Ph170 General Physics

Ch.3 Motion Along a Straight Line

Kinematics

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Topics & Learning Outcomes

1. Position measurement, x(t) or y(t) – this function contains all the information about the motion of an object.
2. Useful concepts: Average velocity, Instantaneous velocity and instantaneous acceleration

Learning outcomes:

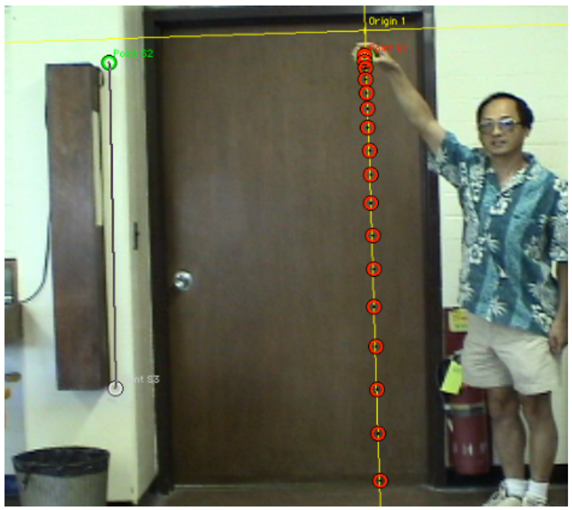
1. Able to deduce instantaneous velocity and instantaneous acceleration from the graph of x(t).

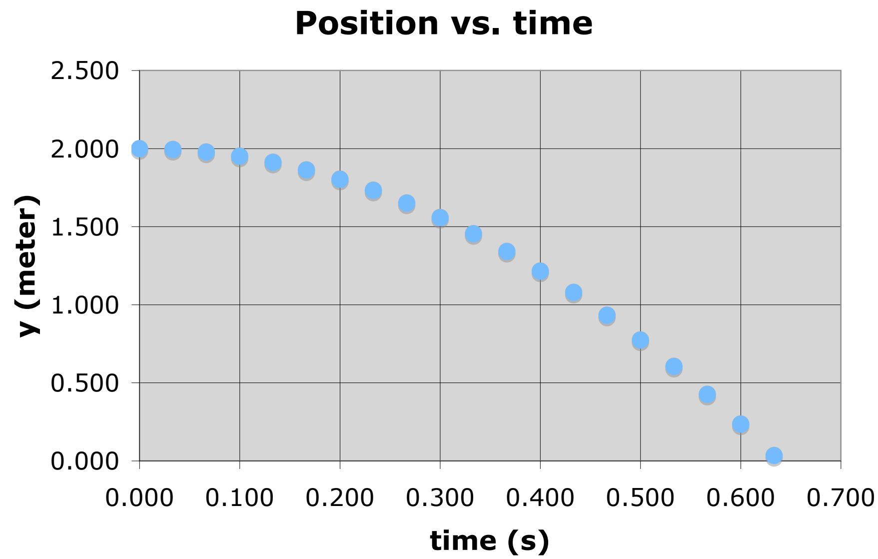
2. Given the function x(t), able to calculate instantaneous velocity, v(t), and Instantaneous acceleration, a(t), by differentiation

3. Given a(t), able to calculate v(t) and y(t) by integration

Position measurement

(1)Set up coordinate system. (2) Measure position from origin.
(3) Graph position vs. time.



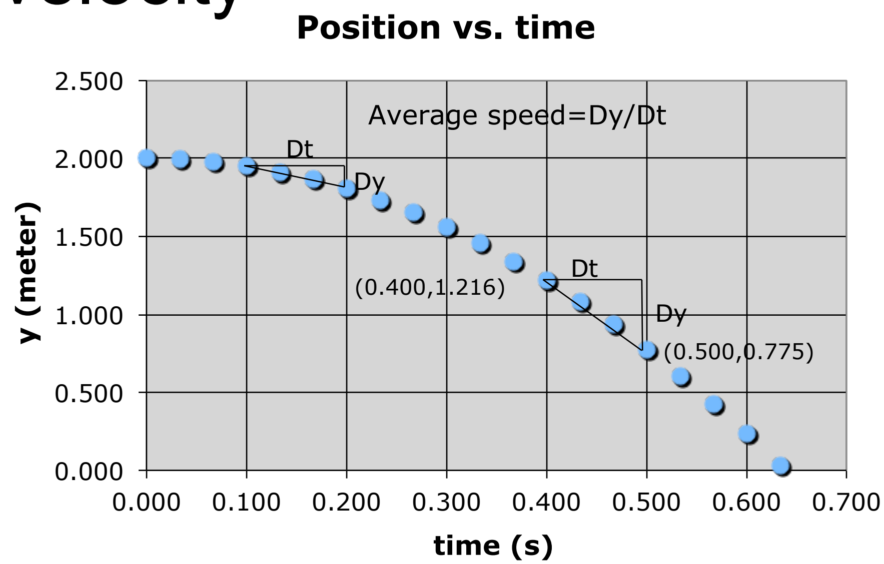


Camcorder: 30 fps
Note: space between images are further apart
=> ball drops faster and faster.

From the graph, we can calculate the average velocity for each time interval.

Activity: Calculate the average velocity for the time interval from 0.4s to 0.5s

Average velocity



Camcorder: 30 fps

$$\text{Average velocity between } t_i \text{ and } t_f = \overline{v} \equiv \frac{y(t_f) - y(t_i)}{t_f - t_i}\hat{j}$$

$$e.g. \text{ between } t_i\text{=0.4s and } t_f\text{=0.5s, } \overline{v} \approx \frac{0.775\text{-}1.216}{0.500\text{-}0.400}\hat{j} = -4.41\frac{m}{s}\hat{j}$$

Average velocity =average slope of y vs. t between t_i and t_f

Why is it called "average" velocity?

How do we calculate instantaneous velocity?

Instantaneous velocity and instantaneous acceleration

- Suppose we fit the camcorder data with a smooth function,

$\vec{y}(t) = (2 - 4.9t^2)\hat{j}$

Instantaneous velocity

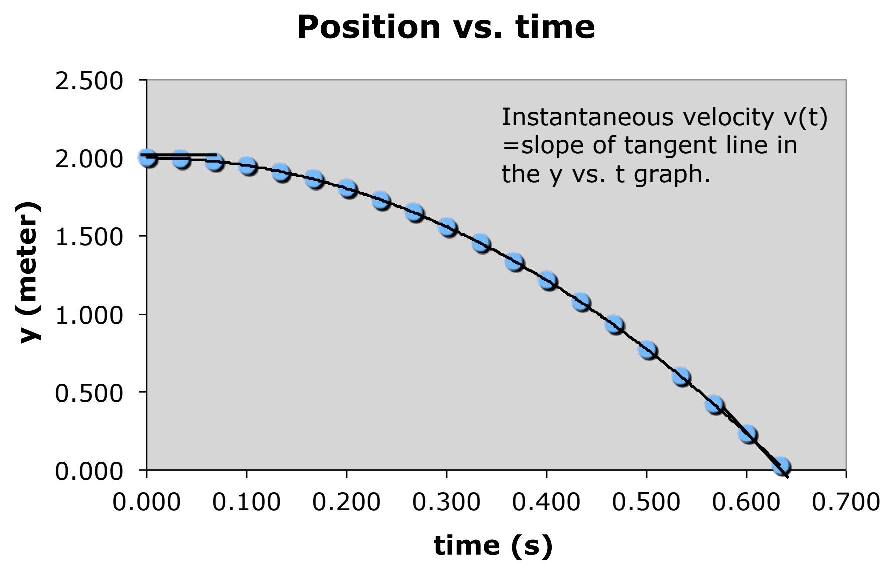
=slope of the tangent line

=first derivative of y wrt t:

$$\Rightarrow \vec{\mathrm{v}}(\mathrm{t}) \equiv \frac{\mathrm{d}\vec{\mathrm{y}}}{\mathrm{dt}} = (-9.8t\ )\hat{\mathrm{j}}$$

Instantaneous acceleration=

$$\vec{\mathrm{a}}(\mathrm{t}) \equiv \frac{d\vec{v}}{dt} = (-9.8\frac{m}{s^2})\hat{j}$$



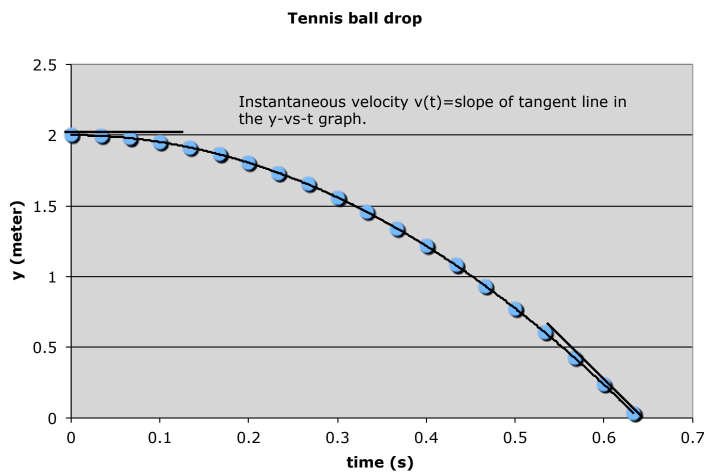
Challenge :

(1) Use these equations find the velocity at t = 0.4s and t = 0.5s.

(Compare them with the average velocity found in the previous slide.)

(2) Find t when the ball hits the ground.

Graphical interpretation of v(t) and a(t)



$$\vec{v}(t) \equiv \frac{d\vec{y}}{dt} = \textit{slope} \text{ of tangent line}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{y}}{dt^2} = \text{second derivative}$$

$$\vec{a}(t) < 0 \Rightarrow y(t) \text{ curves downward}$$

$$\vec{a}(t) > 0 \Rightarrow y(t) \text{ curves upward}$$

Suppose we know a(t), can we find v(t) and y(t)?

- Suppose a(t)=-9.8 m/s^2 (constant in time)
- From the definition of a(t)

$$a(t) \equiv \frac{dv}{dt} \Rightarrow v(t) = \int^{t} a(t')dt' = \int^{t} -9.8dt' = -9.8t + \text{constant}$$

(Meaning of the constant?) $\quad v(t) = -9.8t + v_o$

$$v(t) = \frac{dy}{dt} \Rightarrow y(t) = \int^{t} v(t')dt' = -\frac{9.8}{2}t^2 + v_o t + \text{constant}$$

$$\Rightarrow \mathrm{y(t)} = -\frac{9.8}{2}t^2 + v_o t + y_o$$

(1) Apply this equation to the falling tennis ball.

What is the value for v_o? y_o?

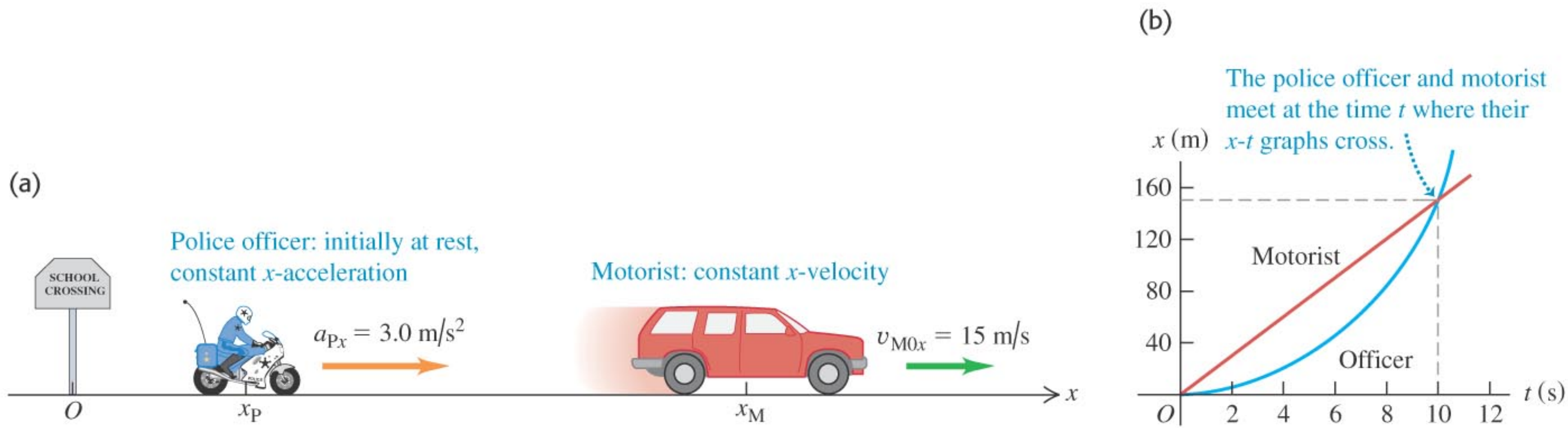
Applications of constant acceleration equations

A boy threw a ball upward with an inital velocity of 3 m/s and initial height of 1.5 m. Find $y(t)$.

*G*iven: The acceleration of the ball is $9.8 \mathrm{m/s^2}$ downward.

Applications of constant acceleration equations

- Example - A motorist travelling at a constant velocity of 15m/s passed by the officer at t=0. The officer immediately accelerates at 3m/s^{2}. When will the officer catch up to the motorist?



Non-constant acceleration

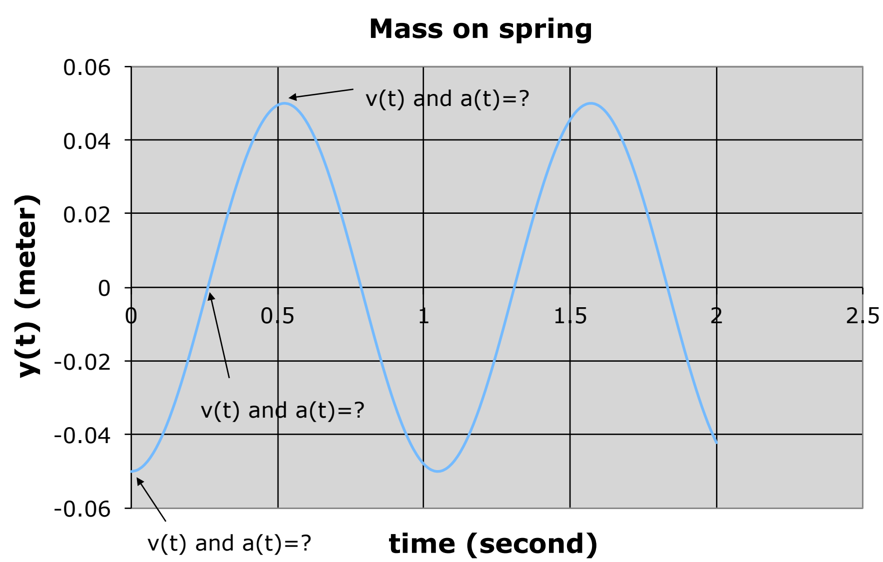
Example of non - constant acceleration - mass on a spring

$y(t) = -0.05\cos(6t)$

Calculate velocity and acceleration.

Extract v(t) and a(t) information from y(t) graph

$$y(t) = -0.05\cos(6t)$$



Summary

- If you know x(t), you have complete information on the motion
- If you know v(t) only, you need to know one position (typically the initial position, x_o) in order to find x(t).
- If you know a(t) only, you need to know one velocity and one position (typically initial velocity, v_o, and initial position, x_o) in order to find x(t).