Ph170- General Physics I

Ch. 2

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Vectors, Scalar Product, Cross Product

- What is a vector?
- What kind of measurements in physics require the use of vectors?

Activity for Vector Representation

(Work with your group - 5 minutes
Class discussion – 5 minutes)

- Example: A man walks 5.0 m eastward and then 6.0 m at 30^o north from east.

(a) How would you draw the individual displacement vector and the net displacement vector in a graph? (graphical representation of vector)

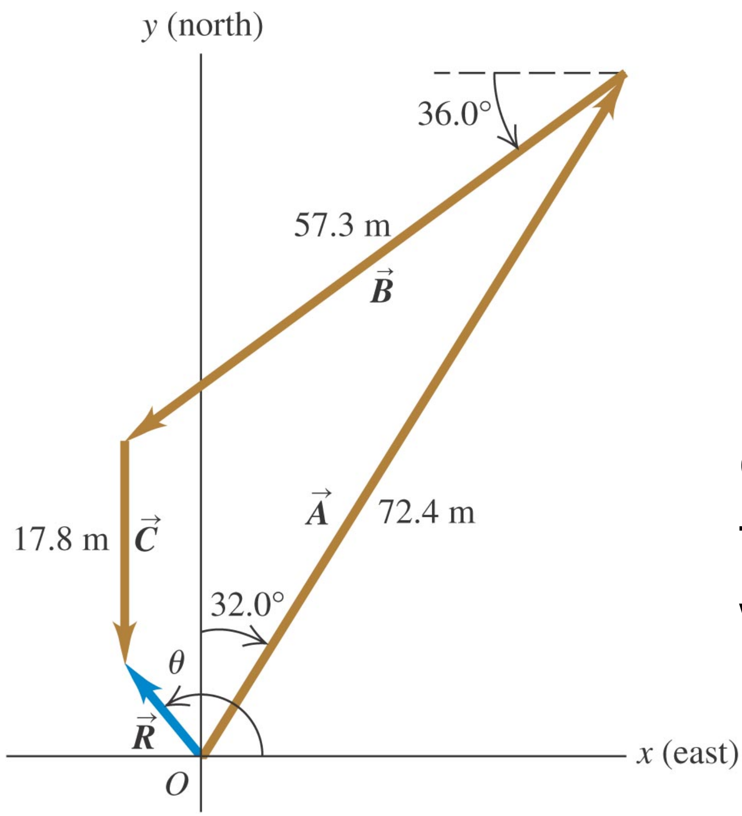
(b) How far is he away from his starting point (magnitude of a vector)?

(c) What is his bearing? (How many degree north from east?)

(d) How did you “add” the two displacement vectors to get the net displacement vector?

Activity: Vector Addition

Geometric Method



Question

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

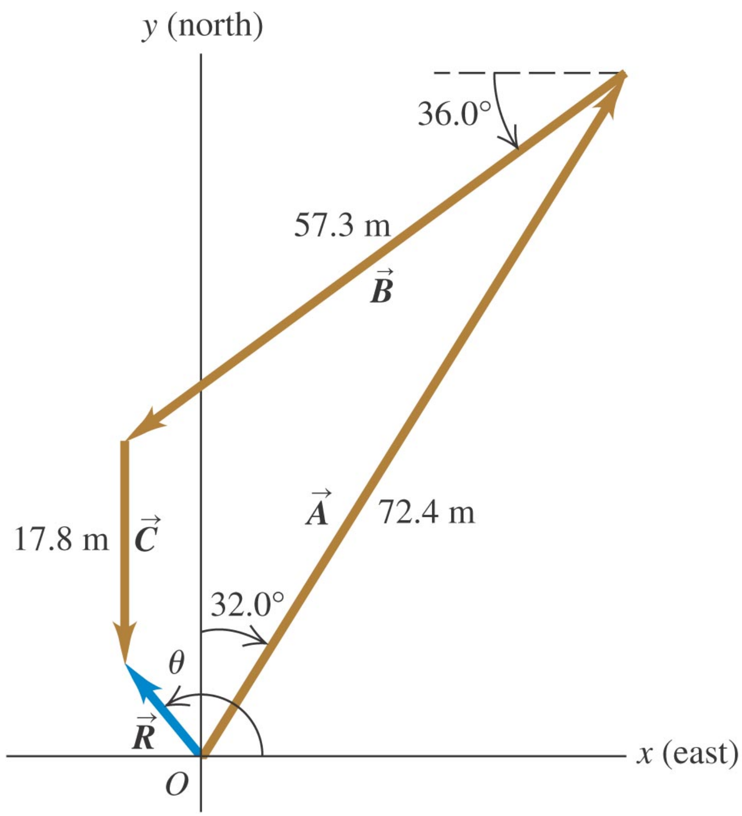
(a) Find the magnitude of $\vec{R}$.

(b) Find the angle θ.

Discuss with your group to come up with an approach to find the answer. What tools would you need?

Vector Addition-Component Method

Geometric Method



$\hat{i}$ is the unit vector representing positive x-direction

$\hat{j}$ is the unit vector representing positive y-direction

Question to students:

$(a)\ \vec{A} = 72.4\cos(32.0^o)\hat{i} + 72.4\sin(32.0^o)\hat{j}$

OR

$(b)\ \vec{A} = 72.4\sin(32.0^o)\hat{i} + 72.4\cos(32.0^o)\hat{j}$

$(a)\ \vec{B} = 57.3\cos(36.0^o)\hat{i} + 57.3\sin(36.0^o)\hat{j}$

OR

$(b)\ \vec{B} = 57.3\sin(36.0^o)\hat{i} + 57.3\cos(36.0^o)\hat{j}$

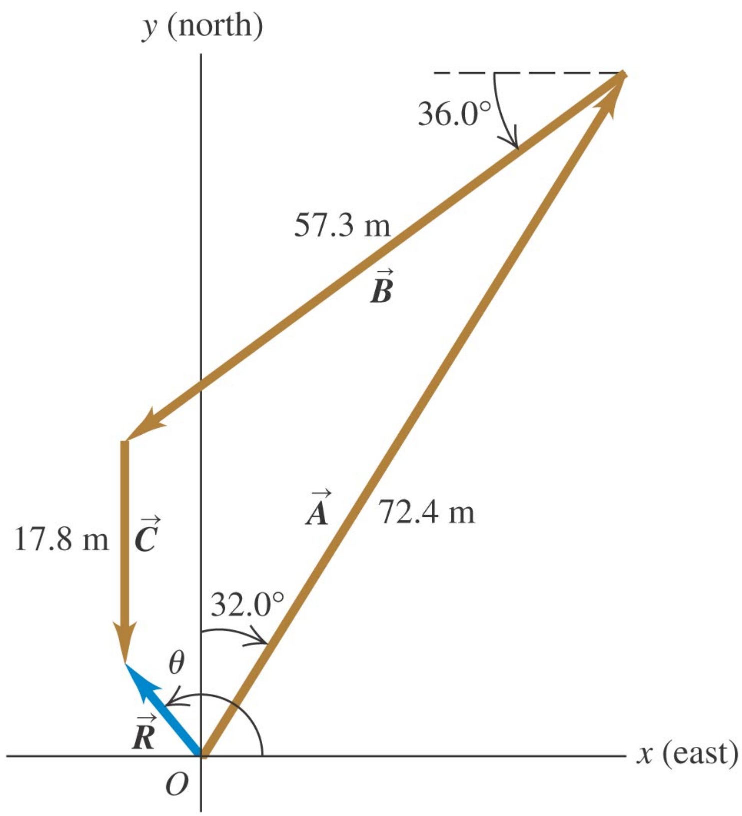
$(a)\ \vec{C} = 17.8\hat{j}$

OR

$(b)\ \vec{C} = -17.8\hat{j}$

Vector Addition - answer

Geometric Method



Component Method

(a) *Express* $\vec{A}, \vec{B},$ *and* $\vec{C}$ *in component form*

(b) *Add* $\vec{A}, \vec{B},$ *and* $\vec{C}$ *to get* $\vec{R}$

$$\vec{A} = (72.4\sin 32^o)\hat{i} + (72.4\cos 32^o)\hat{j}$$
$$= 38.4\hat{i} + 61.4\hat{j}$$
$$\vec{B} = (-57.3\cos 36^o)\hat{i} + (-57.3\sin 36^o)\hat{j}$$
$$= -46.4\hat{i} - 33.7\hat{j}$$
$$\vec{C} = -17.8\hat{j}$$
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = (38.4 - 46.4)\hat{i} + (61.4 - 33.7 - 17.8)\hat{j}$$
$$= -8.0\hat{i} + 9.9\hat{j}$$
$$|\vec{R}| = \sqrt{(-8.0)^2 + (9.9)^2} \approx 13$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{9.9}{-8.0} \Rightarrow \theta = -51^o, 129^o$$

Pick the correct answer for θ.

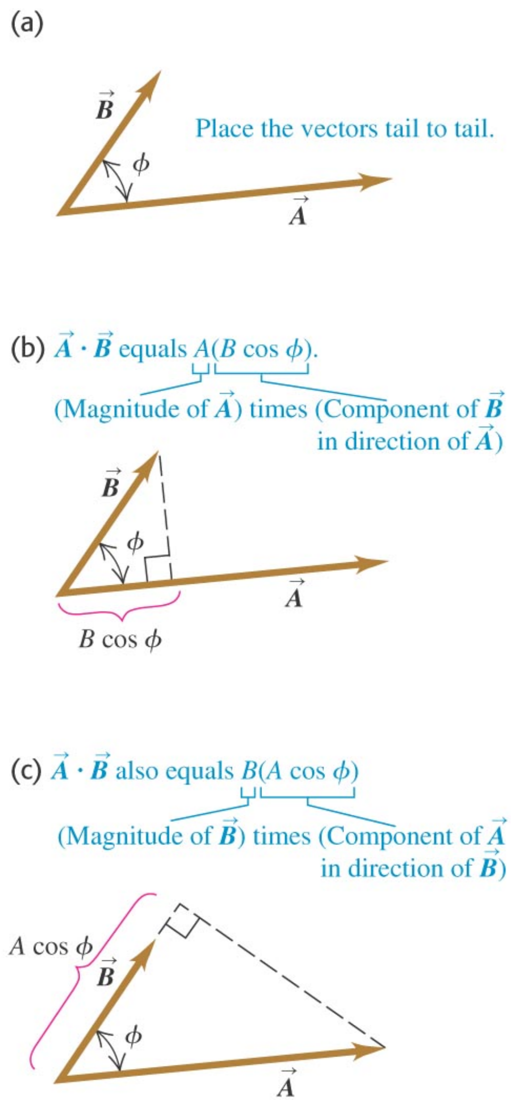
- How can you tell if two vectors are perpendicular to each other if the vectors are expressed in component form?

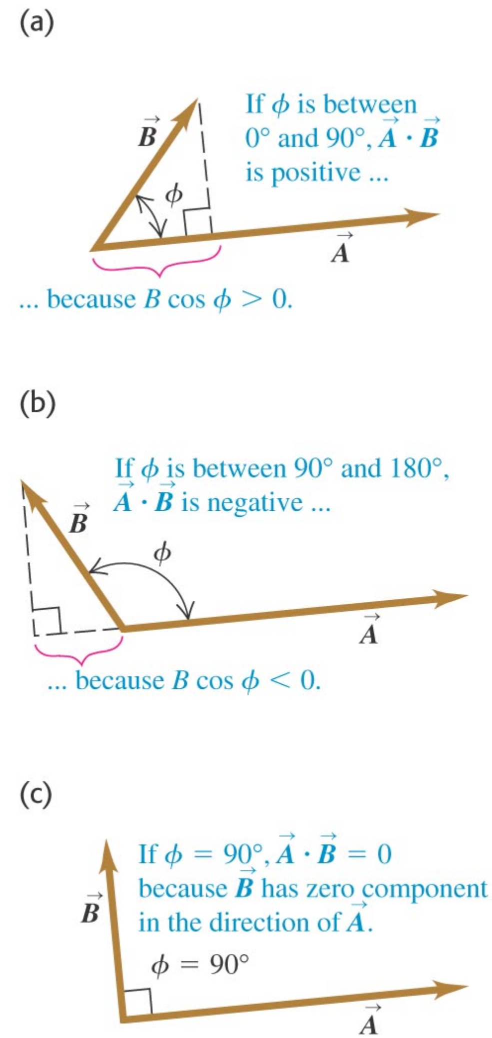
(see next slide)

Scalar product of two vectors

$$Definition: \vec{A} \bullet \vec{B} \equiv |A||B|\cos\phi = \text{ a scalar (i.e. a number not a vector)}$$

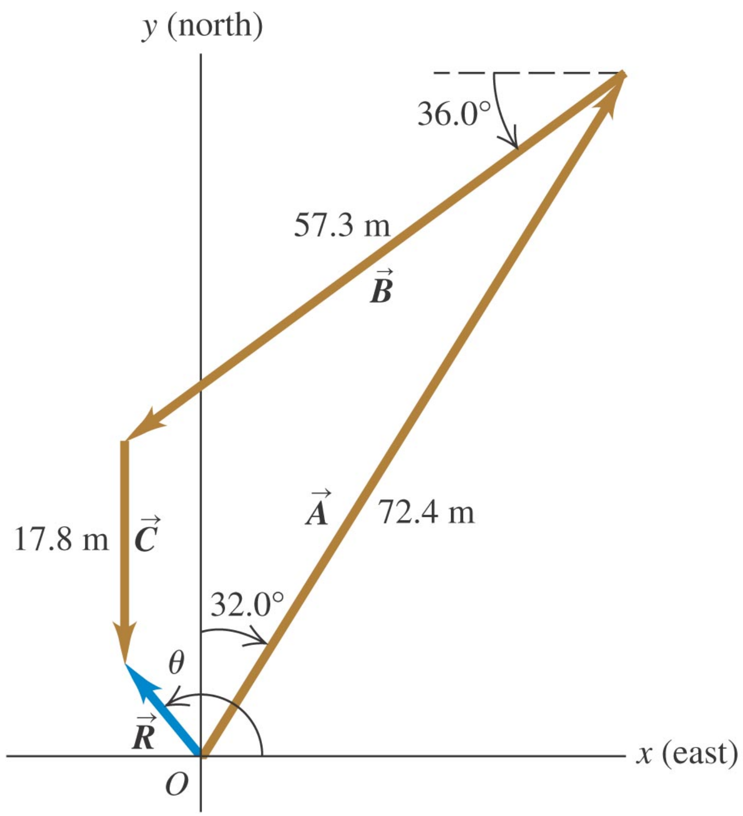
- Also called "dot product."
- These Figures show the **geometric interpretation** of scalar product of two vectors.
- **Scalar product will be used when we study work and energy**.





Activity: Scalar Product

Geometric Method



Compute the scalar product

$$\vec{A} \bullet \vec{B}$$

How to calculate the scalar product of two vectors when then angle between them is not given but the the vectors are given in component form?

$Given: \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Compute $\vec{A} \bullet \vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \bullet \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$= A_xB_x\hat{i} \bullet \hat{i} + A_yB_y\hat{j} \bullet \hat{j} + A_zB_z\hat{k} \bullet \hat{k}$

$+A_xB_y\hat{i} \bullet \hat{j} + other$ cross terms ..

$Note: \hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$

$\hat{i} \bullet \hat{j} = 0$

$\Rightarrow \vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$

In particular: $\vec{A} \bullet \vec{A} = A_xA_x + A_yA_y + A_zA_z = \left|\vec{A}\right|^2$

$Question1:$ Find the scalar product of $\vec{A} = 1\hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$Question2:$ Find the angle between $\vec{A}$ and $\vec{B}$

Find angle between two vectors when the vectors are given in component form (answer)

$$Question1:\ \text{Find the scalar product of } \vec{A} = 1\hat{i} + 2\hat{j} \text{ and } \vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$Question2:\ \text{Find the angle between } \vec{A} \text{ and } \vec{B}$$

$$Solution:$$

$$\vec{A} \bullet \vec{B} = 1*3 + 2*4 + 0*5 = 11$$

$$11 = \vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\theta = \sqrt{1^2 + 2^2}\sqrt{3^2 + 4^2 + 5^2}\cos\theta = 15.8\cos\theta$$

$$\Rightarrow \cos\theta = \frac{11}{15.8} = 0.696 \Rightarrow \theta \approx 45.9^o$$

Practical applications?

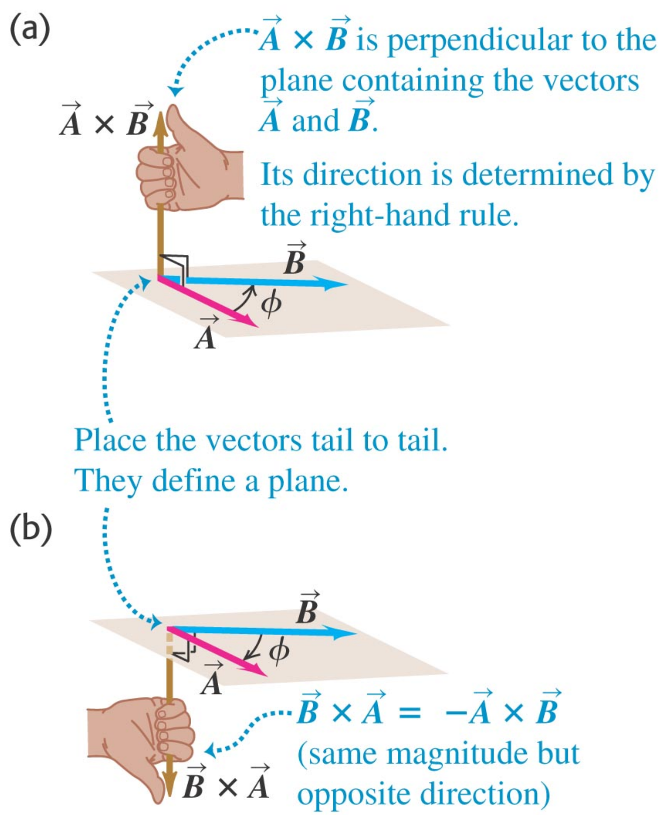
Vector product of two vectors

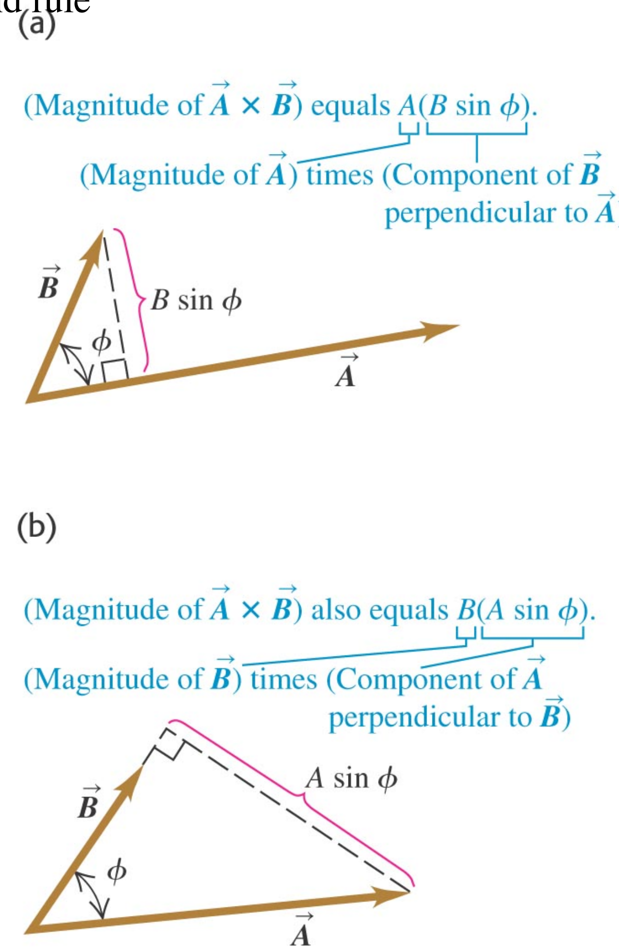
$$Let\ \vec{C} = \vec{A} \times \vec{B}$$

$$Definition: \left|\vec{C}\right| \equiv \left|\vec{A}\right|\left|\vec{B}\right| \sin\phi\ \&$$

Direction of $\vec{C}$ is given by the "right-hand rule"

- Also called "cross product."
- These Figures illustrate the vector cross product.
- **Vector cross product will be used when we study torque and rotational motion.**





Calculation of Vector product - when the vectors are given in component form

$$Given: \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \text{ and } \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\text{Compute } \vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\bullet\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\ \right)$$

$$= A_xB_x(\hat{i}\times\hat{i}) + A_yB_y(\hat{j}\times\hat{j}) + A_zB_z(\hat{k}\times\hat{k})$$

$$+A_xB_y(\hat{i}\times\hat{j}) + A_yB_x(\hat{j}\times\hat{i})other \text{ cross terms ..}$$

$$Note: \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\hat{i}\times\hat{j} = \hat{k}; \quad (\hat{j}\times\hat{i}) = -\hat{k}; \text{ etc.}$$

$$\Rightarrow \vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

$$\text{In particular: } \vec{A}\times\vec{A} = 0 \text{ and } \vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$$

$$Question1: \text{ Find the vector product of } \vec{A} = 1\hat{i} + 2\hat{j} \text{ and } \vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$Question2: \text{What is the area of the parallelogram formed by } \vec{A} \text{ and } \vec{B}?$$

Calculation of Vector product - when the vectors are given in component form

$Question1:$ Find the vector product of $\vec{A} = 1\hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$Question2:$ What is the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$?

$Solution:$

$$\vec{A} \times \vec{B} = 1*4(\hat{i} \times \hat{j}) + 1*5(\hat{i} \times \hat{k}) + 2*3(\hat{j} \times \hat{i}) + 2*5(\hat{j} \times \hat{k})$$

$$= 1*4(\hat{k}) + 1*5(-j) + 2*3(-\hat{k}) + 2*5(\hat{i})$$

$$= 10\hat{i} - 5\hat{j} - 2\hat{k}$$

$$Area = |\vec{A} \times \vec{B}| = \sqrt{10^2 + (-5)^2 + (-2)^2} = \sqrt{129} \approx 11.4$$

Both dot-product and cross-product will be used in this class.