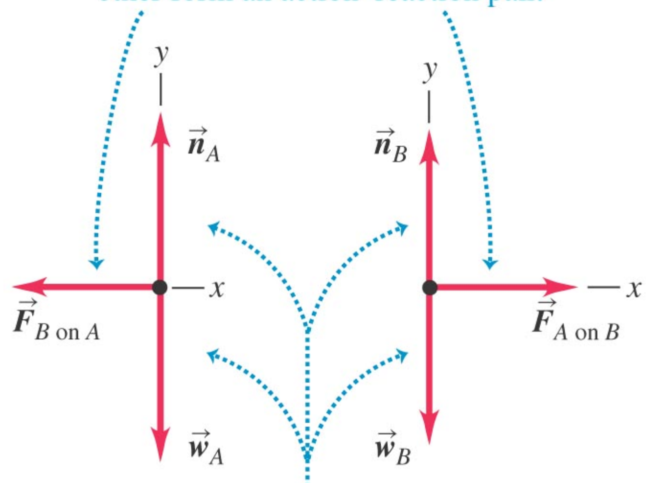


The forces the astronauts exert on each other form an action–reaction pair.

Internal forces (force of A on B and force of B on A) are equal and opposite => produce equal and opposite momenta thus the internal forces do not affect the total momentum; only the external forces do.

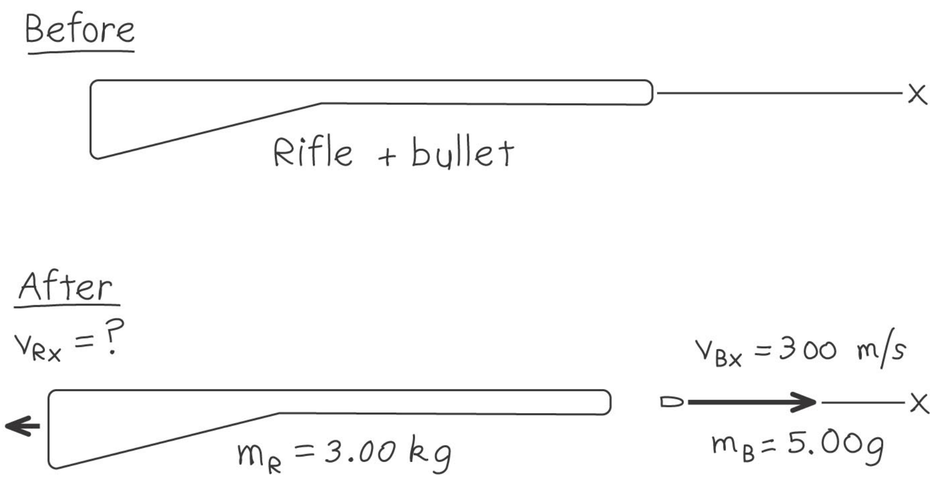


The forces the skaters exert on each other form an action–reaction pair.



Although the normal and gravitational forces are external, their vector sum is zero, so the total momentum is conserved.

An application of conservation of total momentum



The bullet's velocity is 300 m/s wrt the ground

$$(\vec{p}_{total})_i = (\vec{p}_{total})_f$$

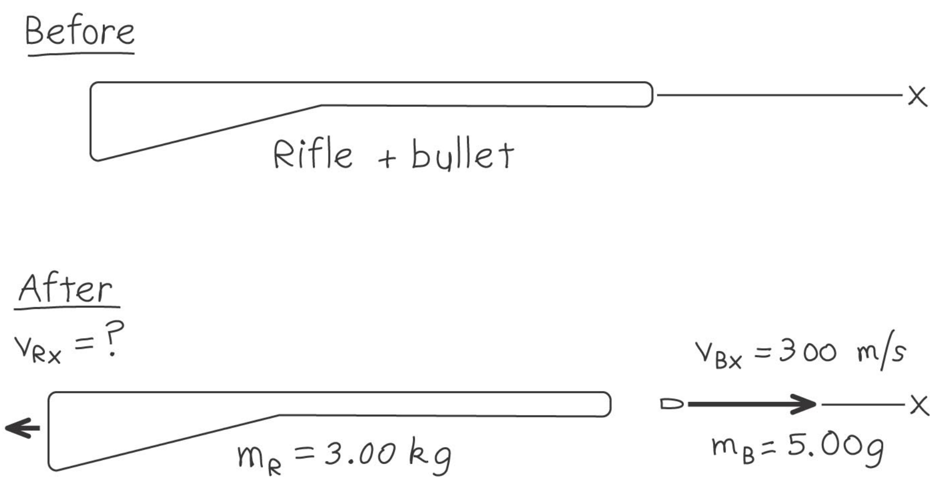
$$0 = \left(m_B \vec{v}_B + m_R \vec{v}_R\right)$$

$$0 = \left((0.005)(300\hat{i}) + (3)\vec{v}_R\right)$$

$$\Rightarrow \vec{v}_R = -0.5\hat{i}\,\frac{m}{s}$$

** Don't forget momentum is a vector

An application of conservation of total momentum



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More correct analysis:

Typically a rifle is specified by its muzzle velocity, i.e. the bullet's velocity wrt the gun. Suppose the muzzle is 300 m/s. Now find the rifle recoil velocity and the bullet's velocity wrt the ground.

$$(\vec{p}_{total})_i = (\vec{p}_{total})_f$$

$$0 = \left(m_B(\vec{v}_{BR} + \vec{v}_{RG}) + m_R\vec{v}_{RG}\right)$$

$$0 = \left((0.005)(300\hat{i} + \vec{v}_{RG}) + (3)\vec{v}_{RG}\right)$$

$$\Rightarrow \vec{v}_{RG} = -\frac{1.5}{3.005}\hat{i}\,\frac{m}{s} \approx -0.499\frac{m}{s}\hat{i}$$

Due to recoil, the bullet's velocity wrt ground is a little bit smaller than the muzzle velocity.

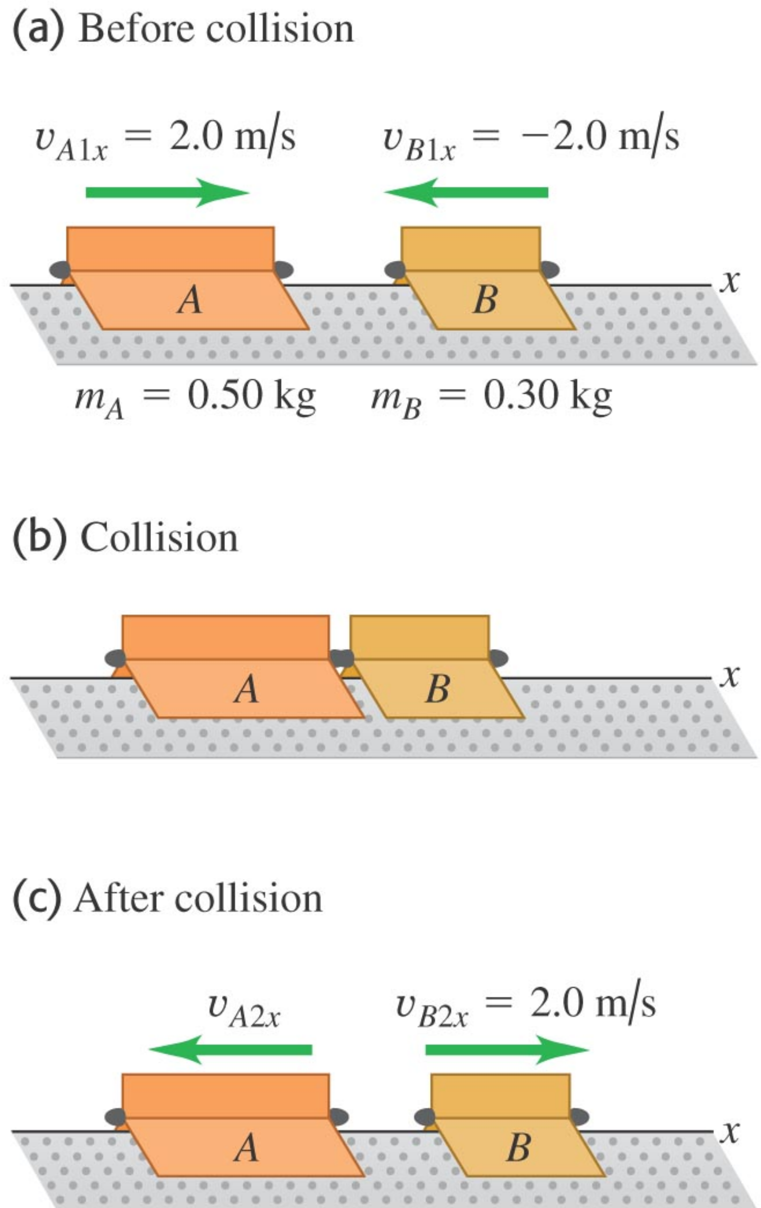
$$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG} = 300\hat{i} - 0.499\hat{i}$$

The correction in this case is very small due to the small mass of the bullet compared to the rifle. Correction is not small for cannons firing heavy cannon balls.

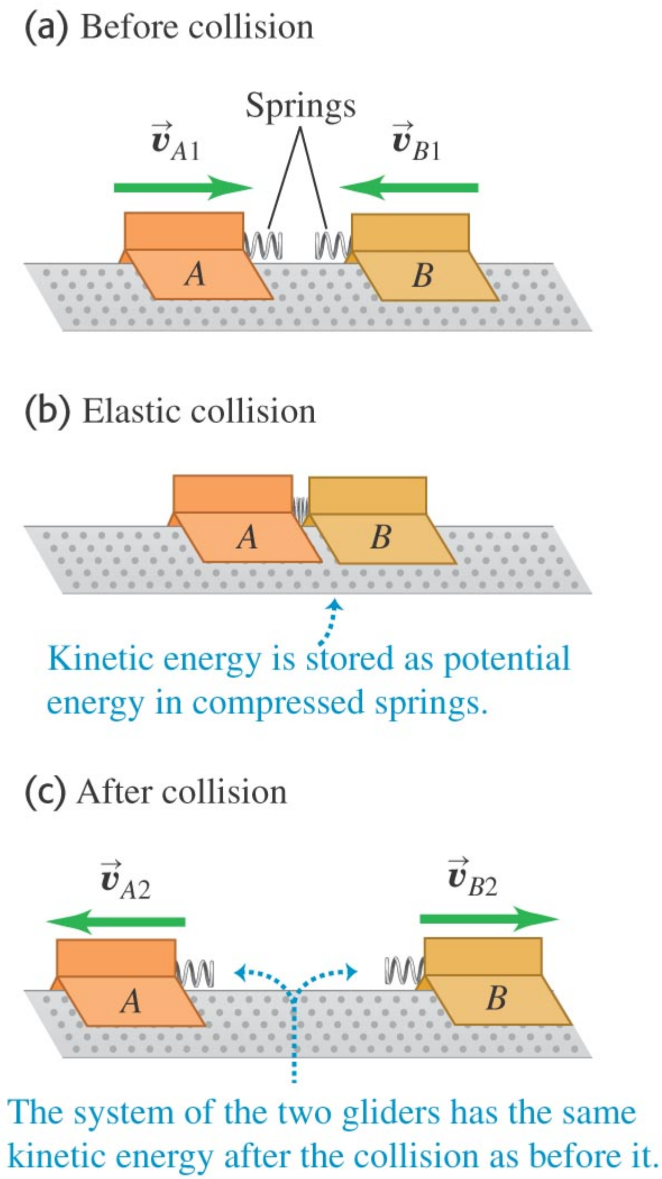
** Don't forget momentum is a vector

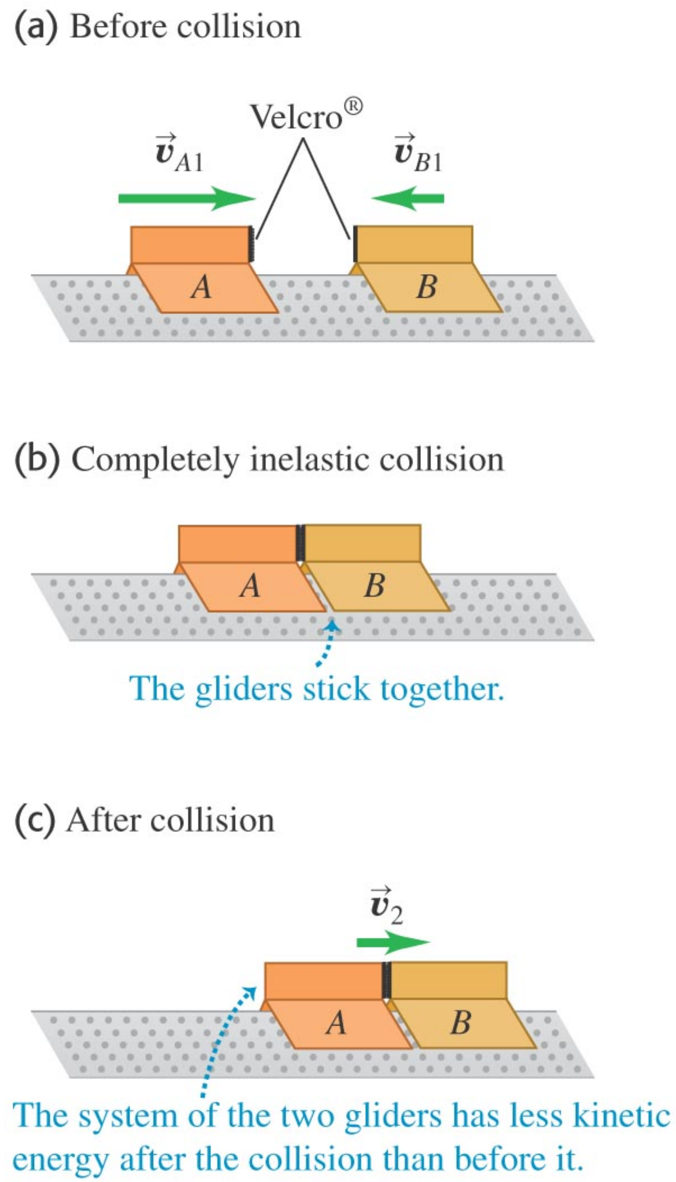
Another example of conservation of total momentum

- Consider the collision depicted
- Find v_{A2x}.
- Also compare the initial kinetic energy and final kinetic energy.



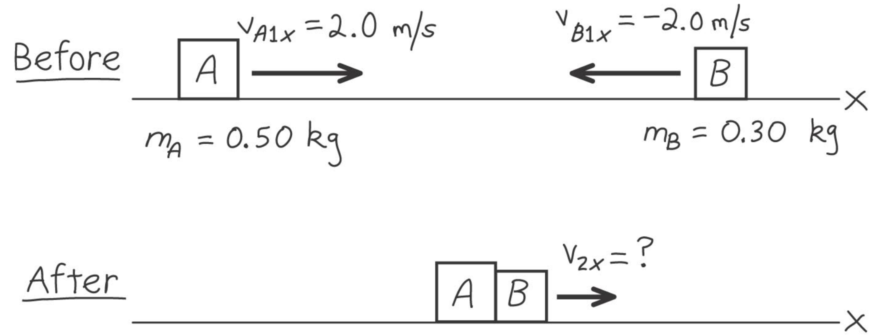
Compare elastic and inelastic collisions





Completely inelastic collisions

- Cars are designed to crumple and absorb as much energy as possible so the passengers do not need to.
- In any inelastic collisions, total momentum is conserved but total kinetic energy is not conserved (some of the kinetic energy went into deforming the cars!)

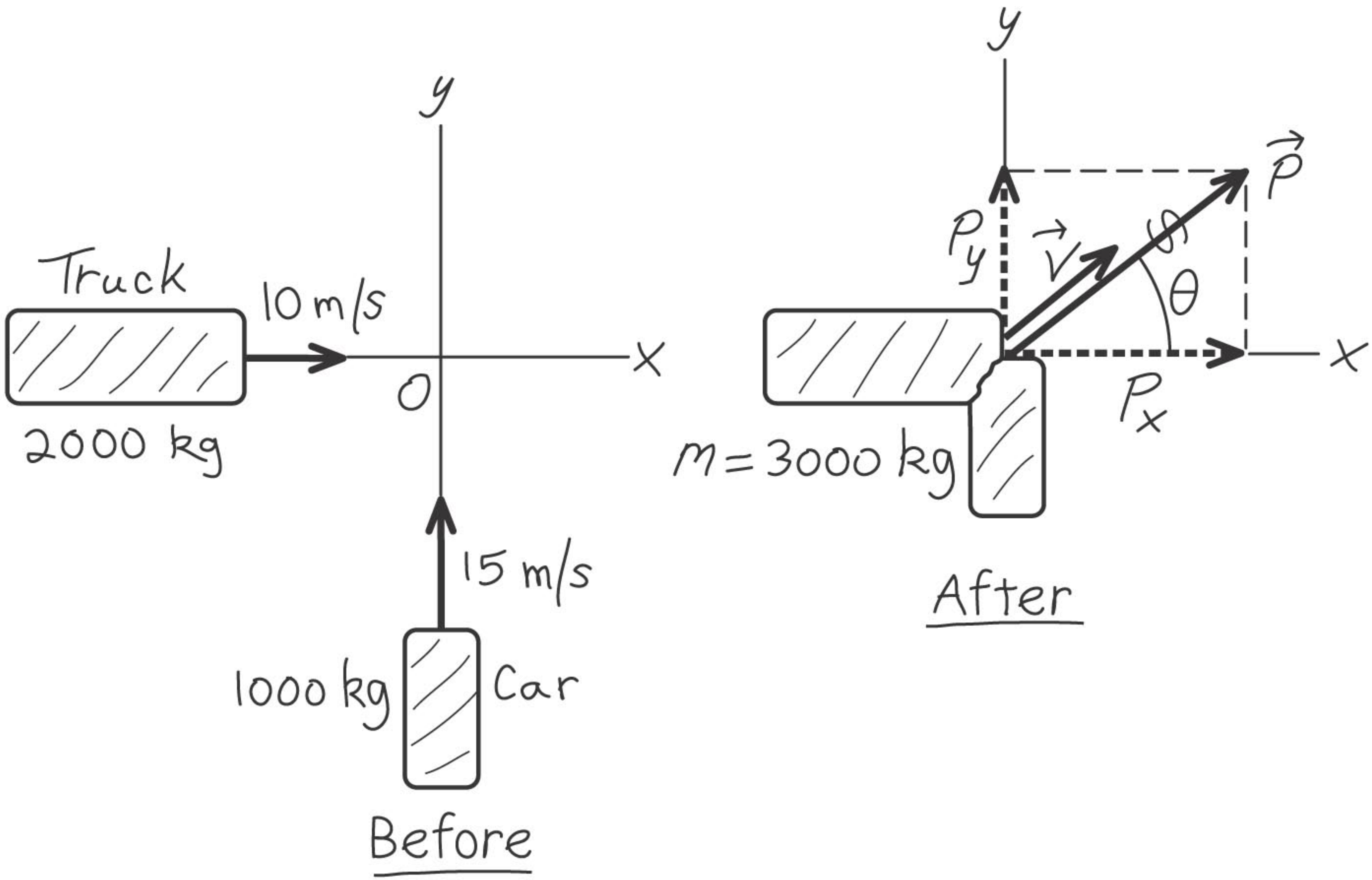


Find the final velocity, the initial and final total kinetic energies.

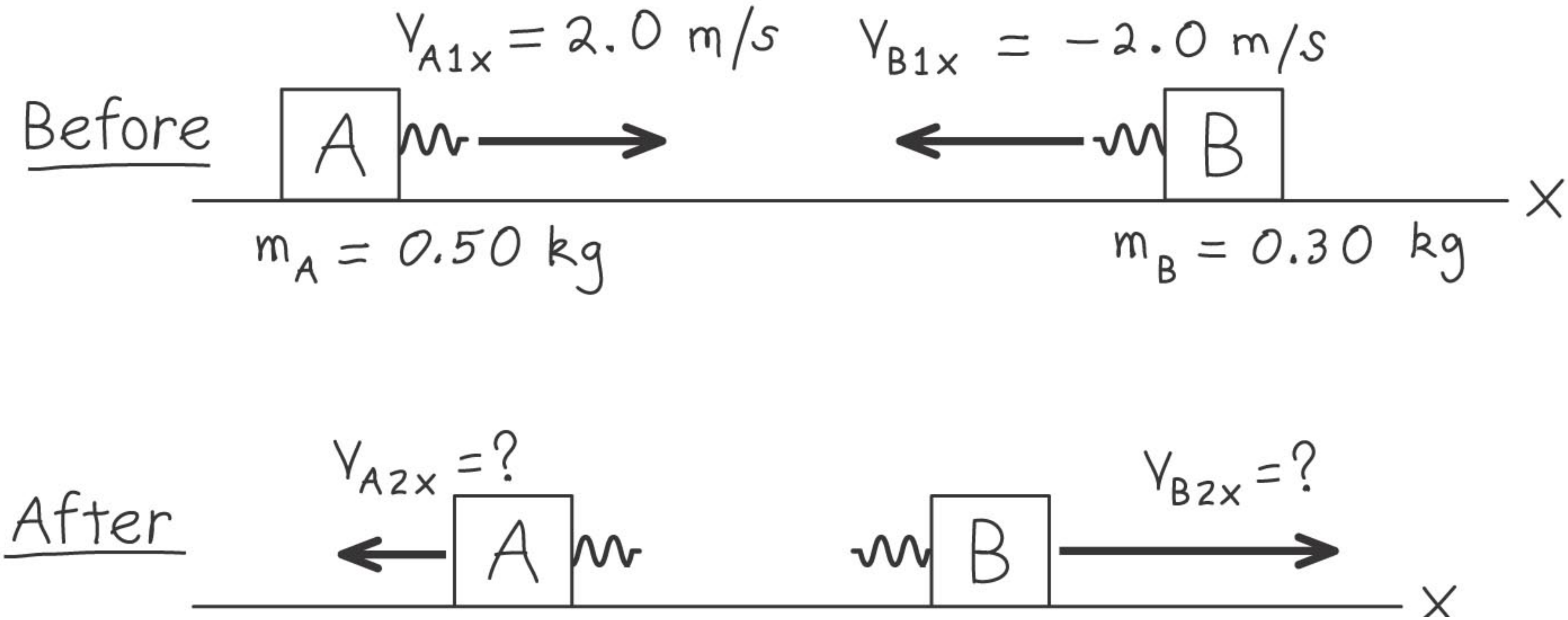
A collision where two object stick together is called completely inelastic collision because it loses the most kinetic energy.

Another completely inelastic collision - automobile accidents

- Refer to figure below, *find v and θ.*



1-D elastic collision - 2 equations and 2 unknowns

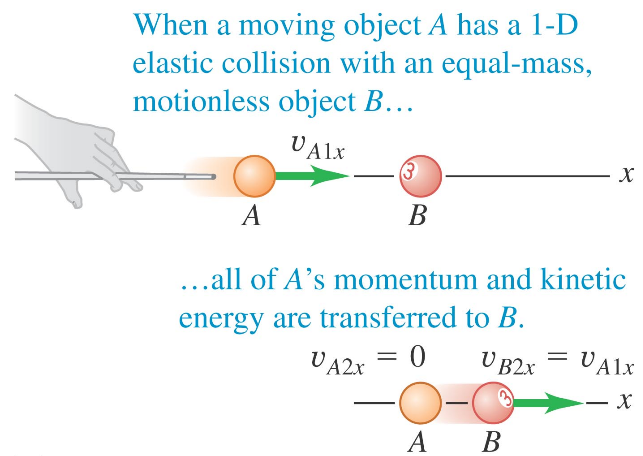


First equation: Conservation of total momentum (x - direction because net $F_x = 0$)

Second equation: Conservation o kinetic energy (elastic collision)

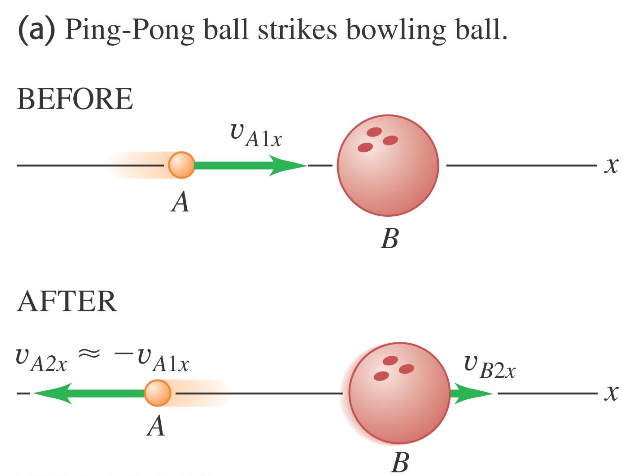
Elastic collisions – qualitative analysis

- There are collisions where you can guess the solutions.
- Two equal masses
- Two very different masses

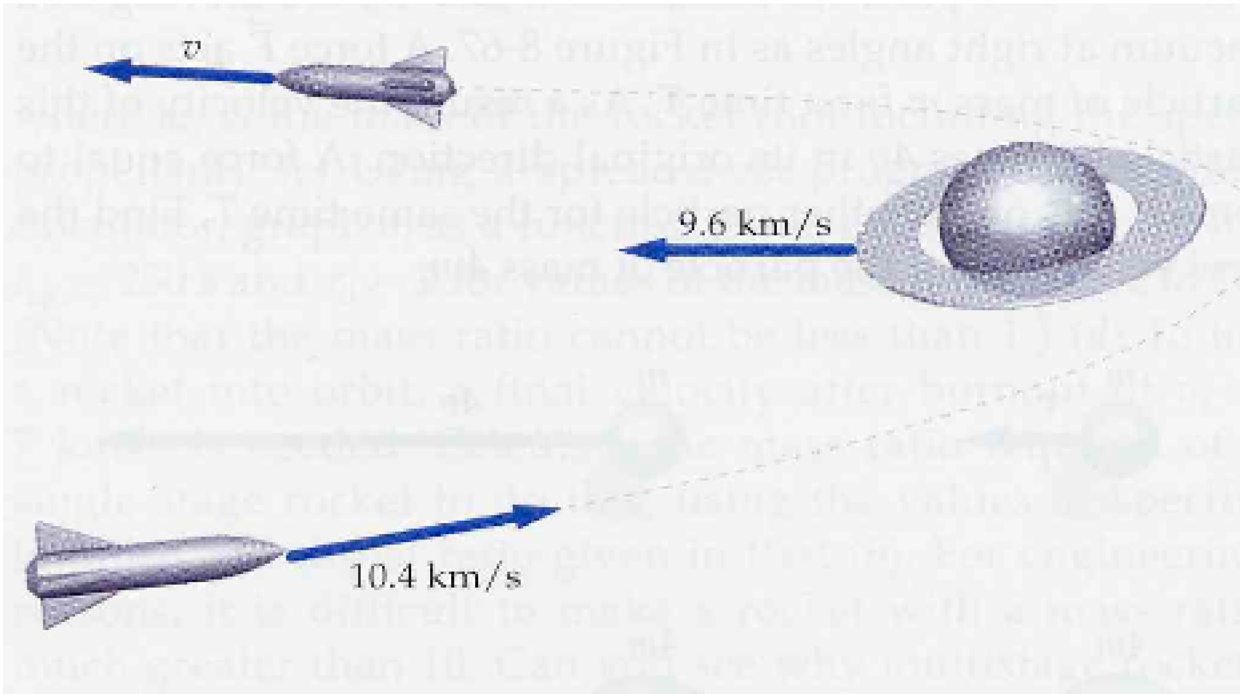


What happen to the ping-pong ball's final velocity if the bowling ball is also initially moving toward the ping-pong ball?

This is the concept behind gravitational sling-shot effect.



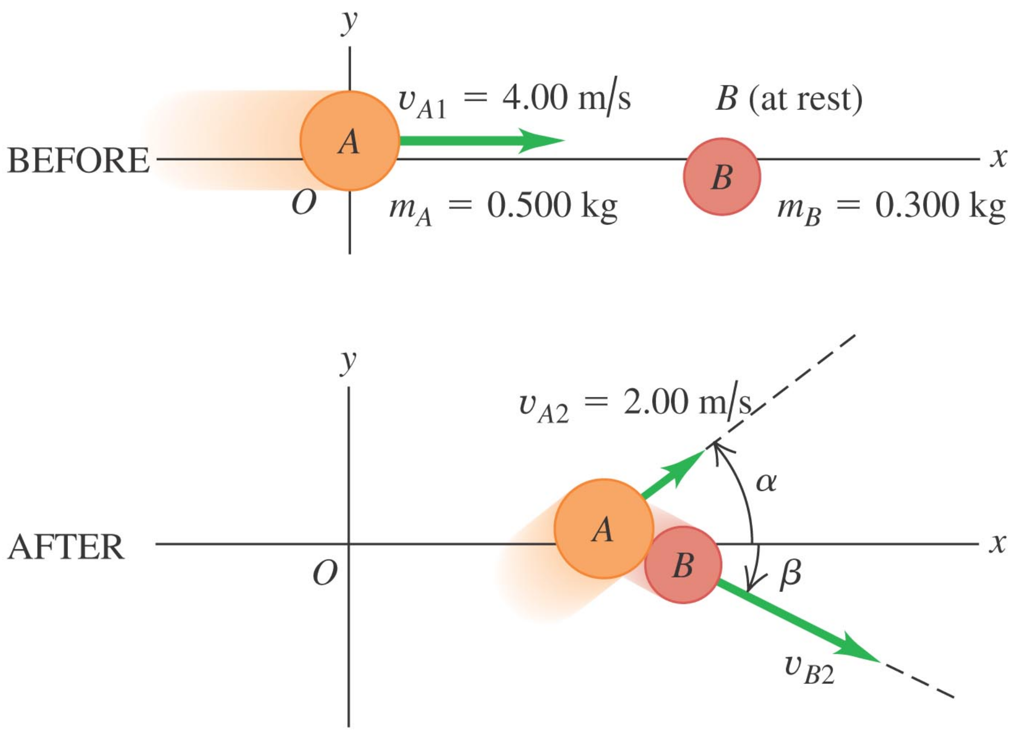
Gravitational "sling-shot" effect



Can be analyzed effectively as 1-D collision

2-D elastic collision - 3 equations and 4 unknowns.

- Consider Example 8.12.



Conservation of total momentum x & y - directions $\Rightarrow$ 2 equation

Conservation of kinetic energy $\Rightarrow$ 1 equation

4 unknowns : $|v_{A2}|, |v_{B2}|, \alpha, \beta$!

(Not enough equation because we treated masses as points)

Example of center of mass motion when $a_{cm}=0$

A wrench spins around and slides across a very smooth table.
The net external force ~0, hence the center of mass (white dot) exhibits a nearly constant velocity=> conservation of momentum.

Definition of Center of mass & its motion

Center of mass position:

$$\text{Defn}: \ \vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$\Rightarrow M_{total}\vec{r}_{cm.} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots$$

Center of mass velocity:

$$M_{total}\vec{v}_{cm.} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots = \text{ total momentum}$$

Center of mass acceleration:

$$M_{total}\vec{a}_{cm.} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots = \text{ net } \vec{\mathrm{F}}_{\text{external}} + \text{net } \vec{\mathrm{F}}_{\text{internal}}$$

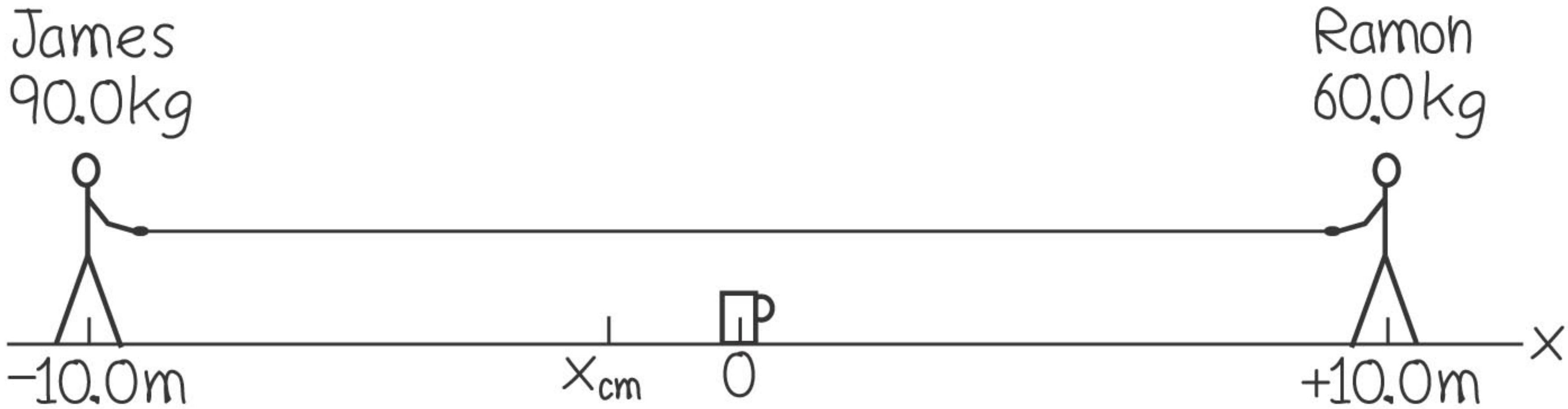
net $\vec{\mathrm{F}}_{\text{internal}} = 0$ according to Newton's third law.

$\Rightarrow$ Center of mass motion is governed by external forces only

Furthermore, if net $\vec{\mathrm{F}}_{\text{external}} = 0$, then $\vec{a}_{cm.} = 0 \Rightarrow M_{total}\vec{v}_{cm} = \text{constant}$

This is a re-statement of conservation of total momentum!!

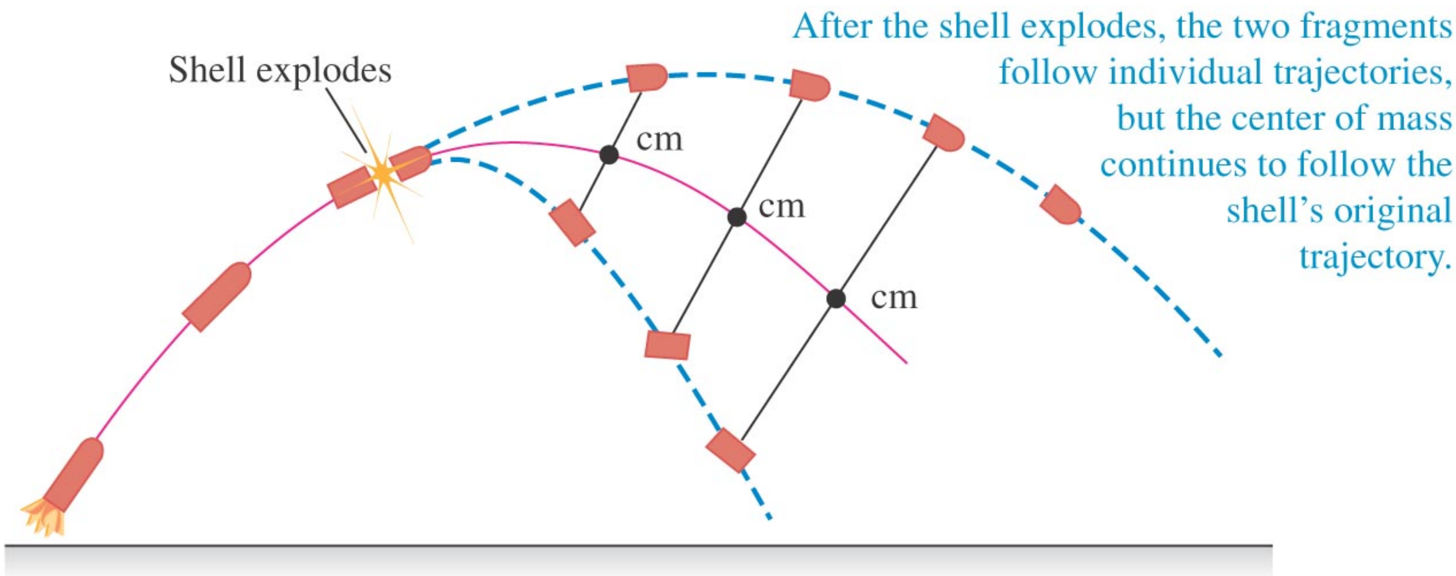
Calculate the center of mass



Find the center of mass

Example of center of mass motion is governed by external force only

(a)

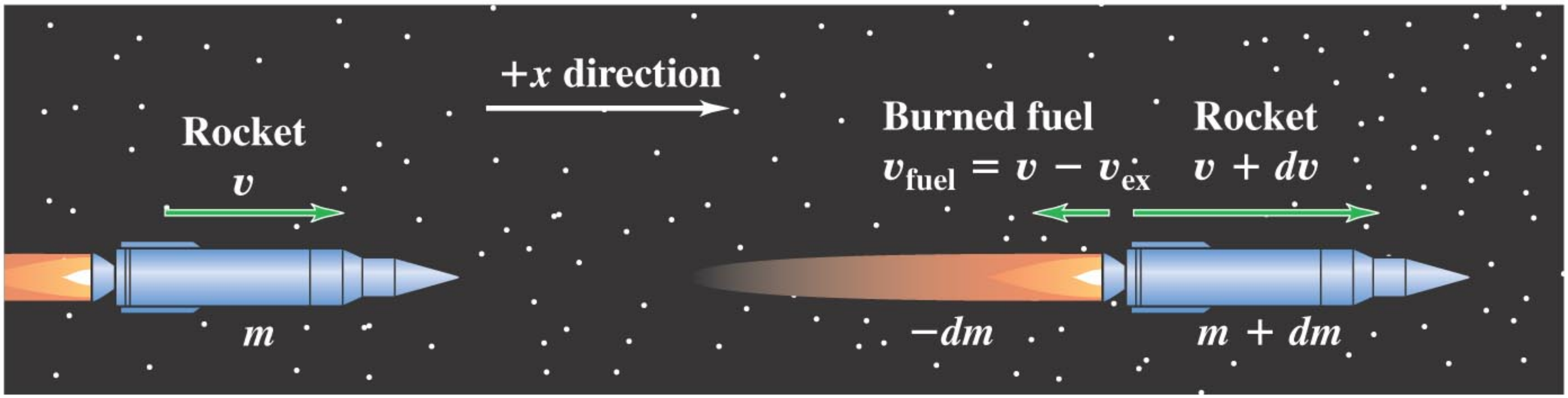


Net external force $= M_{total}\vec{g} \Rightarrow \vec{a}_{cm} = g$

$\Rightarrow$ center of mass motion follows a parabola path even after the shell exploded (because internal forces which cause the explosion do not affect the center of mass motion.) Note: Total momentum is NOTconserved because there is an external force.

Rocket propulsion-example of conservation of total momentum

(a) (b)



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

Set up the conservation of total momentum equation