Chapter 16

Mechanical Waves

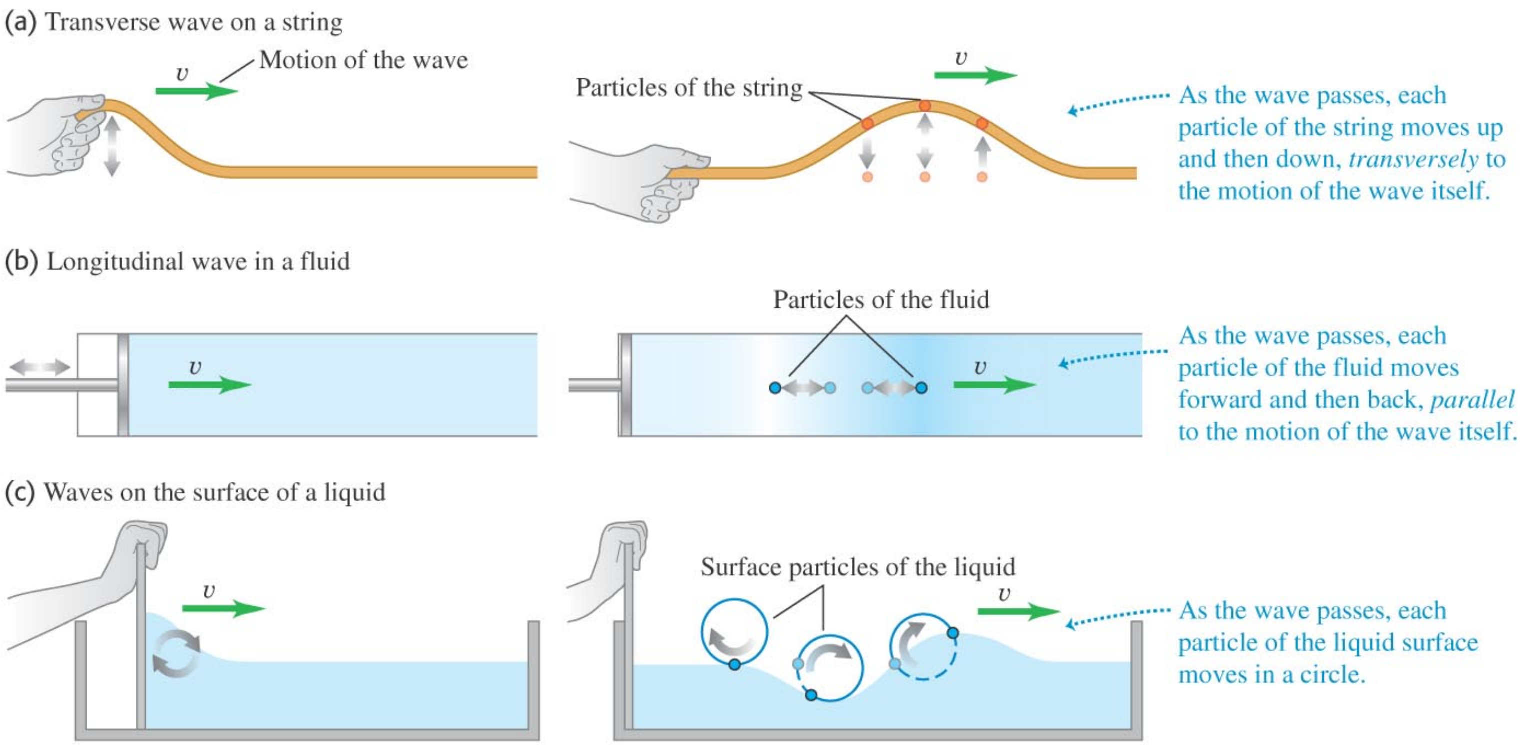
Modified by P. Lam 8/3/2018

Topics for Chapter 16

- Properties of mechanical waves
- Mathematical description of traveling wave
- Energy carried by in traveling wave
- Superposition of waves
- Standing waves

What is a mechanical wave motion?

- Create a **disturbance** in one region of **medium**, the **propagation of this disturbance** to other regions of the medium = **mechanic wave**
- Note: the **medium** must be **elastic** (it has some kind of restoring force) and has **inertia** (mass)
- See examples below (identify the medium and the restoring force(s)):

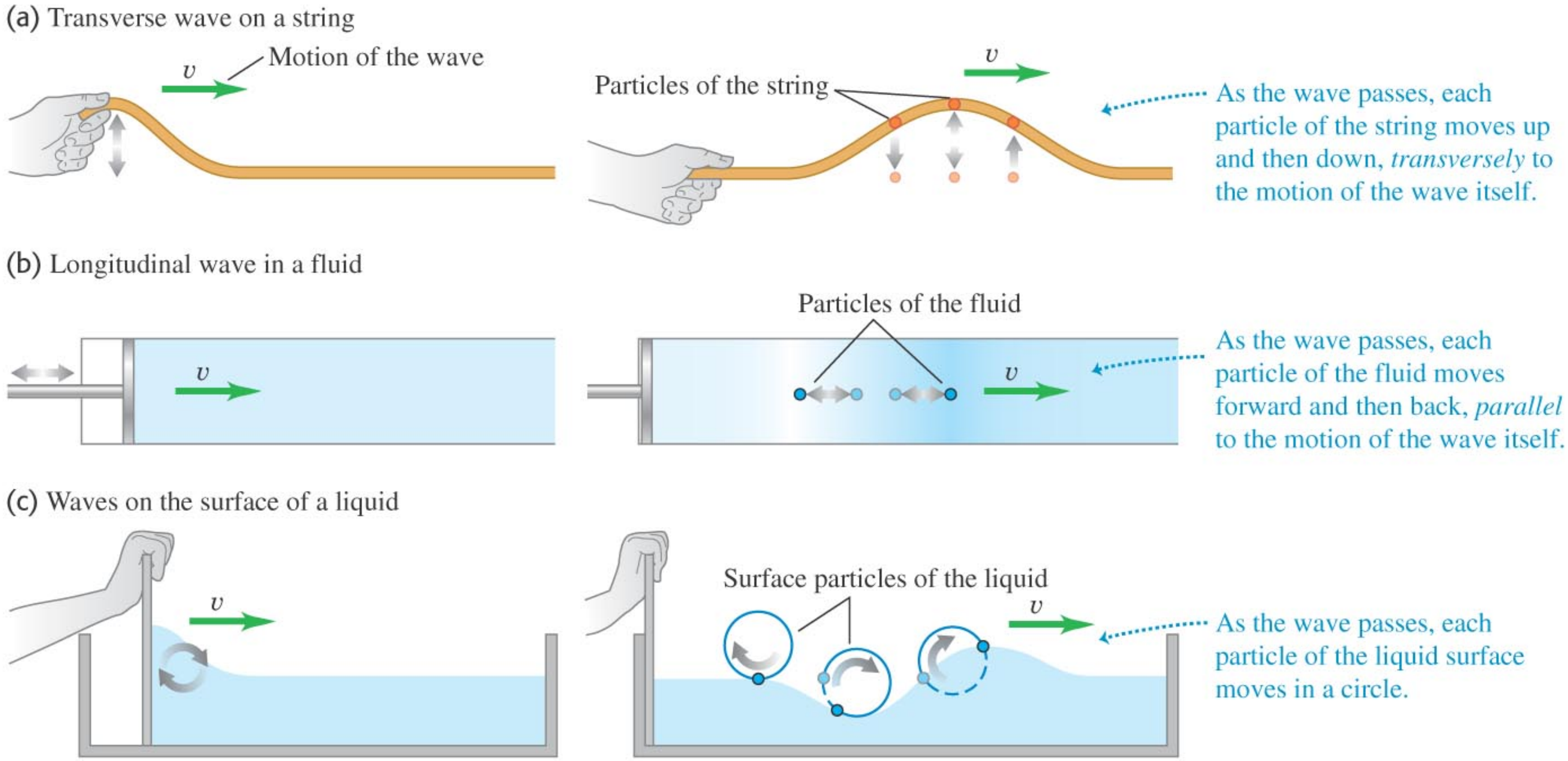


Do all waves require a medium to travel?

- Almost all waves require a medium to travel.
- Exception: Electromagnetic waves (radio wave, microwave, visible light, ultraviolet, X-ray, gamma rays, etc) can travel through empty space (vacuum)

Types of waves

- **Longitudinal waves** - Waves that have disturbance parallel to the direction of wave propagation are called longitudinal wave
- **Transverse waves** -Waves that have disturbance perpendicular to the direction of propagation.
- Identify the waves below as longitudinal or transverse.



Wave speed

- Wave speed depends on the properties of the medium

$$\text{v}=\sqrt{\frac{\text{Magnitude of restoring force}}{\text{Inertia of the medium}}}$$

e.g. wave speed of a rope under tension:

$$\text{v}=\sqrt{\frac{\text{F}}{\mu}};\quad F = tension,\quad \mu = \frac{mass}{length} = \frac{kg}{m}$$

$$\text{check unit: F=kg}\frac{\text{m}}{\text{s}^2}$$

$$\sqrt{\frac{\text{F}}{\mu}} = \sqrt{\frac{kgm}{s^2} \bullet \frac{m}{kg}} = \frac{m}{s}$$

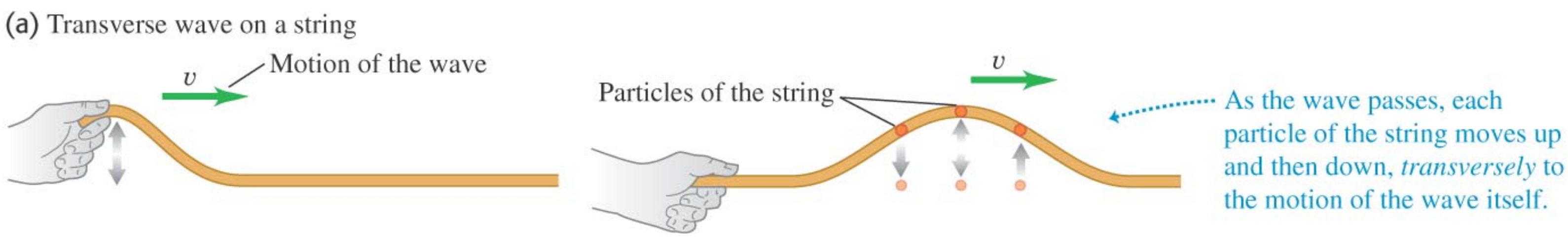
Other examples:

(1) Sound velocity depends on the medium: air vs. solid (both magnitude of restoring force and inertia are different in these two media).

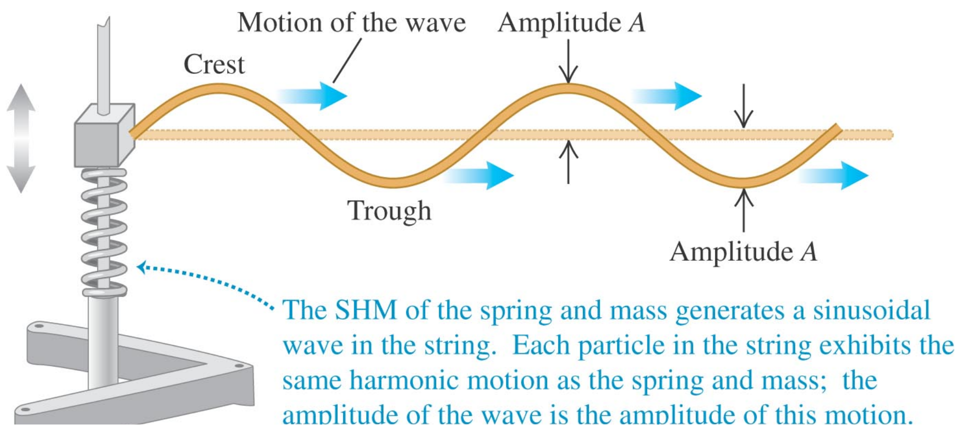
(2) Longitudinal earthquake waves (P-wave) is faster than the transverse earthquake wave (S-wave) - the magnitude of compressional restoring force (P-wave) is greater than the shear restoring force (S-wave).

Travelling Waveforms - examples

- Traveling wave pulse - generated by a pulsed driving force



- Periodic traveling wave (harmonic wave) - generated by a continuous oscillating driving force



Mathematical Description of traveling waves

Mathematically, the amplitude of a wave is described by:

$\vec{A}(x,t) = f(x,t)\hat{n}$

(a)The function f(x,t) describes the shape of the wave at various time.

(b)To describe a travelling wave moving toward the positive x-direction,

f has the following form: f(x,t)=f(x-vt)

e.g. for a periodic traveliing wave, $\mathrm{f}(x,t) = A\cos(x - vt) + B\sin(x - vt)$

(c) How do you describe a travelling wave movng toward the negative x-direction?

(e) The direction of the amplitude is denoted by the unit vector $\hat{\mathrm{n}}$.

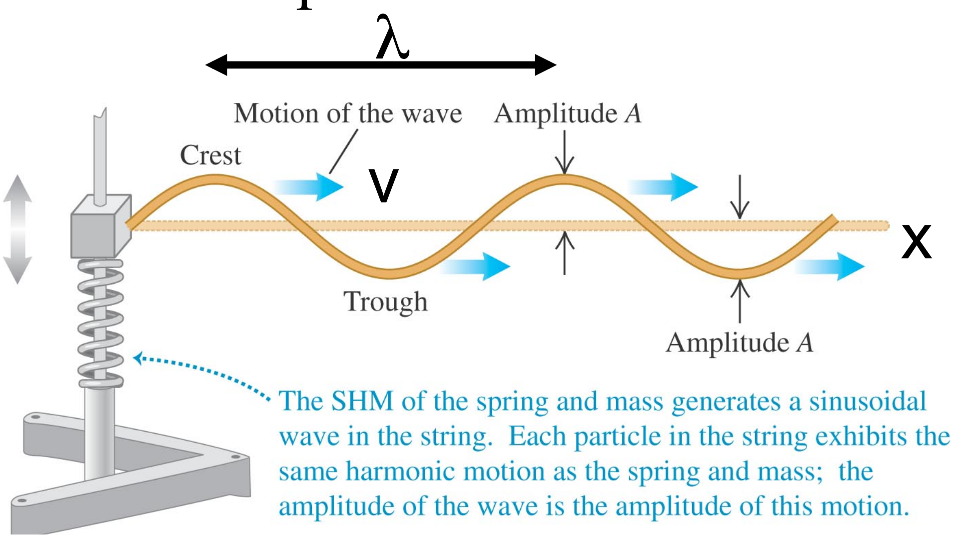
Example: If the wave is longitudinal and it travels along the x-direction,then $\hat{\mathrm{n}}=\hat{\mathrm{i}}$.

If the wave is transverse and and it travels along x-direction, then $\hat{\mathrm{n}}=\hat{\mathrm{j}}$ or $\hat{\mathrm{n}}=\hat{\mathrm{k}}$

Q. Write a general expression for a longitudinal wave travelling in the negative y-direction with a wave speed of 10 m/s.

Transverse Periodic Travelling Wave

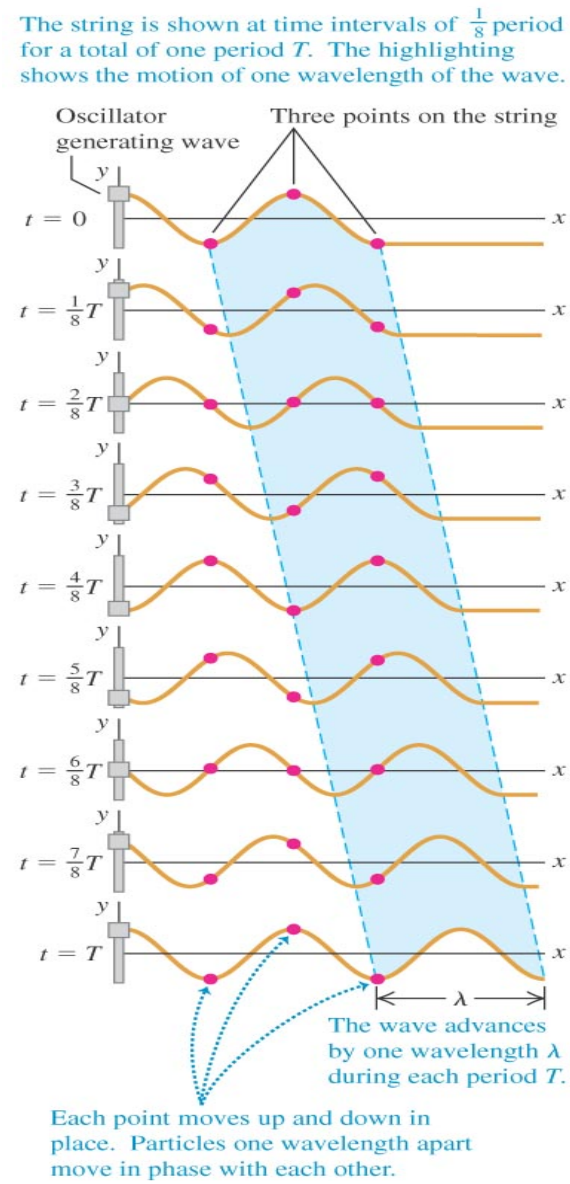
- A detailed look at transverse periodic traveling waves to extract parameters.



$$\vec{A}(x,t) = A\hat{j}\cos\left[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right]$$

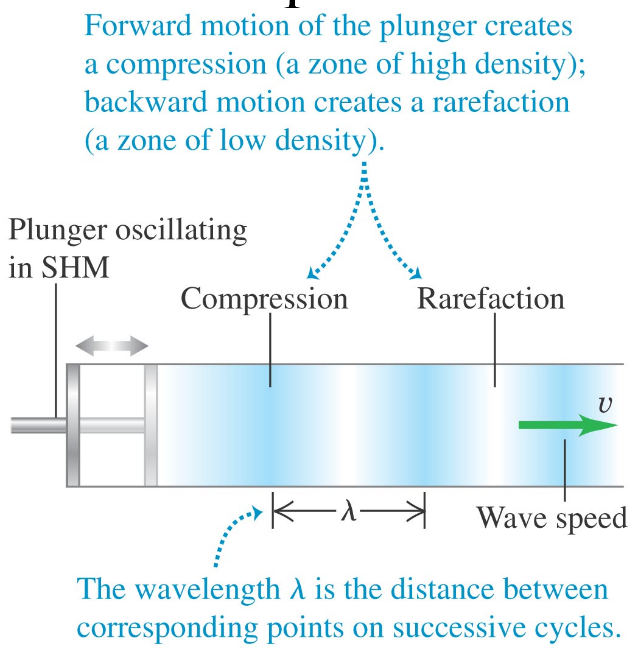
$$= A\hat{j}\cos\left[\frac{2\pi}{\lambda}\left(x - \frac{\lambda}{T}t\right)\right]$$

$$= A\hat{j}\cos\left[\frac{2\pi}{\lambda}(x - vt)\right]; \quad v = \frac{\lambda}{T} = \lambda f$$

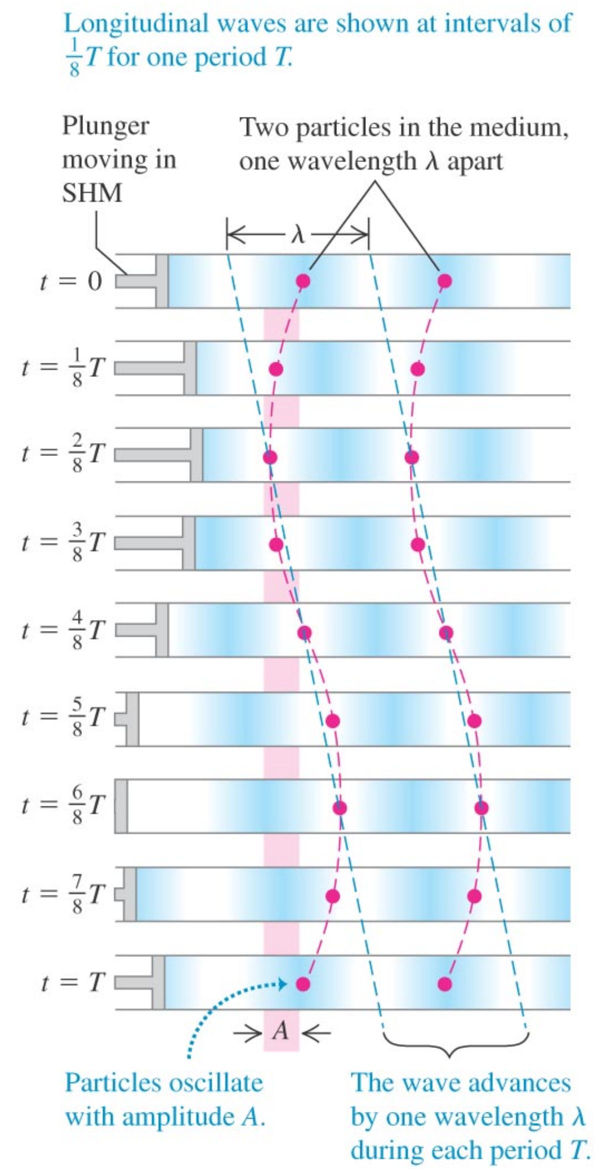


Longitudinal Periodic Travelling Waves

Refer to Example 15.1.



$$\vec{A}(x,t) = A\hat{i}\cos\left[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right]$$



Identify traveling periodic wave parameters

Given the amplitude of a wave as a function of y & t:

$\vec{A}(y,t)=A\hat{i}\cos(4y + 6t)$

(a) Which direction is the wave travelling toward?

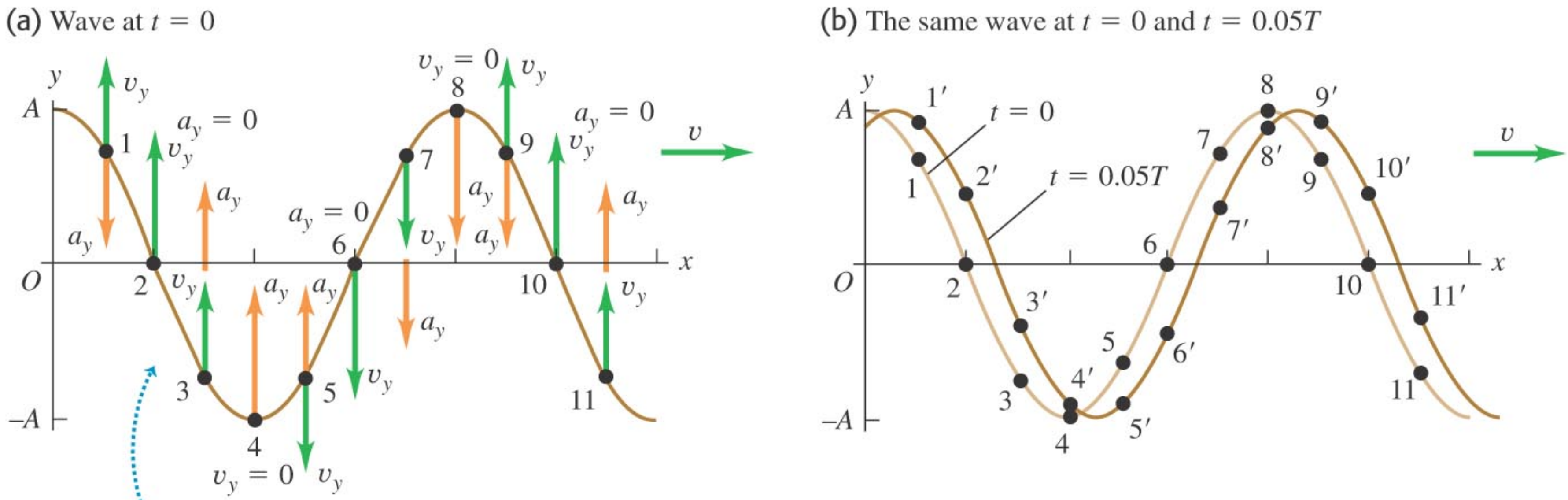
(b) Is it a longitudinal or transverse wave?

(c) What is its wavelength, its period, and its wave velocity?

(d) Write an function that describes a longitudinal travelling periodic wave moving toward the negative x-direction with wavelength = 3 m, wave speed = 6 m/s and maximum amplitude = 0.2m.

Particle velocity vs. wave velocity

- Consider a transverse wave on a rope



• Acceleration a_y at each point on the string is proportional to displacement y at that point.
• Acceleration is upward where string curves upward, downward where string curves downward.

$$\vec{A}(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x-v)t\right]\hat{j}$$

$$\text{Particle velocity} = \frac{\mathrm{d}\vec{A}(x,t)}{dt} = A\frac{2\pi}{\lambda}v\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\hat{j}$$

$$\Rightarrow \text{Maximum particle speed} = |A\frac{2\pi}{\lambda}v| = |A2\pi f| = |A\frac{2\pi}{T}|$$

Energy (or Power) carried by a wave

As the wave travels along the medium, it transports the energy that it carries.

$$\frac{\text{transported kinetic energy}}{\text{time}} = \frac{1}{2}\left(\frac{\text{mass}}{\text{length}}\right)\left(\text{particle velocity}\right)^2 \bullet (\text{ wave velocity})$$

Example: Periodic wave

$$= \frac{1}{2}\mu\left(A\omega\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2 \bullet v$$

$$= \frac{1}{2}\mu v\omega^2 A^2\left(\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2$$

The $\frac{\text{transported potential energy}}{\textit{time}}$ give exactly the same term

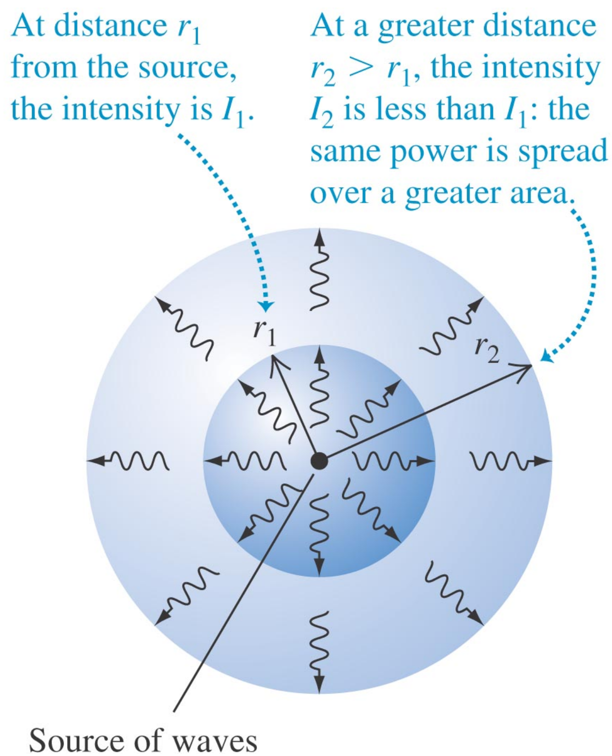
$$\Rightarrow \text{Total instantaneous power=P=}\mu v\omega^2 A^2\left(\sin\left[\frac{2\pi}{\lambda}(x-v)t\right]\right)^2$$

$$\Rightarrow \text{Average power=}\frac{1}{2}\mu v\omega^2 A^2 \quad \text{(because sine square = 1/2)}$$

Wave intensity

- Go beyond the wave on a string and visualize, say … a sound wave spreading from a speaker. That wave has intensity dropping as $1/r^2$ due to conservation of energy.

$$\text{Intensity} = \frac{\text{Energy/time}}{\text{area}} = \frac{Power}{4\pi r^2}$$



Exercise

Given the Earth-Sun distance = d =1.5×10^{11} m

And the Earth radius = R =6.4×10^{6} m.

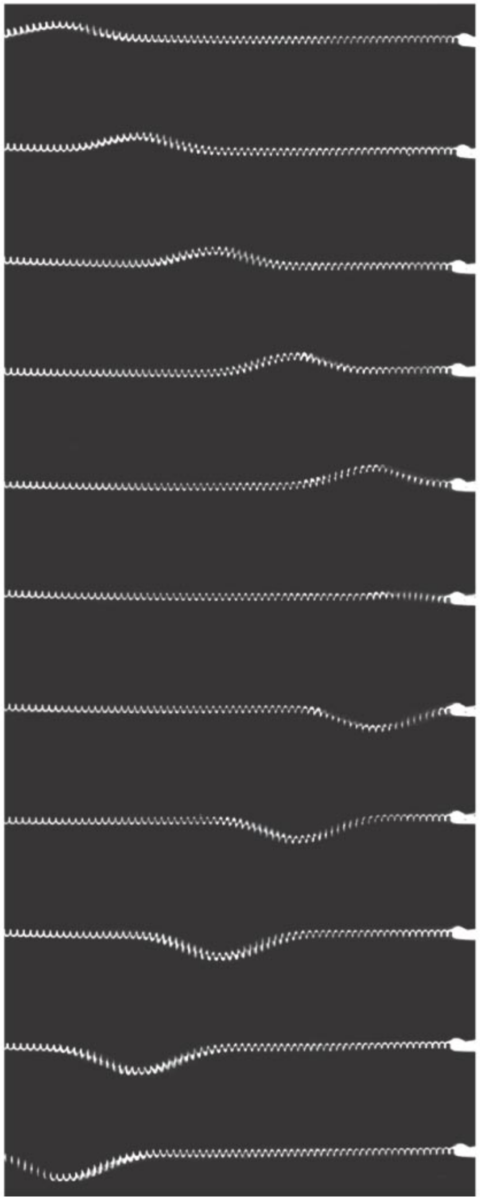
Estimate the percentage of the Sun's power is

absorbed by the Earth (assuming all light wave hitting the Earth are absorbed, i.e. no reflected waves)

Non-dispersive vs dispersive medium

- A non-dispersive medium is one where all wavelengths have same wave velocity. For example: ALL electromagnetic waves traveling in vaccuum have the same wave velocity ~ $3x10^8$ m/s). Another example is wave along a tight rope is is *approximately* non-dispersive,

$$v = \sqrt{\frac{F}{\mu}} \quad \text{independent of } \lambda$$



In a non-dispersive medium, a wave pulse does not disperse (spread out) as it travels, see figure on the right. Mathematical explanation on next slide

Mathematical connection between wave pulse and periodic waves

A wave pulse can be thought of as a superposition of many periodic waves with various wavelengths and frequencies - Fourier’ s Theorem.

Any fucntion f(x) can be expressed as a sum of cosine and sine fucntions of various wavelengths:

$$f(x) = \sum_{\lambda} A_{\lambda} \cos\left(\frac{2\pi}{\lambda} x\right) + B_{\lambda} \sin\left(\frac{2\pi}{\lambda} x\right)$$

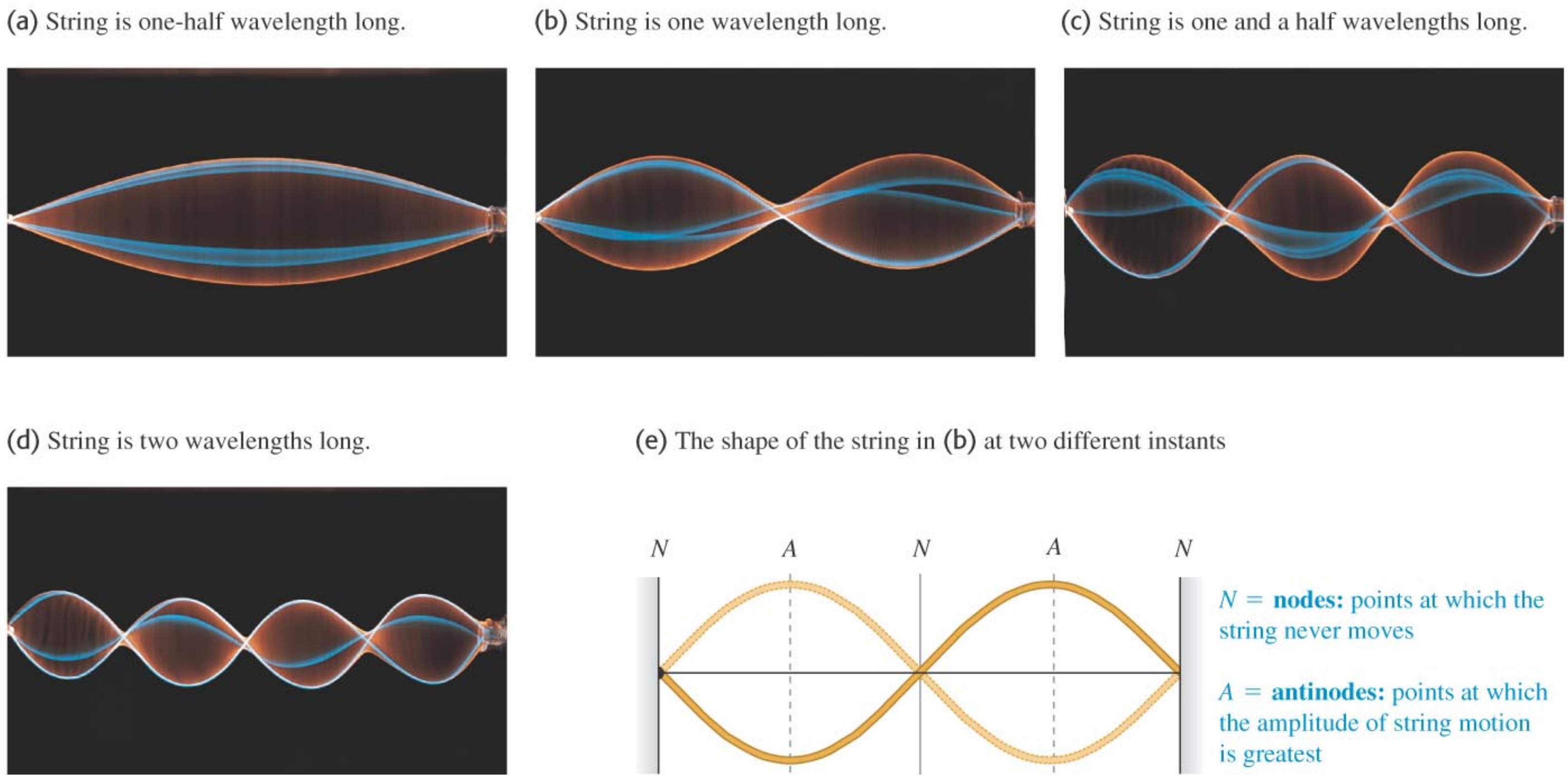
In a non-dispersive medium, all wavelengths have the same Velocity => the shape of the pulse is unchanged with time in analogous to a marching band where every member marches at the same velocity

Non-dispersive vs dispersive medium

- Almost all media are dispersive (the only true non-dispersive medium is the vaccum!)
- For example: Electromagnetic waves (light) travels inside a piece glass have wave velocities depending on the wavelength of light; red and blue light have different wave velocities => white light entering a piece of glass (a prism) will disperse into different colors.
- Dispersion of wave pulse prevents digital signal to travel long distance; eventually the pulse shape will be distorted.

Standing waves on a string - resonance frequencies

- Although a wave traveling along a string can have any wavelength, a string with both ends fixed have certain "preferred" wavelengths (or preferred frequencies, called resonance frequencies) - (e.g. guitar).

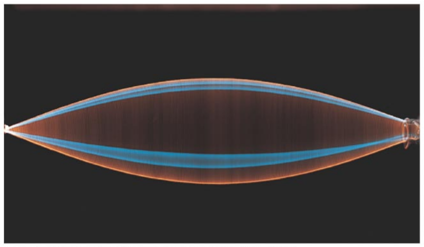


Calculation of resonance wavelength (or frequencies)

Given : The length of a guiter string is 0.3m.

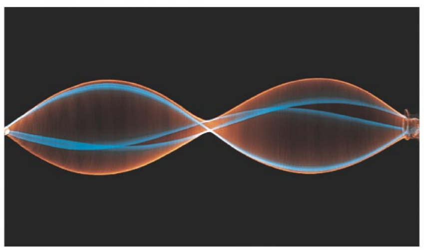
Find the first three "resonance" wavelengths.

(a) String is one-half wavelength long.



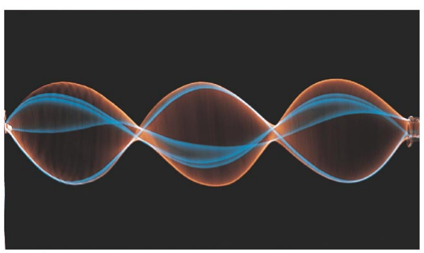
$$\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L = 2(0.3) = 0.6m$$

(b) String is one wavelength long.



$$\lambda_2 = L \Rightarrow \lambda_2 = 0.3m$$

(c) String is one and a half wavelengths long.



$$\frac{3\lambda_3}{2} = L \Rightarrow \lambda_3 = \frac{2}{3}L = \frac{2}{3}(0.3) = 0.2m$$

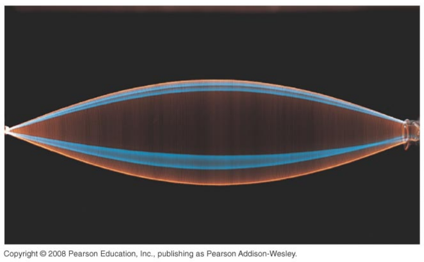
Calculation of resonance wavelength (normal modes)

Given : The length of a guiter string is 0.3m.

Find the first three "resonance" frequencies.

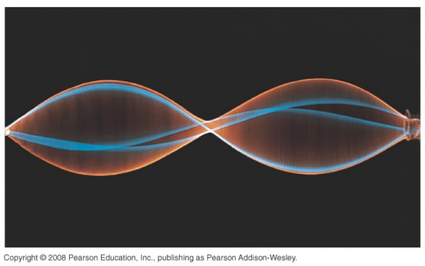
What additional information do we need?

(a) String is one-half wavelength long.



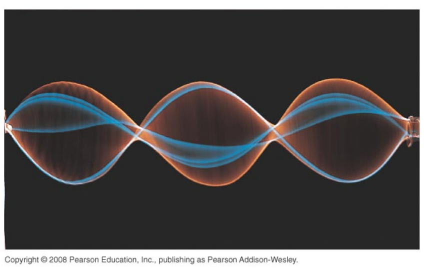
$$\frac{\lambda_1}{2} = L \Rightarrow \lambda_1 = 2L = 2(0.3) = 0.6m$$

(b) String is one wavelength long.



$$\lambda_2 = L \Rightarrow \lambda_2 = 0.3m$$

(c) String is one and a half wavelengths long.



$$\frac{3\lambda_3}{2} = L \Rightarrow \lambda_3 = \frac{2}{3}L = \frac{2}{3}(0.3) = 0.2m$$