Chapter 7

Work and Kinetic Energy

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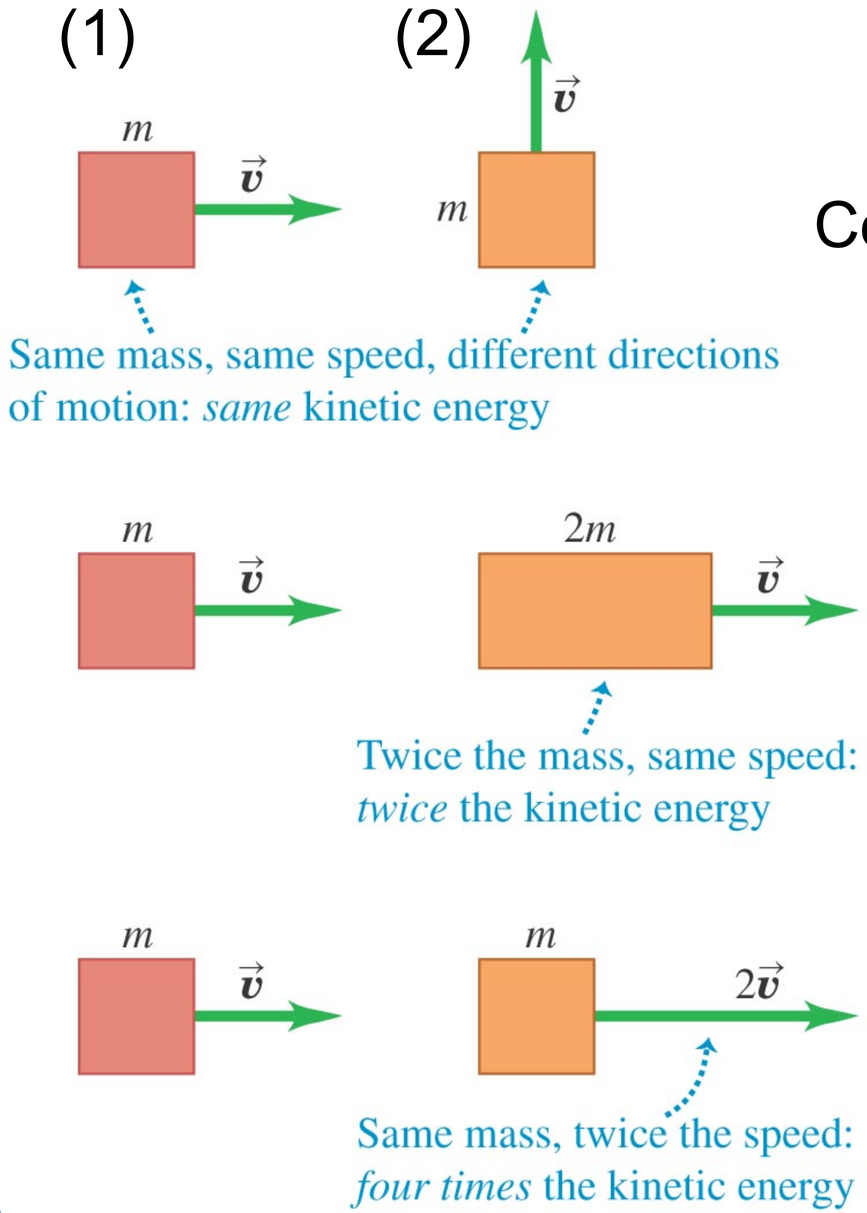
Learning Goals for Chapter 7

- To understand the concept of kinetic energy (energy of motion)
- To understand the meaning of work done by a force.
- To apply the work-kinetic energy theorem to solve problems in mechanics
- To understand the concept of power in physics (rate of doing work).

Definition of kinetic energy

- Qualitative concept: Kinetic energy (K) is the energy associated with how fast an object is moving and its mass; the faster the object's speed (v), the higher its kinetic energy; the heavier it's mass (m), the greater the kinetic energy.
- Quantitative definition: $K={}^{1}/_{2}mv^{2}$
- Unit of kinetic energy = kg m^2/s^2=1 Joule
- ***Kinetic energy is NOT a vector; it is scalar (just a number; it does not depend on direction of motion)***

Compare the kinetic energy of different bodies

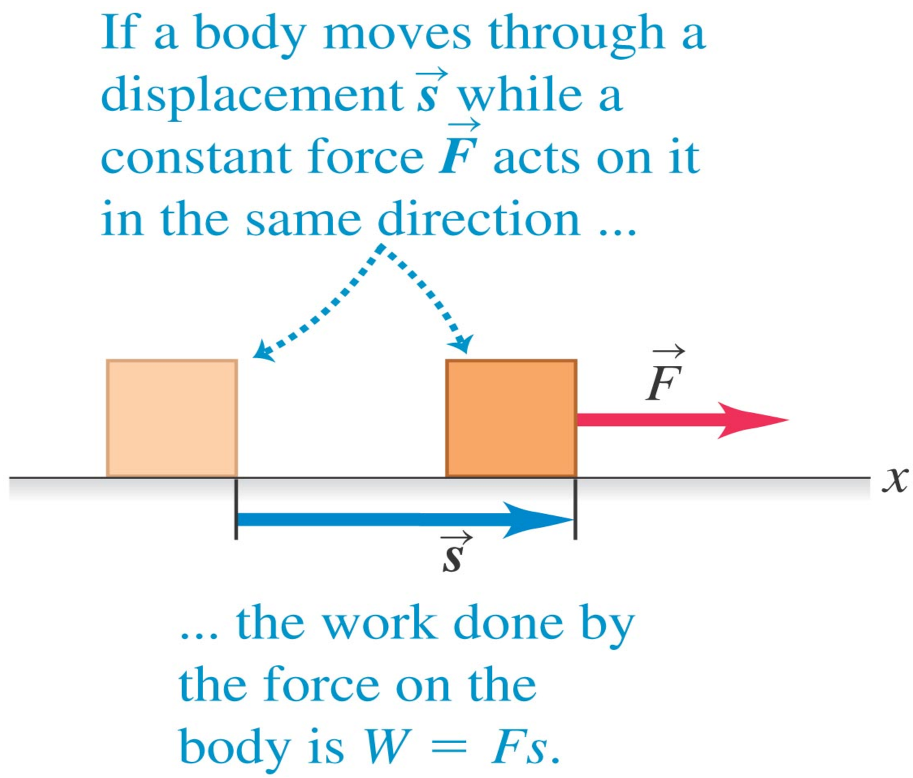


$$K \equiv \frac{1}{2} m \,|\vec{v}\,|^2 = \frac{1}{2} m(\vec{v} \bullet \vec{v})$$

Compare K_2 to K_1

Work, a force applied over a distance

- Definition of work: W=Fs (this definition is valid for the situation where a *constant* force is applied in the same direction that the object moves; more general definition will be given in later slides)



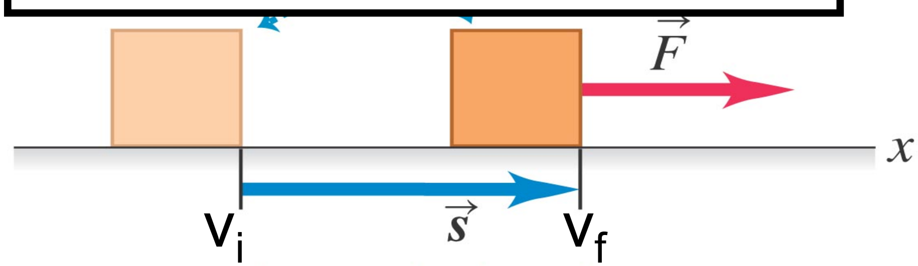
Work-Kinetic Energy Theorem

- Concept: A force applied parallel to the velocity of an object will cause the object to speed up, hence increasing the object's kinetic energy. The amount of kinetic energy change is related to the work done by the force; derivable from Newton's Second Law and kinematics.

$$\textit{Kinetics}: \; v_f^2 = v_i^2 + 2as$$

$$\text{2nd Law} \Rightarrow v_f^2 = v_i^2 + 2\frac{F}{m}s$$

$$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = Fs$$



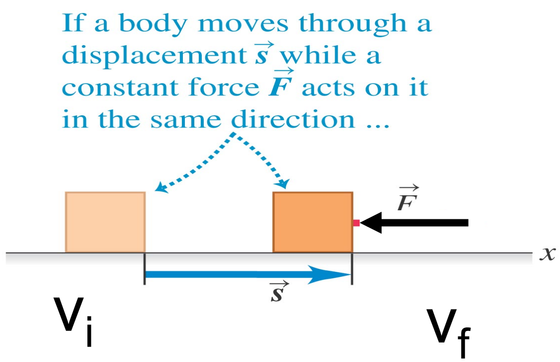
... the work done by the force on the body is $W = Fs$.

Work-Kinetic Energy Theorem

If the force is pointing opposite to the displacement (example, friction) then the object slows down

$$\Rightarrow K_f < K_i \Rightarrow K_f - K_i = W < 0$$

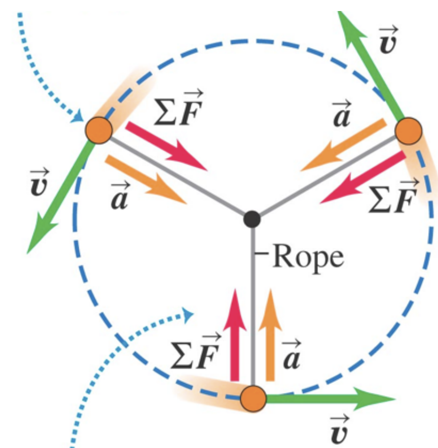
$$W = \vec{F} \bullet \vec{s} < 0$$



If the force is perpendicular to the displacement,
then the force only changes the direction of the object's velocity without changing the object's speed (such as uniform circular motion)

$$\Rightarrow K_f = K_i \Rightarrow K_f - K_i = W = 0$$

$$W = \vec{F} \bullet \vec{s} = 0$$



The following Work-Kinetic Energy Theorem can accounts for all these situations:

$$K_f - K_i = \vec{F} \bullet \vec{s} \quad \text{(dot-product)}$$

Work-Kinetic Energy Theorem- quantitative

$$K_f - K_i = W \Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \vec{F} \bullet \vec{s}$$

(valid for constant force and straight displacment)

Example problems using W-K Theorem:

(a) Drop a ball from a height of 10 m, what is its speed when it reaches the ground? (Assume no air resistance)

(b) With air resistance, the ball's final speed is only 80% of what is calculated in part a. How much work is done by air resistance?

(c) Toss a ball upward with an initial speed of 3 m/s, how high will it reach? (Assume no air resistance)

Note: W-K Theorem cannot answer questions about time such as *when* will the ball reach the maximum height?

More Example: Work-Kinetic energy Theorem

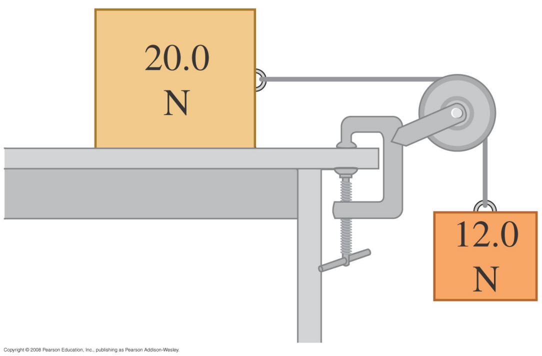
The system is initially at rest.

When the 12.0N weight has dropped by 2m, what is its speed?

(a) Assume no friction between the table and the block.

(b) Assume the coefficient of kinetic friction is 0.2

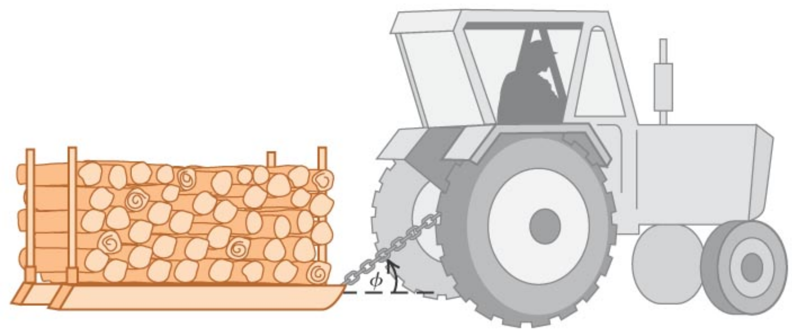
Let $g=10m/s^2$



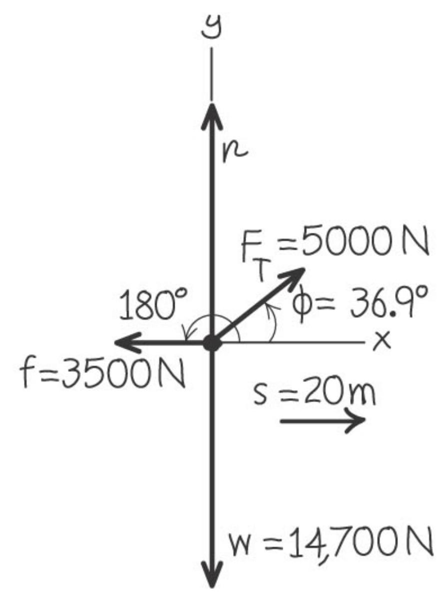
Stepwise solution of work done by several forces

- Suppose the tractor pulls the load over a distance of 20 m along x-direction.
- Find the work done by all the forces indicated in the figure.
- Cos (36.9^0) ~0.8
- If the initial speed is zero, what is the final speed of the load?

(a)



(b) Free-body diagram for sled



More Example: Work-Kinetic energy Theorem

A block (mass=m) slides down an incline to the bottom The initial height is h.

(a) How much work was done on the block by gravity?

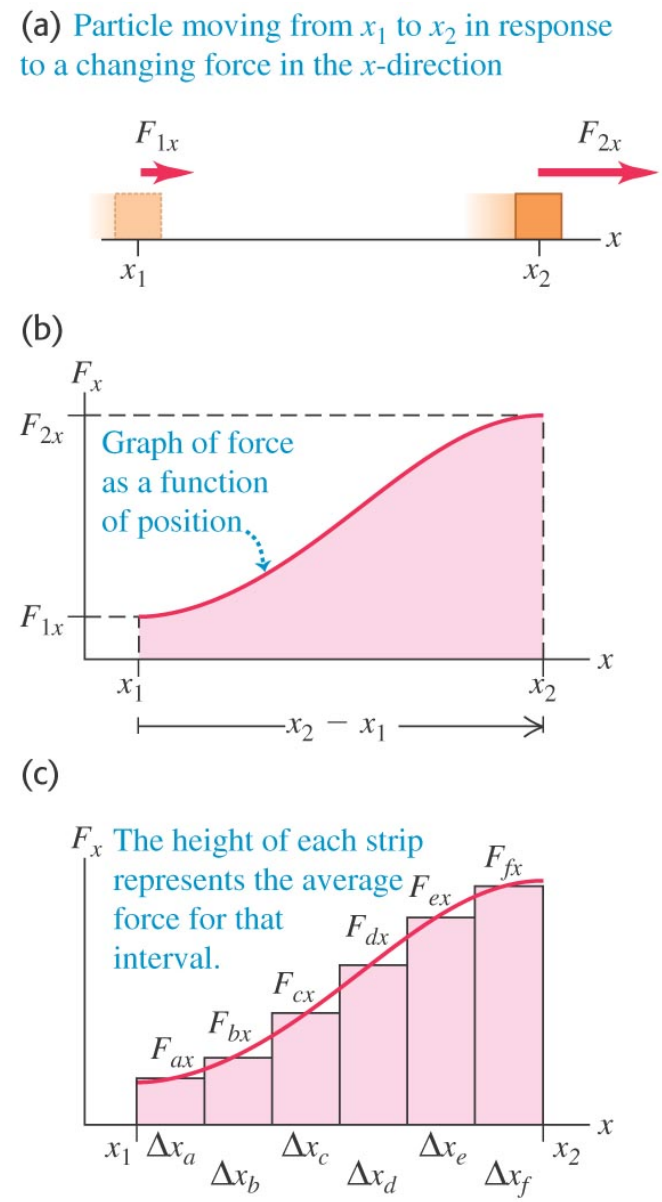
(b) Assume the incline is frictionless, find the speed of the block when it reaches the bottom of the incline? What concept/method would you use to find the answer?

(c) Assume the coefficient of kinetic friction is μ_k

How much work was done on the block by friction? How would you use Work-Kinetic Energy Theorem to find the speed of the block at the bottom?

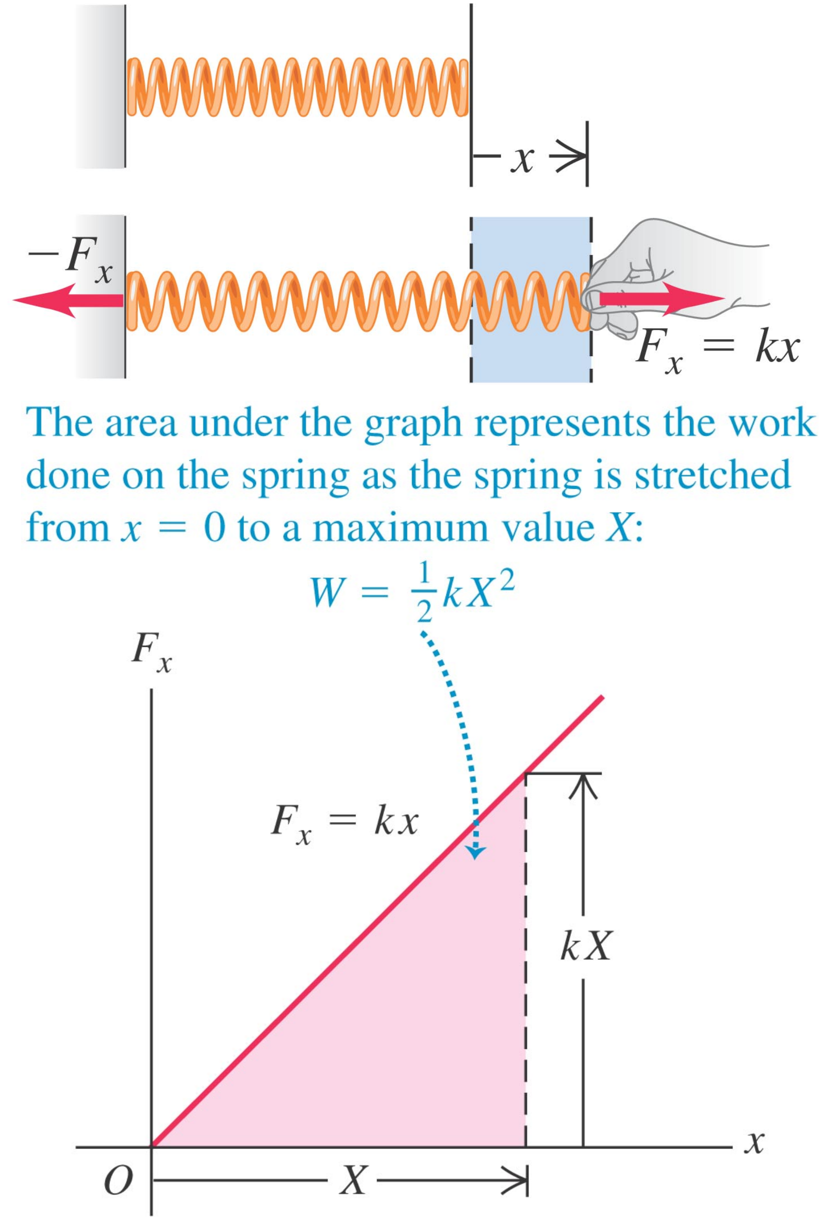
Work done by a force which varies with position - 1-D

$$W = \int_{x_i}^{x_f} F\,dx$$

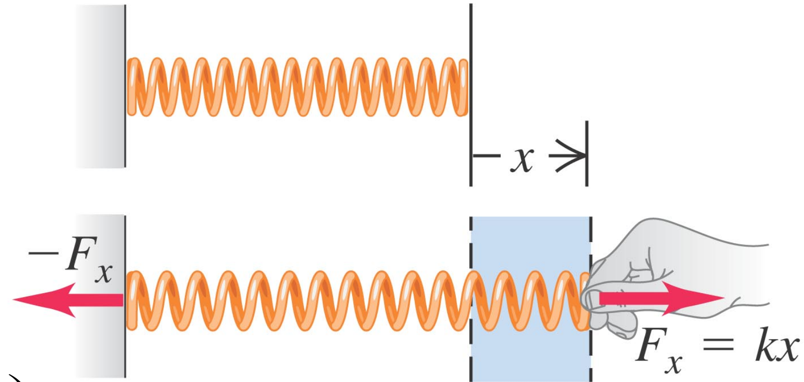


Example of a varying force in 1-Dim. - spring

- The force applied to an ideal spring will be proportional to its stretch.
- The graph of the force F_x (exerted by you) versus stretch x will yield a slope of k, the spring constant.



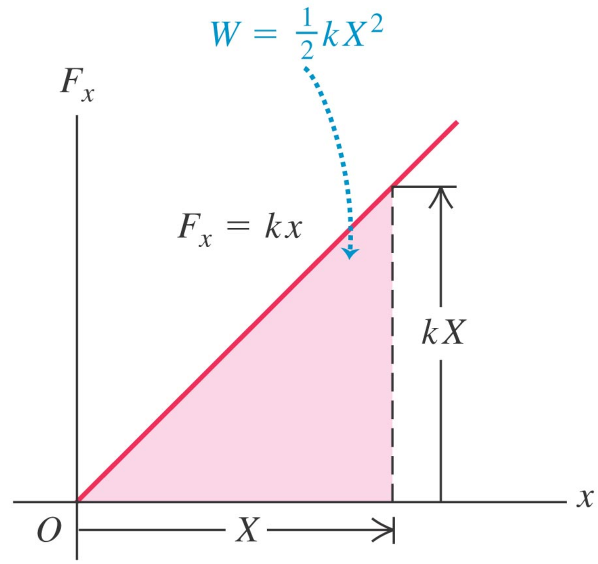
Distinguish the work done by you vs the work done by the spring



$$W_{\text{done by you}} = \int_{x_i}^{x_f} kx\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$W_{\text{done by spring}} = \int_{x_i}^{x_f} -kx\,dx = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:



Example: A spring stetches 1 cm when a 10-kg mass hangs on it.

(A) What is the spring constant?

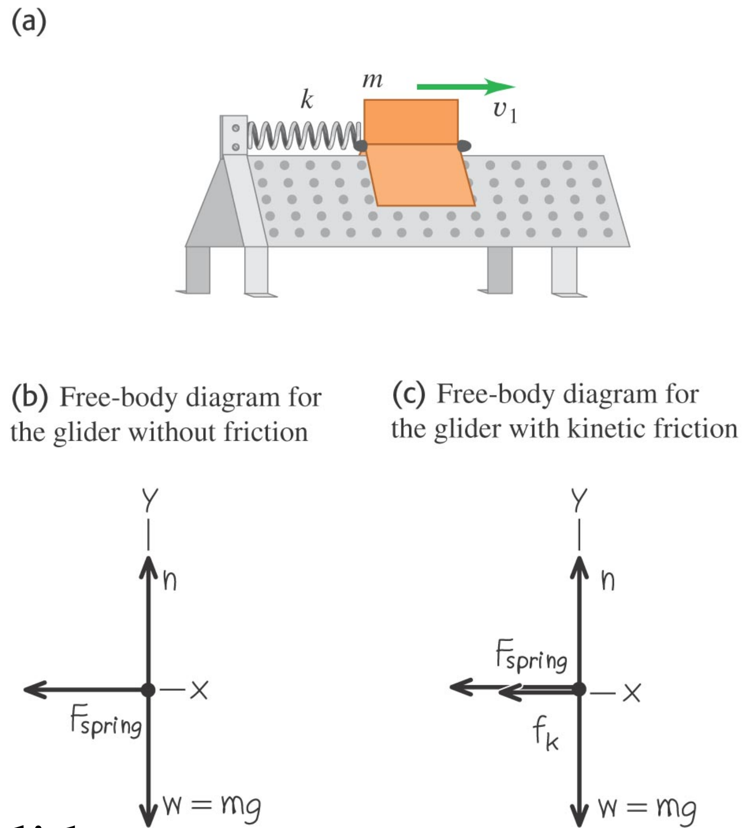
(B) How much work is done by gravity on the mass?

(C) How much work is done by the spring on the mass?

(*D*) Verifiy the work-kinetic energy theorem

Motion with a varying force—Example 6.7

- Refer to Example 6.7.
- Assume no friction.



Given: m = 0.1kg, k = 20N/m,

initial speed $v_i = 1.5$m/s,

spring initially unstretch.

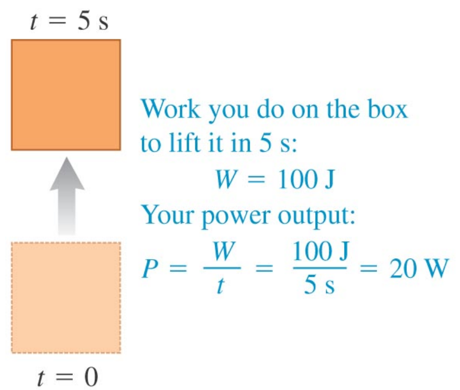
Find the max. distance that the glider will move to the right.

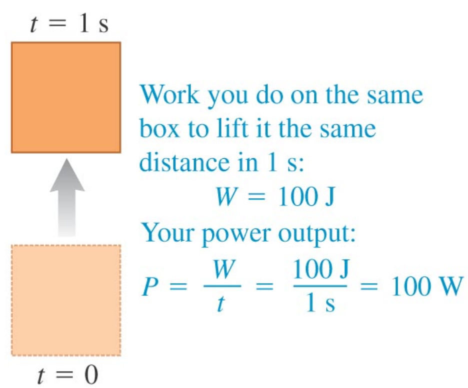
How much is one Joule?

If you lift a 1 kg mass (~2.2 pounds) upward for a distance of 1 meter, then you have performed $(1kg)(9.8m/s^2)(1\ meter)=9.8$ Joules of work.

Watt about power?

- Dividing the work done by the time that passed determines the power $P \equiv \frac{W}{\Delta t} = \frac{\vec{F} \bullet \vec{s}}{\Delta t} = \vec{F} \bullet \vec{\mathrm{v}}$
- Unit of power =Joule/s is called Watt, named after James Watt.
- What is the unit kilowatt-hour?
- How much energy does a 100 Watt bulb use in one hour?
- Ans: 100 J/s * (3600s)=360,000 J
- (This is equivalent to lifting a 1kg mass upward 1 meter ~ 36,000 times!!!)





Example: Calculating Power

An 10-kg object is dropped from a height of 20 m to the ground.

(a) How much work were done by the Earth's gravity on the object?

(b) What is the average power done by gravity? (Assume no air resistance)

(c) What is the instantaneous power at t=1s? (Assume no air resistance)