

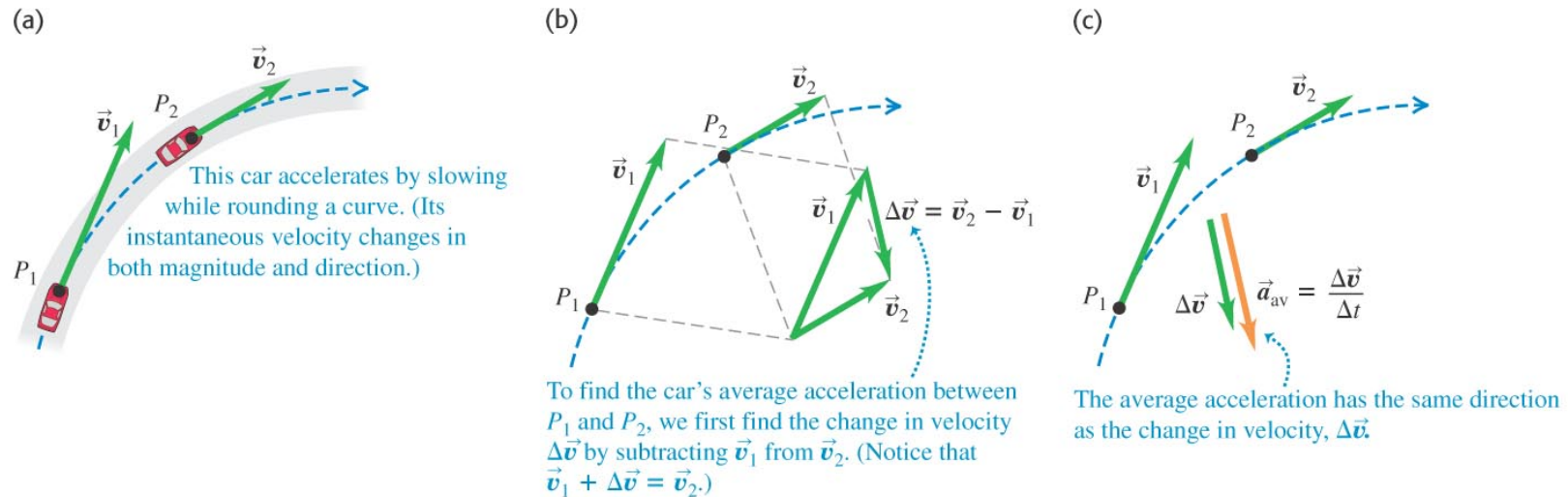
Chapter 4

Motion in Two and Three Dimensions

Pui Lam

Learning Goals for Chapter 4

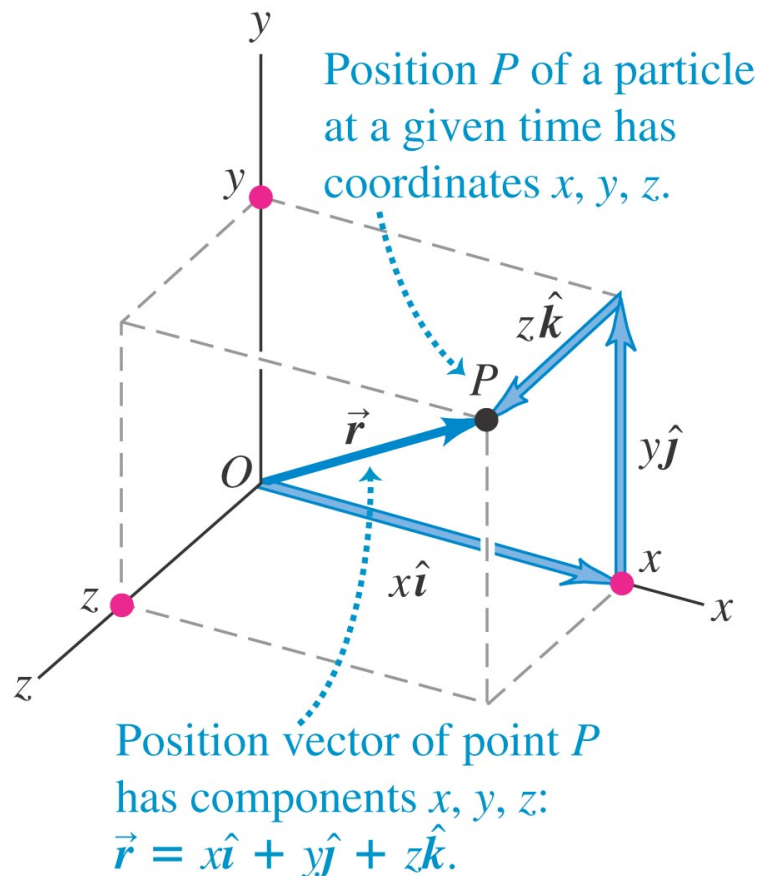
- Recognize that in 2 or 3-dimensions that *the velocity vector and the acceleration vector need not be parallel.*



- Be familiar with the following 2-D examples:
 - projectile motion
 - uniform and non-uniform circular motion
 - general curve motion
- Know how to calculate relative velocity

Position relative to the origin—Figure 3.1

- For general motion in 3-dimension, the position vector relative to your chosen origin will have components in x , y , and z directions. The path of a particle is generally a curve in 3-D space.

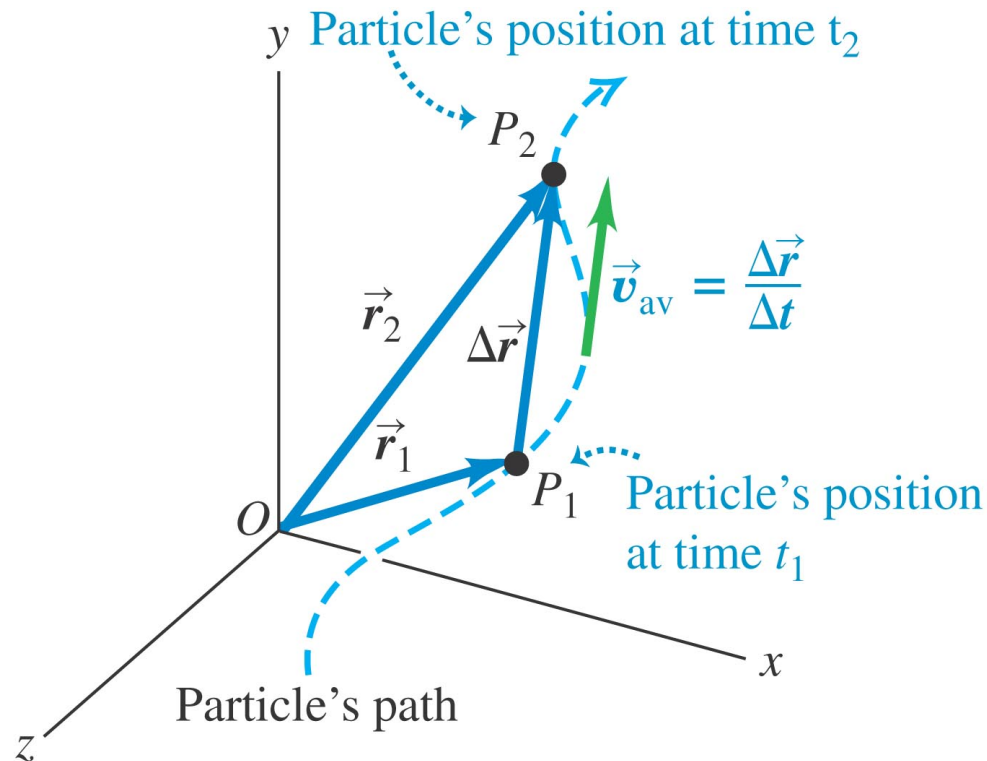


$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

Q : How do calculate the instantaneous velocity, $\vec{v}(t)$, and instantaneous acceleration, $\vec{a}(t)$?

Average velocity and Instantaneous velocity—Figure 3.2

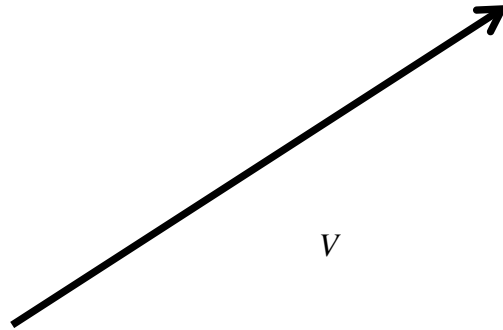
- The instantaneous velocity at a given location is a vector tangent to the path.



** Instantaneous velocity $= \vec{v} \equiv \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$

Velocity (vector) vs. Speed

- Velocity is a vector. Example

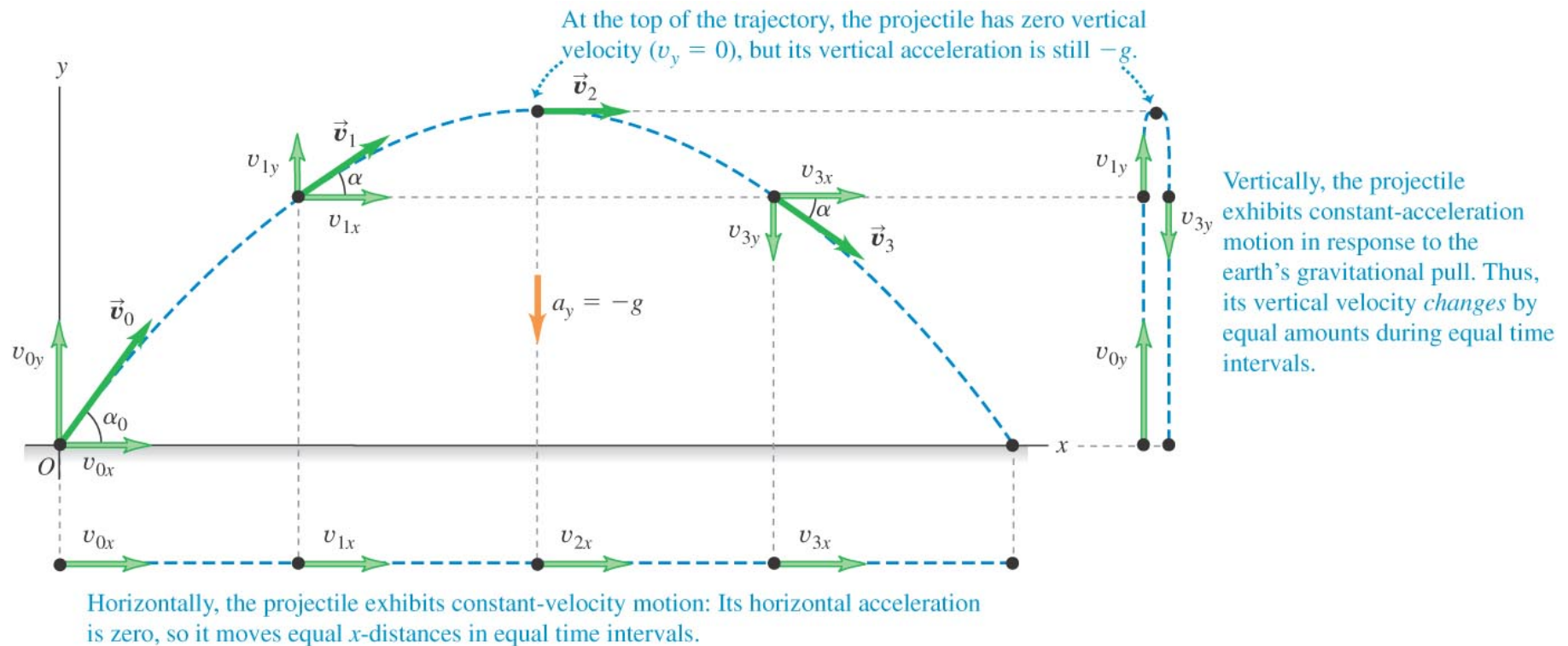


$$\text{Given } \vec{v} \equiv \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = (3\hat{i} + 4\hat{j} + 5\hat{k})\frac{m}{s}$$

How to find the speed?

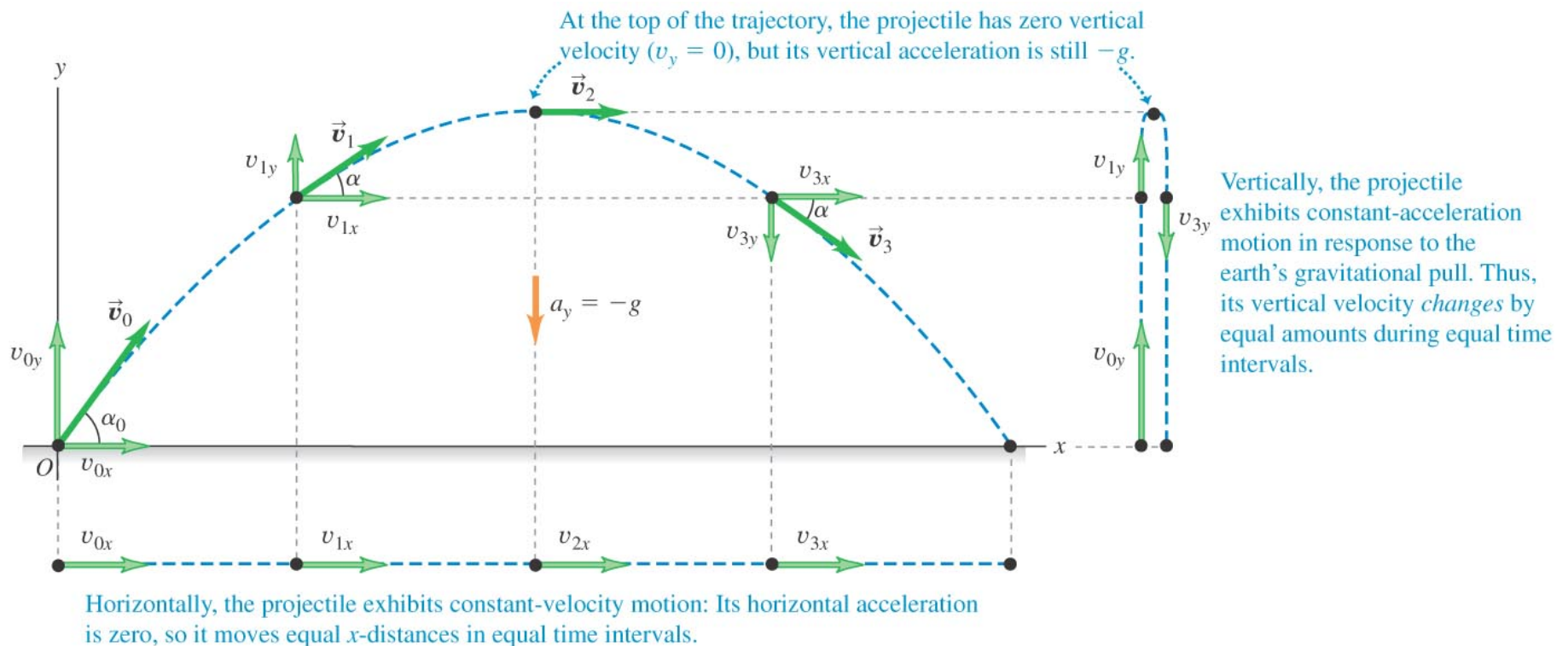
Example of a 2-D motion - Projectile motion

- A projectile is any object that follows a path determined by the effects of gravity, air resistance, and wind, given an initial velocity.
- *If air resistance and wind are negligible, then projectile motion = motion under constant acceleration, $a_x=0$ and $a_y=-g$*



Equations for Projectile Motion (neglect air-resistance)

- Find $x(t)$ and $y(t)$ for a projectile motion given its initial position and initial velocity.
- Calculate the subsequent velocity as a function of time.

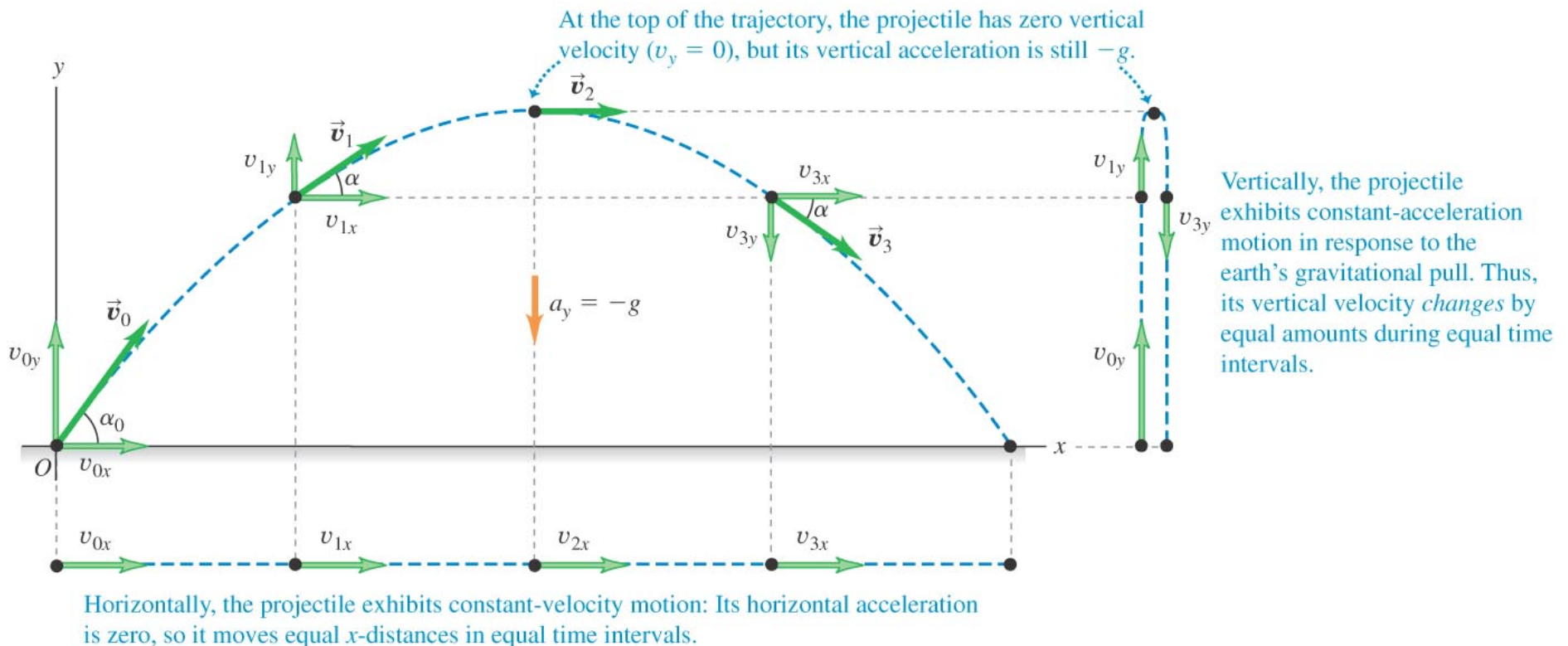


Projectile motion - numerical example I

Q. Given : $|\vec{v}_0| = 10 \text{ m/s}$, $\alpha_0 = 30^\circ$, and

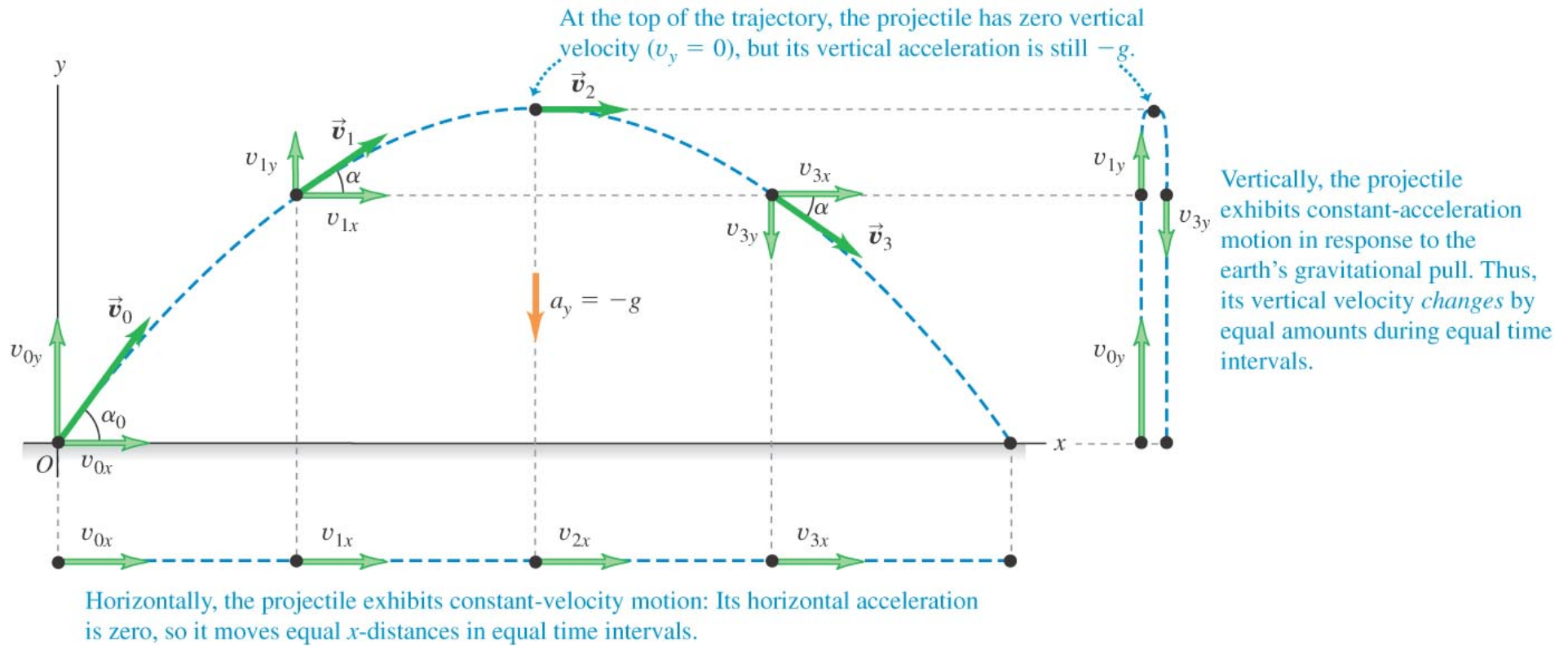
$g \sim 10 \text{ m/s}^2$. (Assume no air resistance)

- (1) Find the time it reaches the top.
- (2) Find the velocity vector at the top
- (3) Find the range.
- (4) Find the velocity vector when it returns to the ground



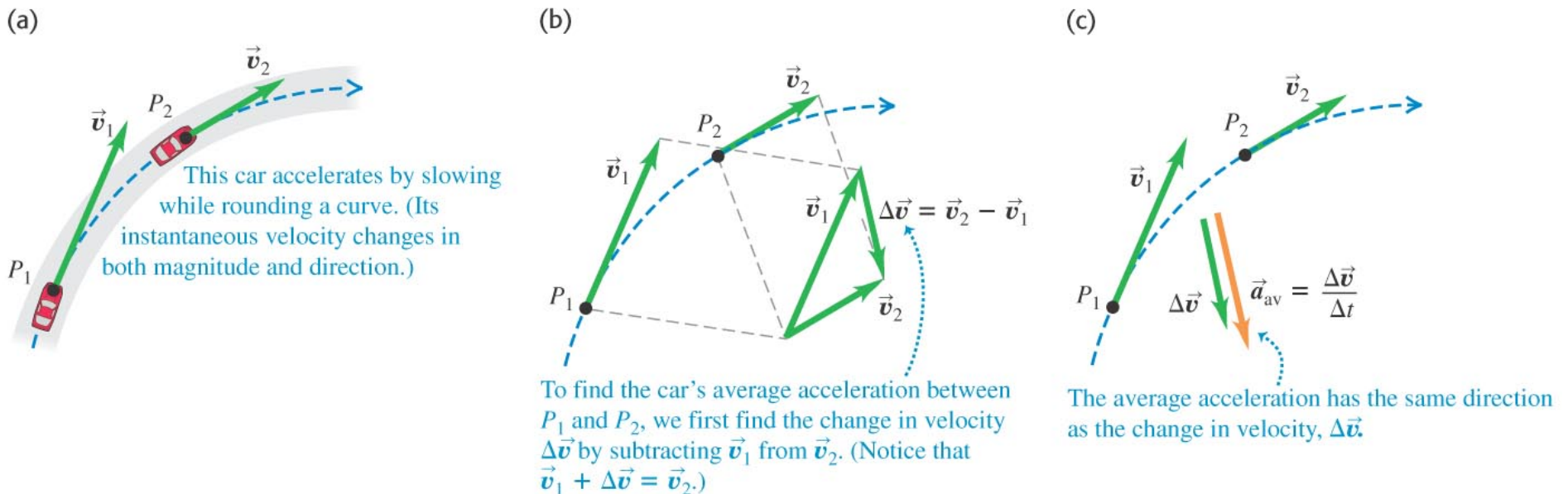
Effect of air resistance

Q. Sketch the path if there is air - resistance.



The instantaneous acceleration vector—Figure 3.6

- *The acceleration vector is non-zero as long as there is a change in the velocity vector.*
- The change can be either the magnitude OR the direction of the velocity, OR both.
- In 2- and 3-dimension, acceleration vector needs not be in the same direction of velocity vector. Example: Car going around a curve.



Uniform circular motion and centripetal acceleration

For circular motion:

$$x(t)=r\cos\theta(t), \quad y(t)=r\sin\theta(t)$$

For uniform circular motion:

$$\theta \text{ increases uniformly} \Rightarrow \theta(t)=\omega t$$

$$\Rightarrow x(t)=r\cos\omega t, \quad y(t)=r\sin\omega t$$

$$(\text{e.g. } 10 \text{ rpm} \Rightarrow \omega = 10(2\pi \text{ rad.}) / 60 \text{ s})$$

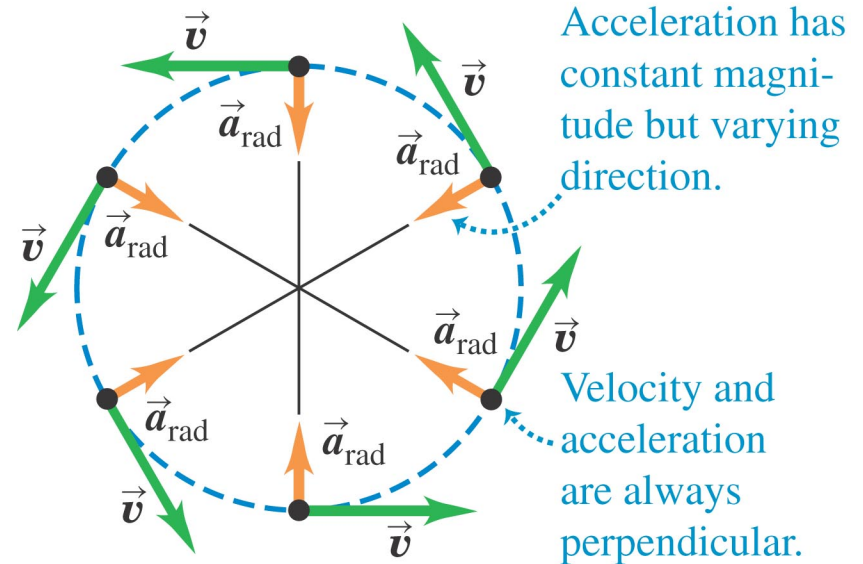
From these, one can calc; $\vec{v}(t)$ and $\vec{a}(t)$
(You fill in the details)

Result:

$$|\vec{a}_c| = \frac{|\vec{v}|^2}{r}; \quad |\vec{v}| = \text{magnitude of velocity} = \text{speed}$$

Note: In **uniform** circular motion, the “speed” is unchanged, but the velocity vector changes \Rightarrow there is an acceleration. The acceleration is called “centripetal acceleration” – pointing to the center.

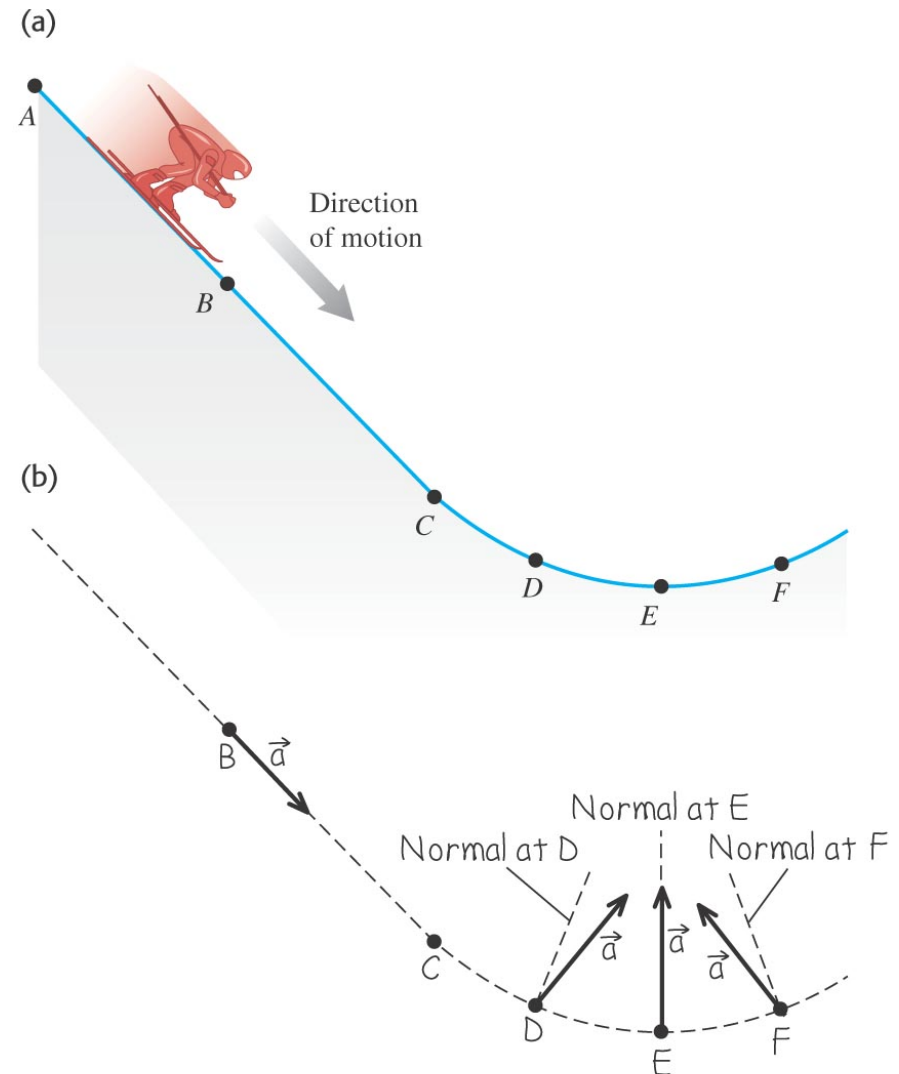
(a) Uniform circular motion



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

Example of non-uniform circular motion

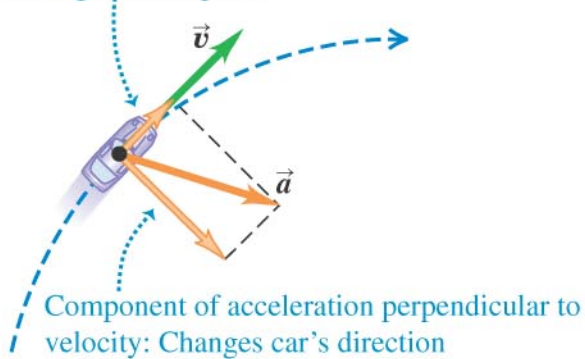
- “Circular motion” doesn’t mean a complete circle, could be part of a circle or any curve.
- In non-uniform circular motion, the speed is NOT constant, hence there is also a **tangential acceleration**, in addition to the centripetal acceleration.



Acceleration – daily usage vs. physics usage

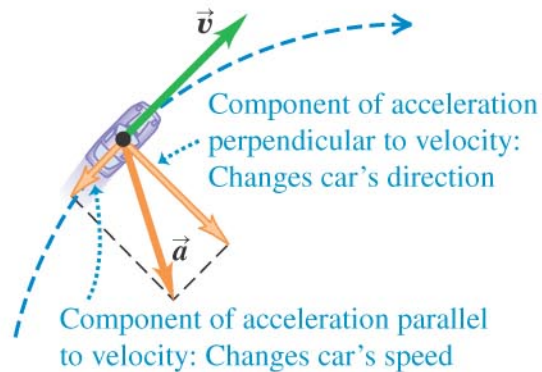
Car speeding up along a circular path

Component of acceleration parallel to velocity:
Changes car's speed



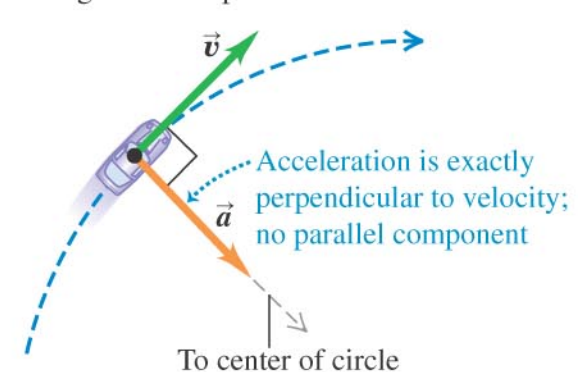
$$\vec{v} \cdot \vec{a} > 0 \Rightarrow \text{speeding up}$$

Car slowing down along a circular path



$$\vec{v} \cdot \vec{a} < 0 \Rightarrow \text{slowing down}$$

Uniform circular motion: Constant speed along a circular path



$$\vec{v} \cdot \vec{a} = 0 \Rightarrow \text{constant speed}$$

In daily language, acceleration means speeding up and deceleration means slowing down.

In Physics, acceleration (vector) = change of velocity (vector) wrt time.

An object has a non-zero acceleration (vector) whenever there is a change in velocity (vector); the object can be speeding up, slowing down, or keeping the same speed.

Relative velocity on a straight road

- What is the velocity vector of the Truck with respect to you?

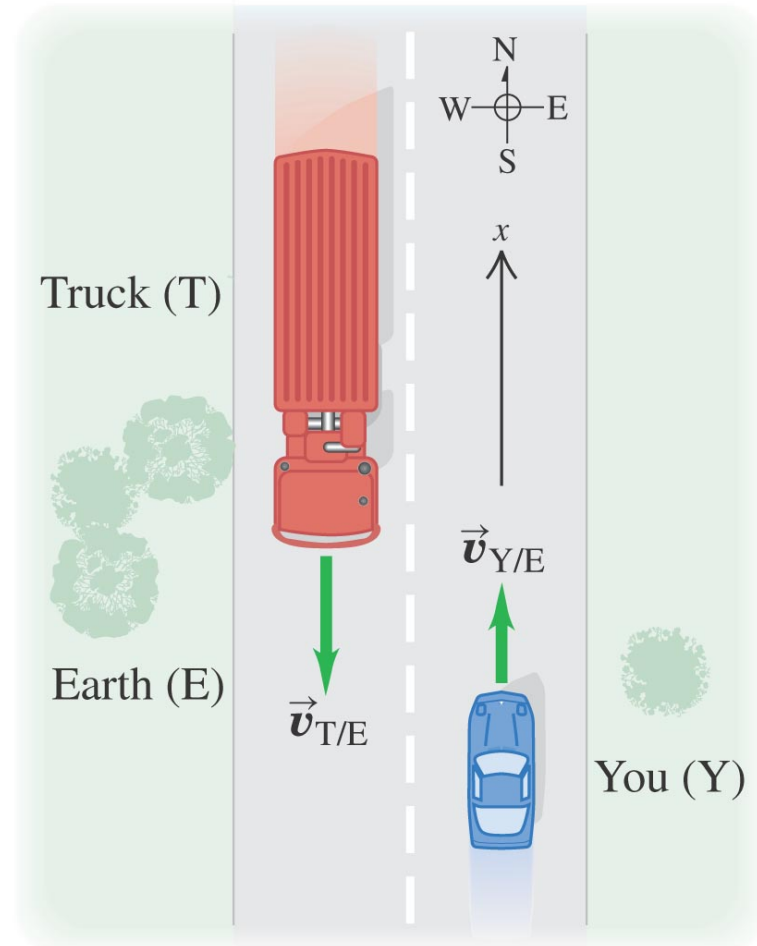
$$\text{Let } \vec{V}_{T/E} = -20\hat{j}; \quad \vec{V}_{Y/E} = +30\hat{j}$$

$$\vec{V}_{T/Y} = \vec{V}_{T/E} + \vec{V}_{E/Y}$$

$$\Rightarrow \vec{V}_{T/Y} = \vec{V}_{T/E} - \vec{V}_{Y/E}$$

$$= -20 - 30 = -50\hat{j}$$

Same formula applies for
2- or 3-dimensions.

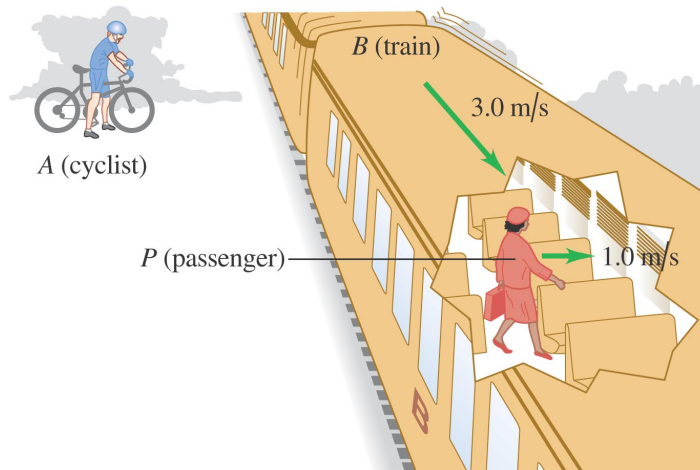


$$\text{Example : } \vec{V}_{T/E} = -20\hat{j}, \vec{V}_{Y/E} = 30\hat{j}$$

Relative velocity in two or three dimensions

- Find the velocity of the passenger with respect to the bicyclist (Ignore the middle diagram).

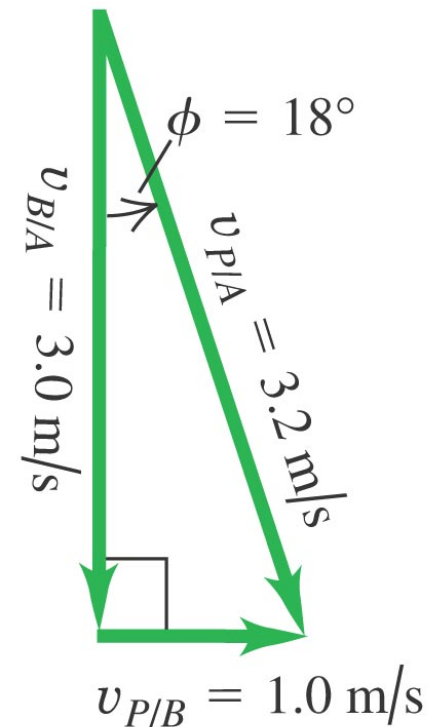
(a)



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

(c) Relative velocities
(seen from above)



Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley

Question: The compass of an airplane indicates that it is headed north and the plane is moving at 240km/hr through the air. If there is wind of 100km/hr from west to east, what is the velocity of the plane relative to the ground?