Chapter 14

Fluid Mechanics

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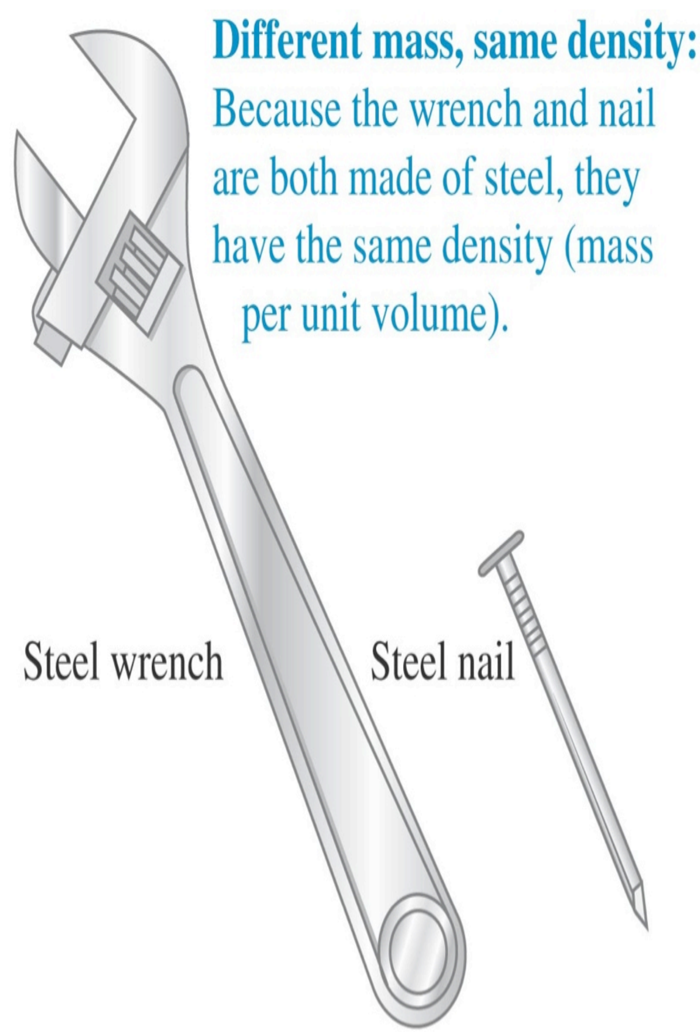
Learning Goals for Chapter 14

- Understand the difference between mass and density
- Study pressure in a fluid at rest – hydrostatics
- Study buoyancy
- Study fluid in motion - hydrodynamics

Density of a substance - definition

- Definition: Density = mass per volume; it is an intrinsic property of the substance, it does not depends on the size or shape of the object.

- Density values are sometimes divided by the density of water to be tabulated as an unit-less quantity called specific gravity. Example: A substance whose specific gravity =2 => its density is twice that of water.



Densities of common substances—Table 14.1

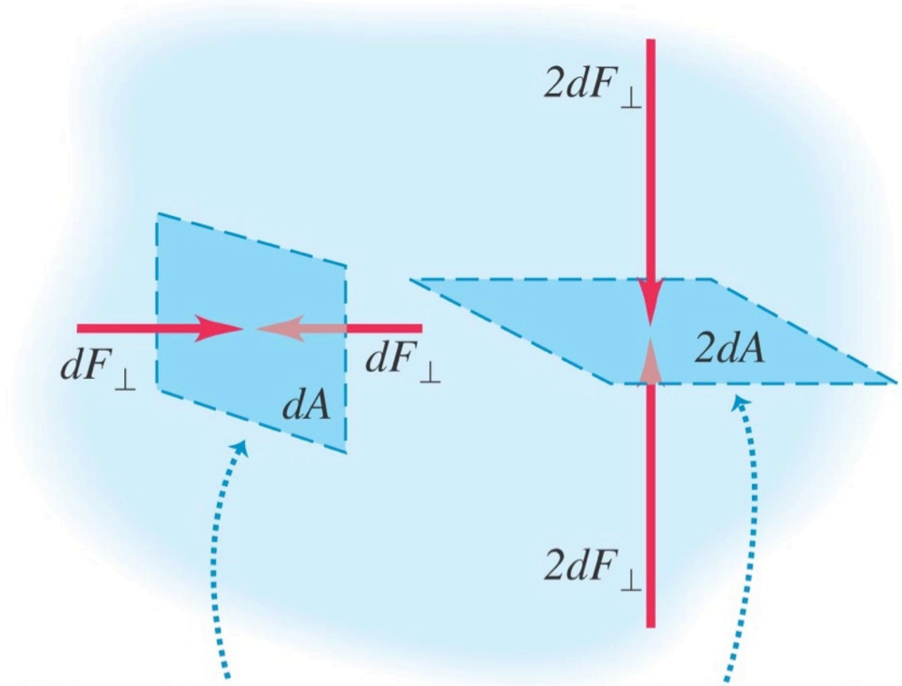
Table 14.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

Pressure in a fluid

- We know that an object floats in a fluid if the object's density is less than that of the fluid. => There must a force supporting the object to balance of the way of the object. Where does that force from?
- Answer: The force of fluid molecules striking the object (fluid pressure)
- Pressure = force per area.
- SI unit for pressure= N/ m^2=Pascal.



Although these two surfaces differ in area and orientation, the pressure on them (force divided by area) is the same.

Note that pressure is a scalar—it has no direction.

Fluid at rest - hydrostatic pressure

Although fluid pressure intrinsically depends on the speed of the fluid molecules, under static equilbirum with gravity,

the fluid pressure depends only on the fluid density, its depth, and g.

Consider a column of fluid from surface to a depth d.

Let p_o = pressure at the surface due to air molcules

Let p = pressure at the depth d due to fluid molcules

Let ρ=density of fluid (assumed ρ=constant, independ of depth; approximated valid for liquids, notvalid for gas)

net $\vec{F}=0 \Rightarrow pA - p_o A - (\rho A d)g = 0 \Rightarrow \mathrm{p} = \mathrm{p}_o + \rho g d$

$\Rightarrow$ fluid pressure increases with depth

If density does depend on depth, then need calculus:

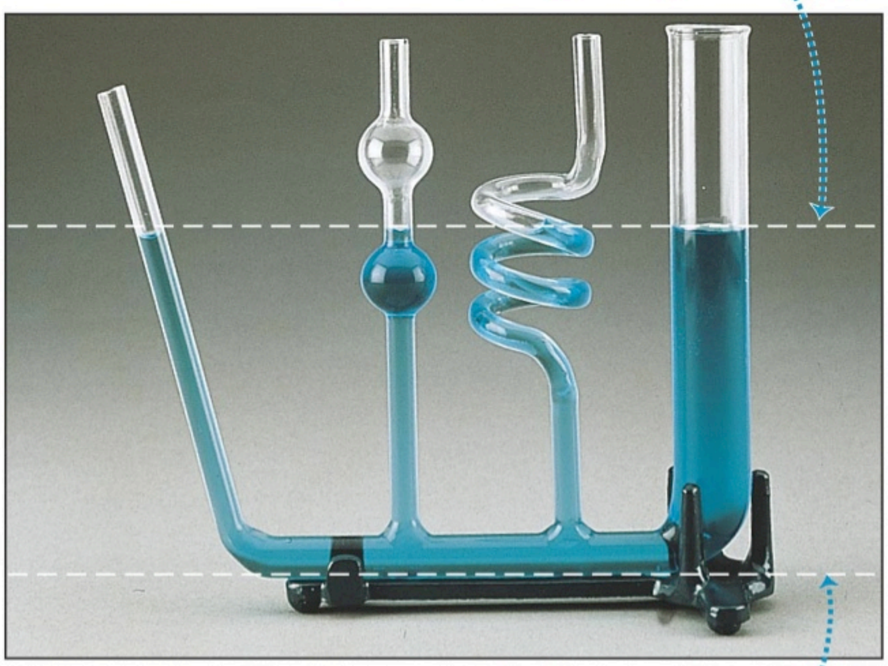
$\Rightarrow$ $-\mathrm{dpA} = \mathrm{mg} = (\rho \mathrm{Ady})\mathrm{g} \Rightarrow \dfrac{dp}{dy} = -\rho(y)g$

$\Rightarrow$ Pressure decreases with height (same as pressure increases with depth)

Pressure, depth, and Pascal's Law

- In a uniform fluid : "Same depth => same pressure"

The pressure at the top of each liquid column is atmospheric pressure, p_0.

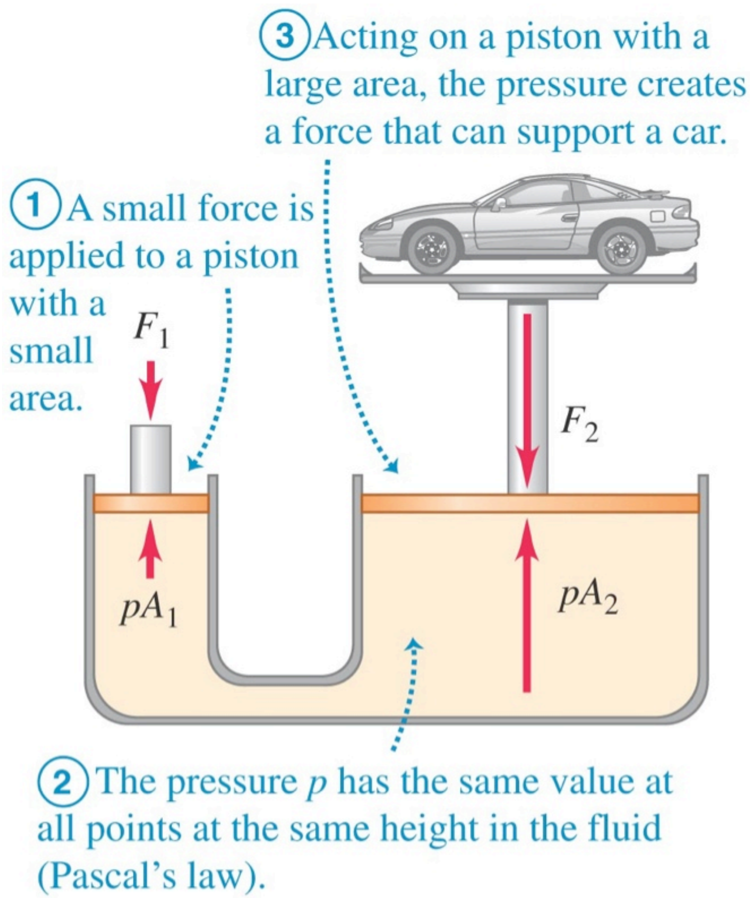


The pressure at the bottom of each liquid column has the same value p.

The difference between p and p_0 is ρgh, where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

Pressure, depth, and Pascal's Law

Consider a practical application in Figure 14.8- Pascal's Law.



$Given: A_1 / A_2 = 0.01$ and

the weight of the car is 2,000 lbs.

What is the amount force (F_1)

need to keep the car lifted?

Measuring atmospheric pressure

Measuring atmospheric pressure
using a mercury barometer.
"Same depth => same pressure"

$P_{outside} = P_{inside}$

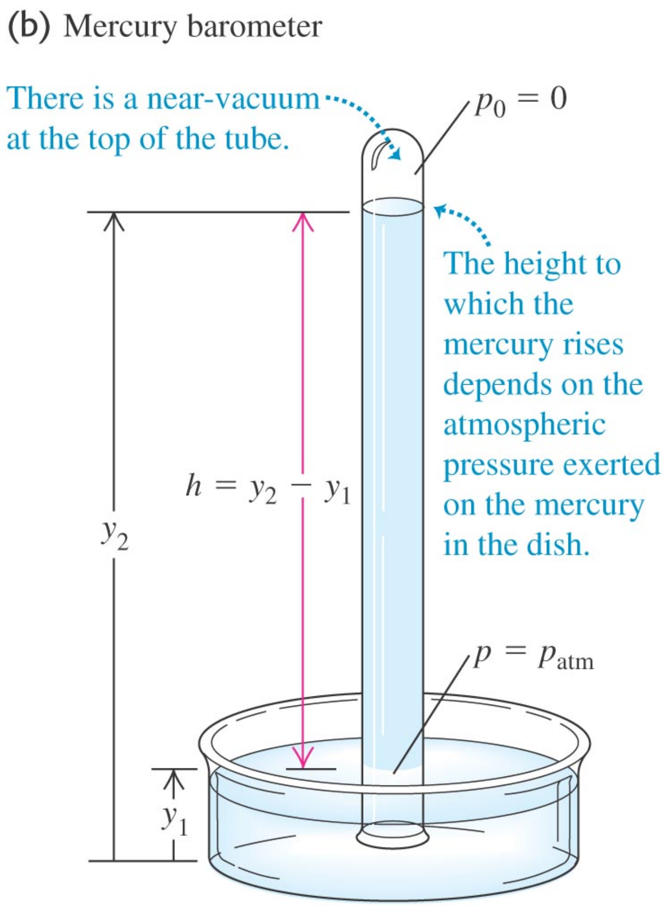
$P_{atm} = P_o + \rho g h \cong \rho g h$

Knowing ρ and g, measuring h
$\Rightarrow$ deduce P_{atm}.

$E.g.\ \rho_{mercury} = 13.6x10^3 kg/m^3, g = 9.8 m/s^2,$

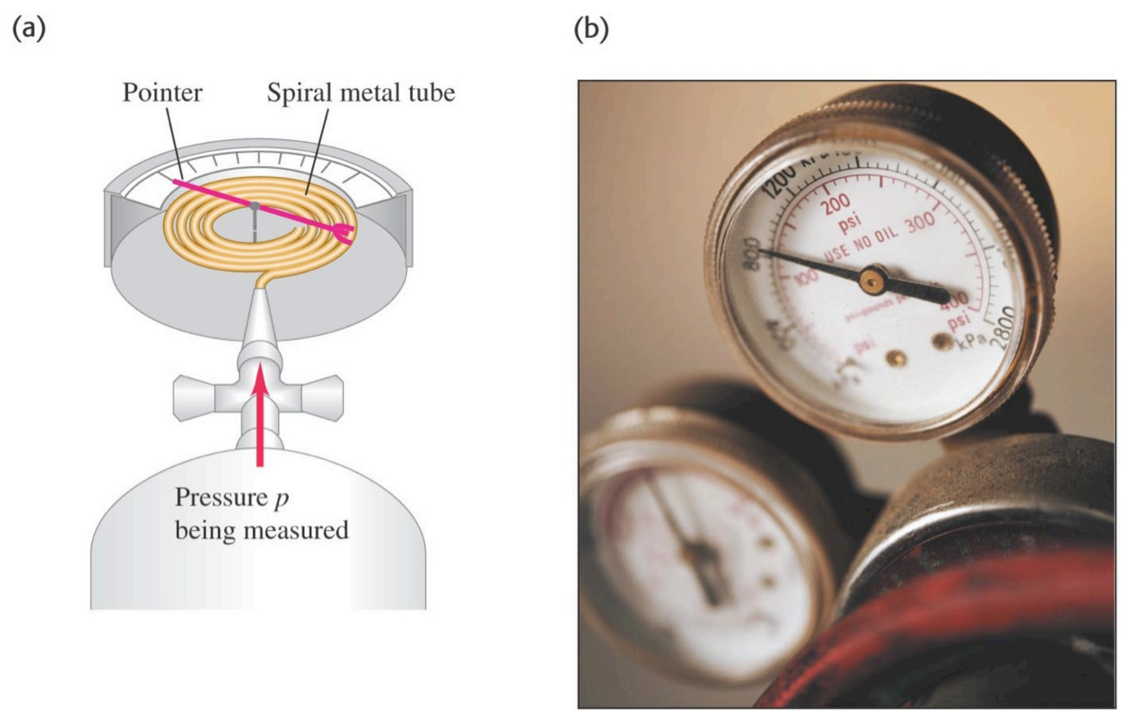
$h = .76m \Rightarrow\ P_{atm} = \rho g h = 1.01x10^5 N/m^2$

If the fluid in the barometer were water (ρ=$10^3 kg/m^3$), then how high is the column of water at 1 atmospheric pressure?



Gauge pressure

- Typically these gauges measure ‘gauge pressure’
- When the absolute presssure of gas inside is 1 atmospheric pressure, the gauge reads zero.
- The gauge reading represent pressure above atmospheric pressure.



Buoyancy and Archimedes Principle

- The buoyancy force (B) comes from the net effect that the pressure pushing up from the bottom of the object is greater the pressure pushing down on an object from the top (remember: pressure increases with depth).
- Numerically the buoynacy force equals to the weight of the displaced fluid
- $B = \rho_{fluid} V_d g$, V_d = volume of fluid displaced.
- An object floats if the buoyancy force (B) balances the weight

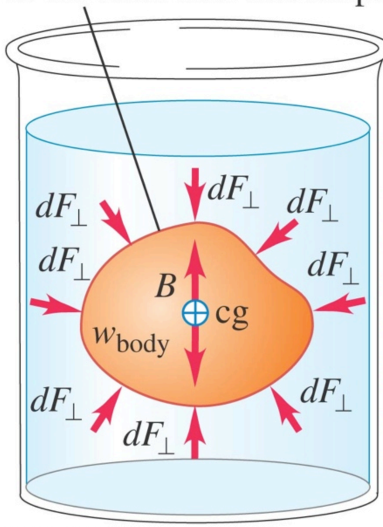
An object floats when $B = w$

$$\Rightarrow \rho_{fluid} V_d g = \rho_{object} V_o g$$

$$\Rightarrow \frac{\rho_{object}}{\rho_{fluid}} = \frac{V_d}{V_o}$$

Example: If the displaced volume is 1/2 of the object volume then the density of the object is 1/2 the density of the fluid. Since the displaced volume cannot be greater than the object's volume, an object float when $\rho_{object} < \rho_{fluid}$.

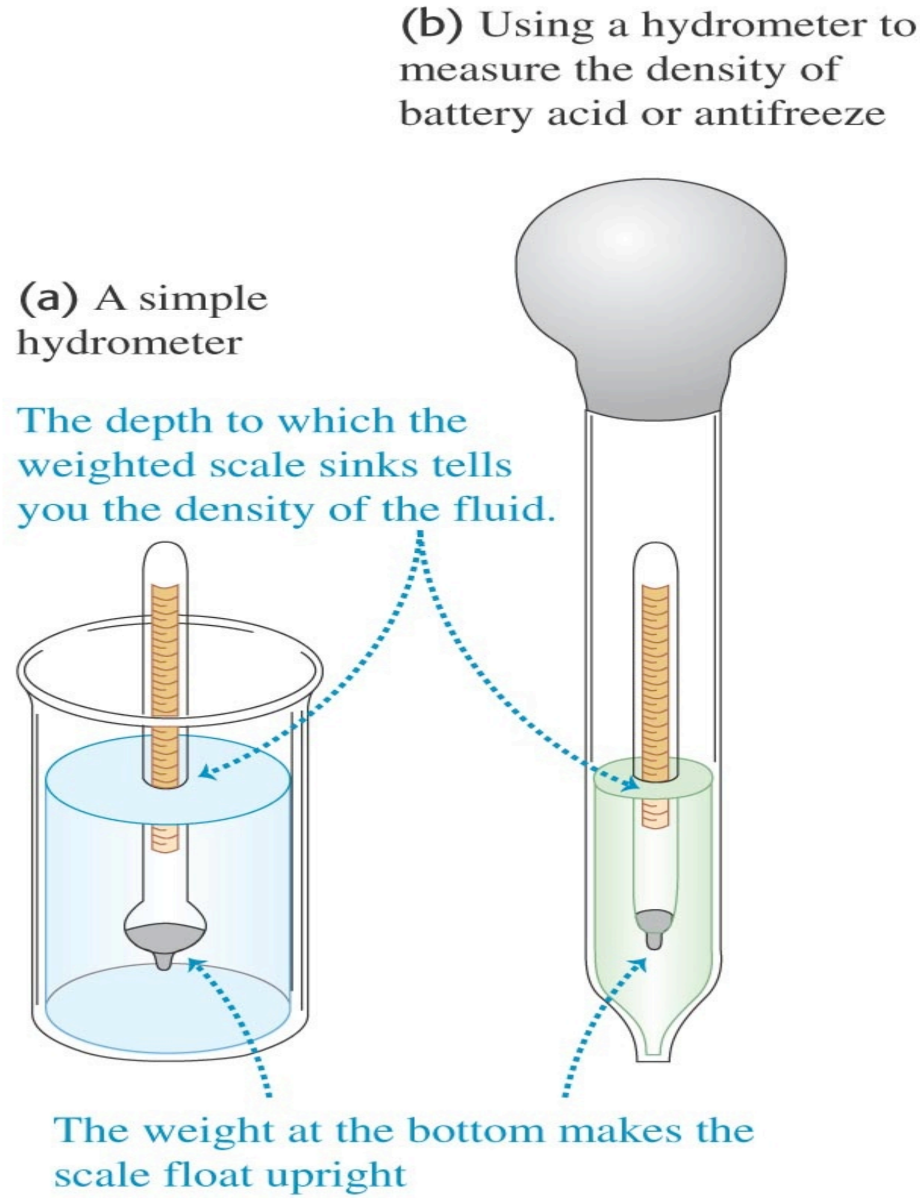
(b) Fluid element replaced with solid body of the same size and shape



The forces due to pressure are the same, so the body must be acted upon by the same buoyant force as the fluid element, *regardless of the body's weight.*

Measuring the density of a liquid

- A hydrometer measures the density of a liquid by how much it floats on the liquid; the scale on hydrometer is calibrated against a liquid with known density (such as water)
- This device is based on the buoyancy principle - see next slide.

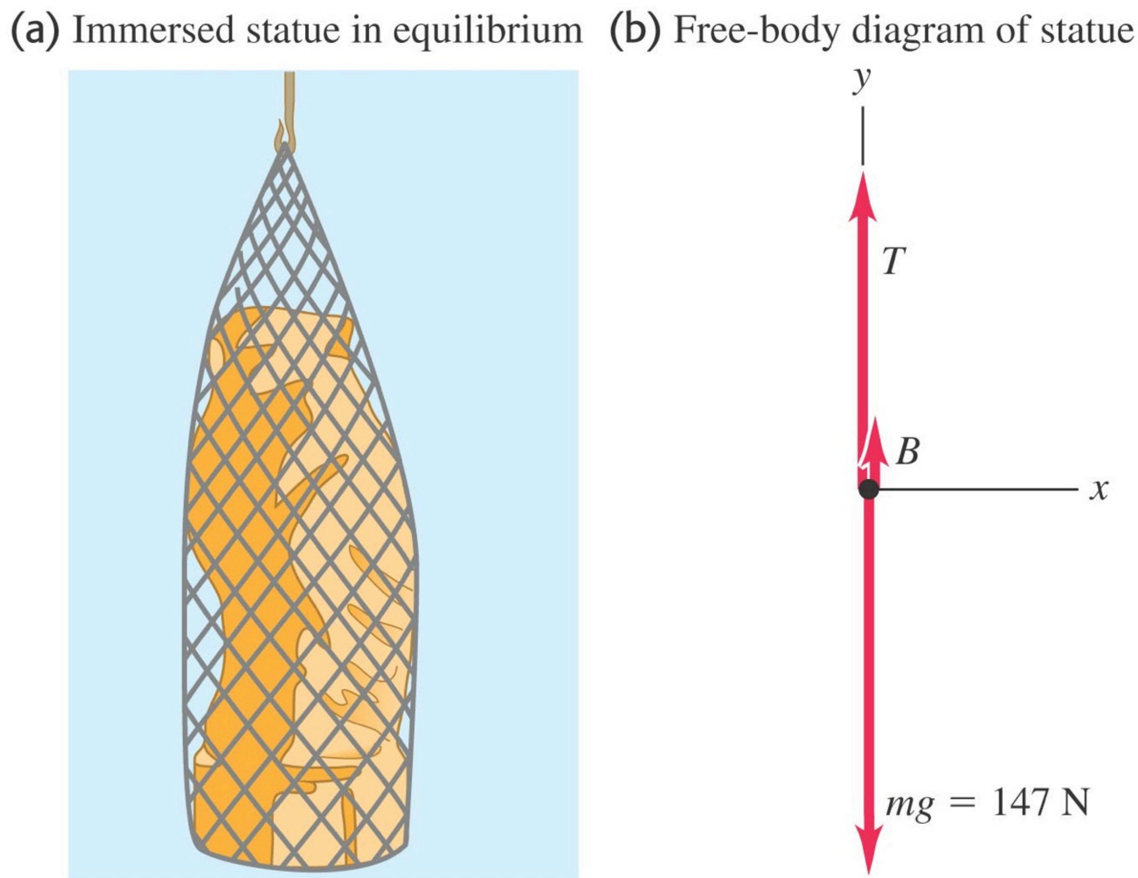


Weighing an elephant using Archimedes Principle

- A rectangular boat with cross-sectional area = 20 m^2 is floating on water. After an elephant stepped into the boat, the boat sinks down by 0.15m. Find the weight of the elephant. (Given the density of water is 10^3 kg/m^3).

There is a buoyancy even when the object sinks

- Example 14.5.
- A 15-kg gold statue is being raised from a sunken ship. What is the tension in the cable when the statue is at rest and fully submerged in seawater?

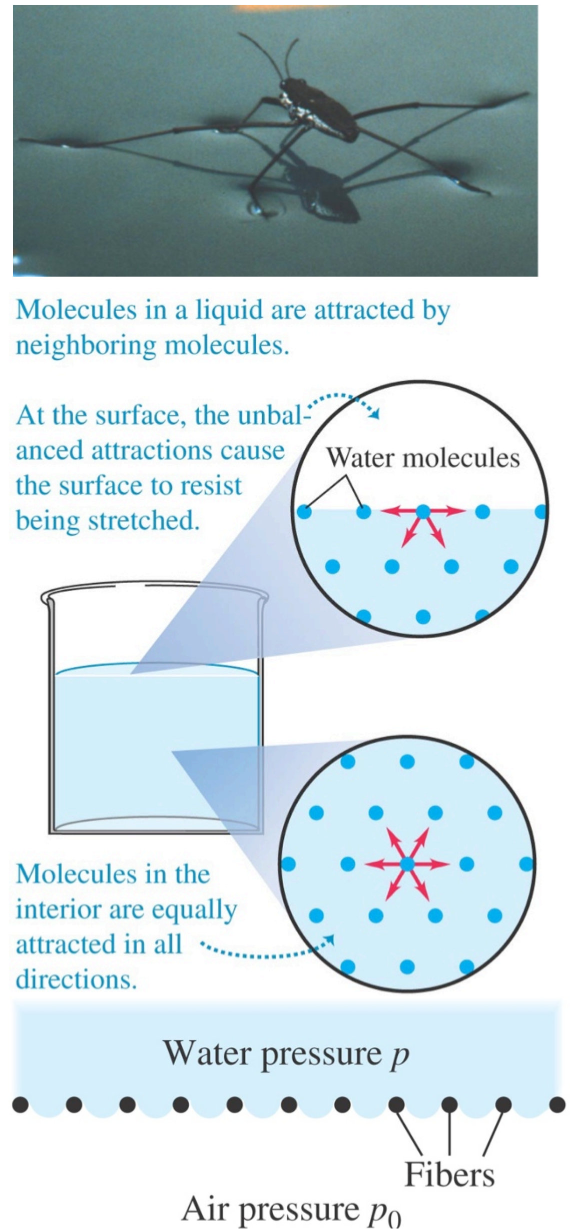


Given :

$$\rho_{gold} = 19.3x10^3 kg/m^3; \quad \rho_{seawater} = 1.03x10^3 kg/m^3$$

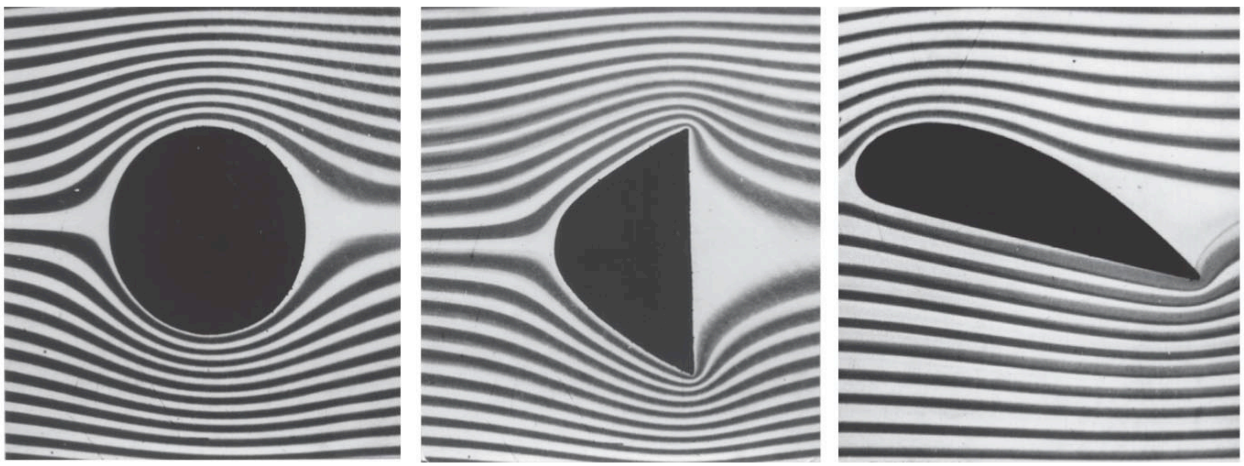
Surface tension

- How is it that water striders can walk on water (although they are more dense than the water)?
- Refer to Figure 14.15 for the water strider and then Figures 14.16 and 14.17 to see what's occurring from a molecular perspective.



Fluid flow - types

- The flow lines at left in Figure 14.20 are laminar.
- The flow at the top of Figure 14.21 is turbulent.



Fluid flow -flow rate and conservation of mass

Flow rate = mass of fluid crossing an area / time

$\Rightarrow$ Flow rate = $\rho A v$

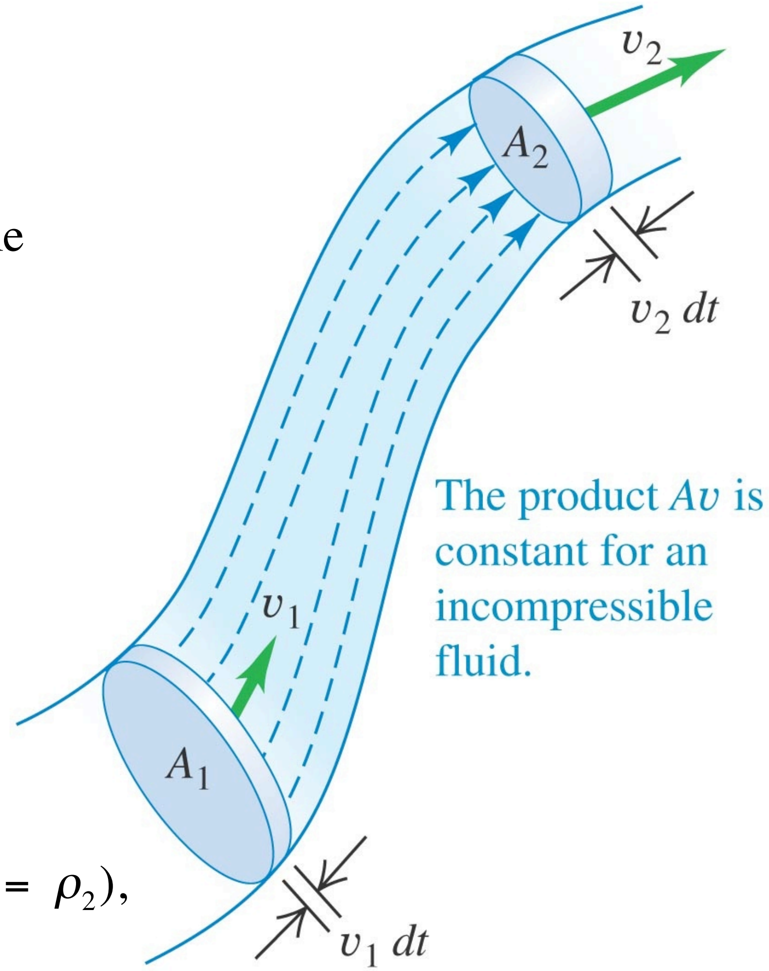
Conservation of mass => flow rate remain the same at different part of the pipe.

$\Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

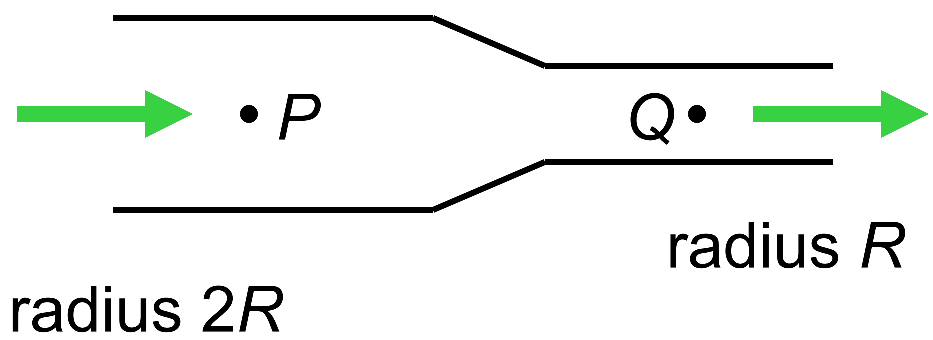
Gases are very compressible $\Rightarrow \rho_1$ and ρ_2 can be quite different.

Most liquids are not very compressible, $\rho_1 \approx \rho_2$.
If we approximate liquids as "imcompressible" ($\rho_1 = \rho_2$),
then $A_1 v_1 = A_2 v_2$.

The conservation of mass equation is also called the continuity equation.



An incompressible fluid flows through a pipe of varying radius (shown in cross-section). The flow rate at point P is 10 m^3/s. What is the flow rate at point Q?



Bernoulli's equation-assumes laminar flow

- Similar to work-kinetic energy theorem:

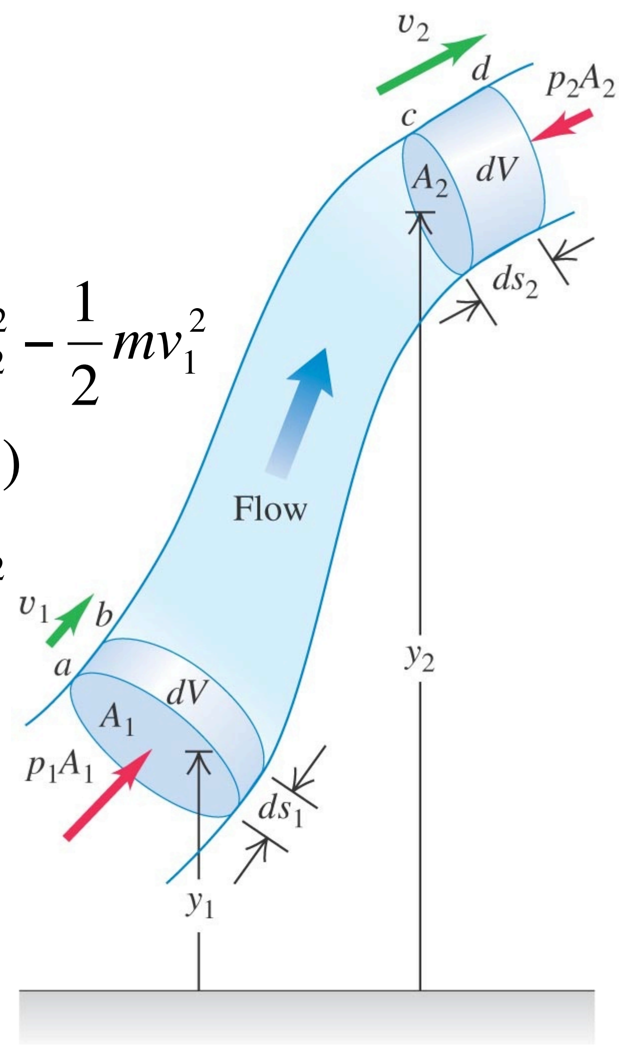
Work = change of kinetic energy

$$(-P_2A_2ds_2 + P_1A_1ds_1) + (mgy_1 - mgy_2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$(Note: m = \rho A_1 ds_1 = \rho A_2 ds_2 \Rightarrow A_1 ds_1 = A_2 ds_2)$$

$$\Rightarrow (-P_2 + P_1) + (\rho g y_1 - \rho g y_2) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

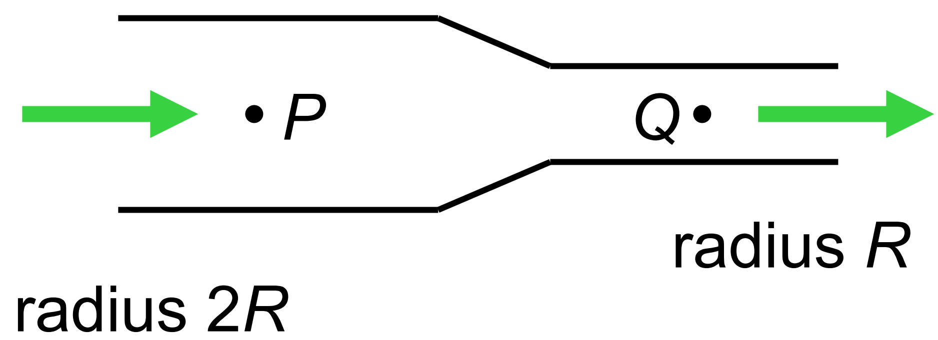
$$\Rightarrow P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$



An incompressible fluid flows through a pipe of varying radius (shown in cross-section). The flow rate at point P is 10 m^3/s. The density of the fluid is 10^3 kg/m^3.

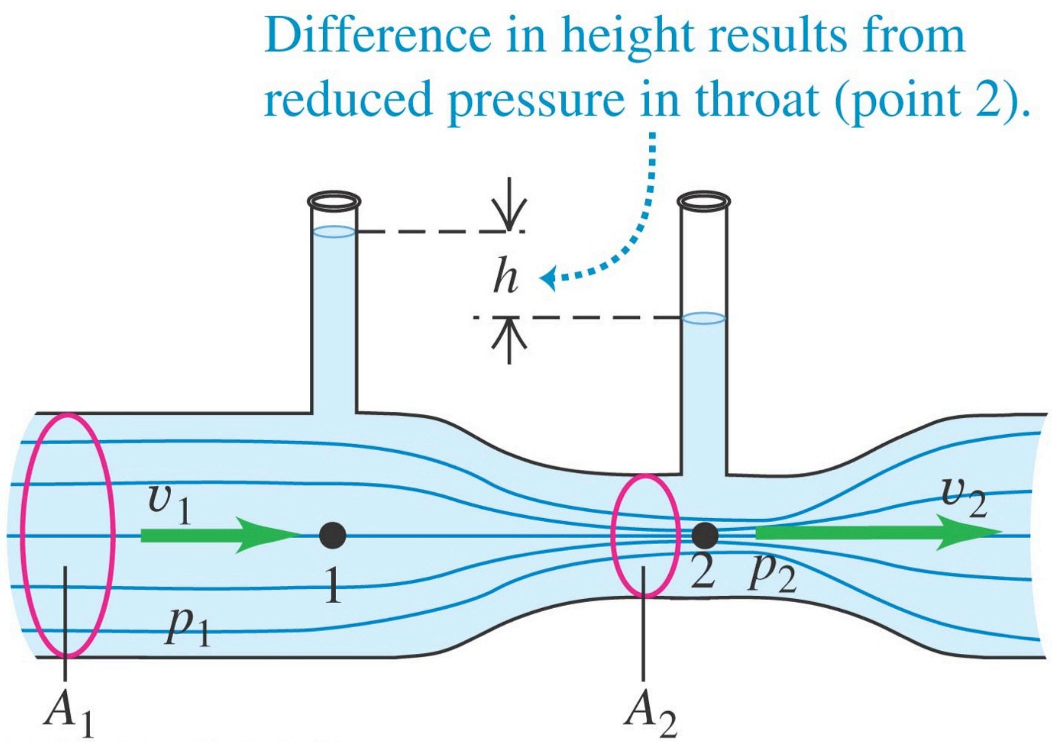
(a) Which point has a higher pressure? P or Q

(b) What is the pressure difference?



The Venturi meter (Bernoulli's Eq.+ continuity Eq.)

- Consider Example 14.9.



$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_2 > v_1$$

Bernoulli's equation

$$\Rightarrow P_2 < P_1$$

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1}{A_2}\right)^2 v_1^2 - \frac{1}{2}\rho v_1^2$$

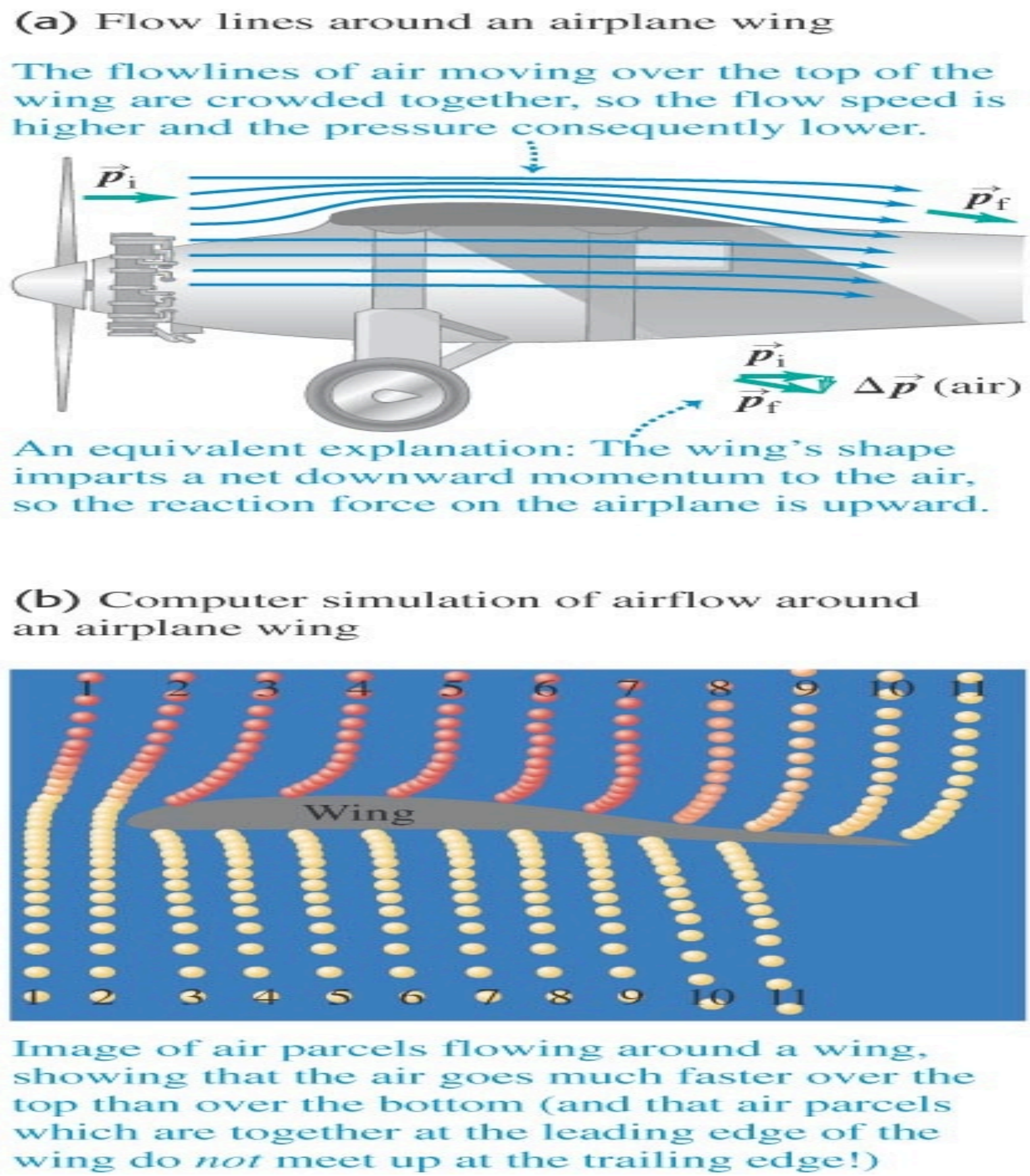
$$P_1 - P_2 = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]v_1^2$$

Measure the pressure difference => find v_1

A large bucket filled with water has a small hole at the bottom. The depth of water is 0.5m. What is the velocity of the water coming out the hole?

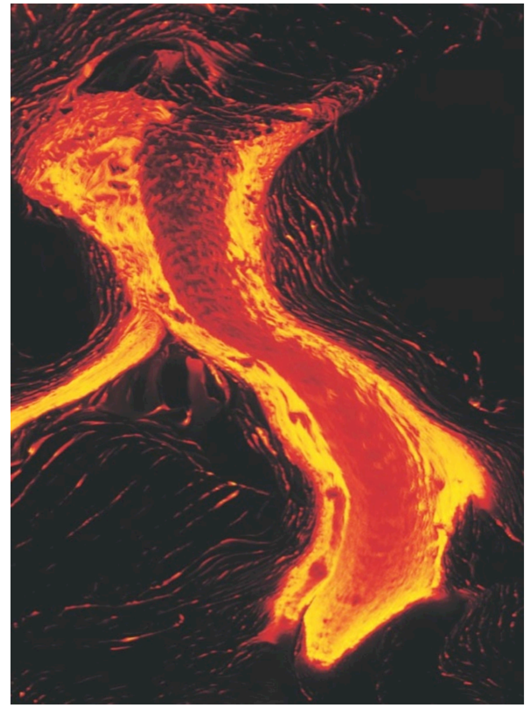
Lift on an airplane wing

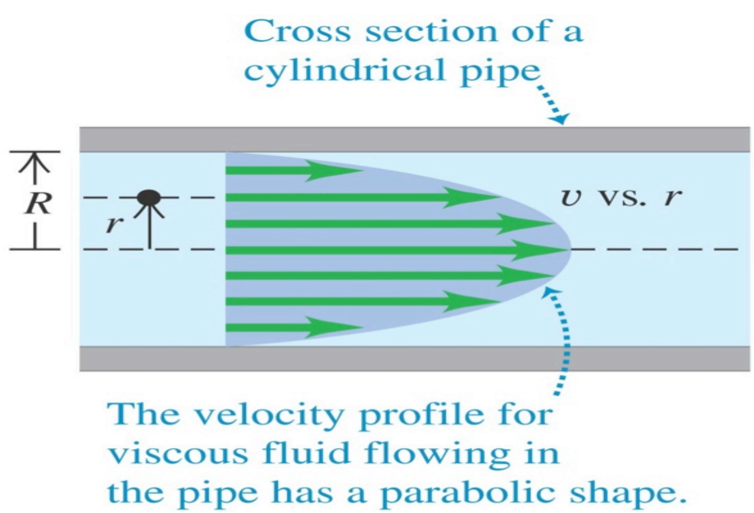
- Refer to Conceptual Example 14.10.



Viscosity and turbulence—Figures 14.28, 14.29

- In real fluids (as compared to idealized model), molecules can attract or repel one another and can interact with container walls give rise to viscosity. Molecular interactions can also result in turbulence.

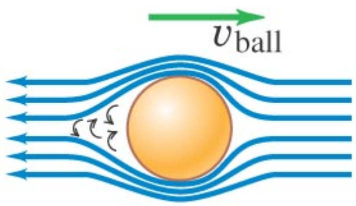




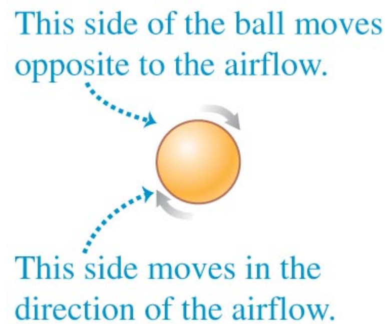
A curve ball (Bernoulli's equation applied to sports)

- Bernoulli's equation allows us to explain why a curve ball would curve.

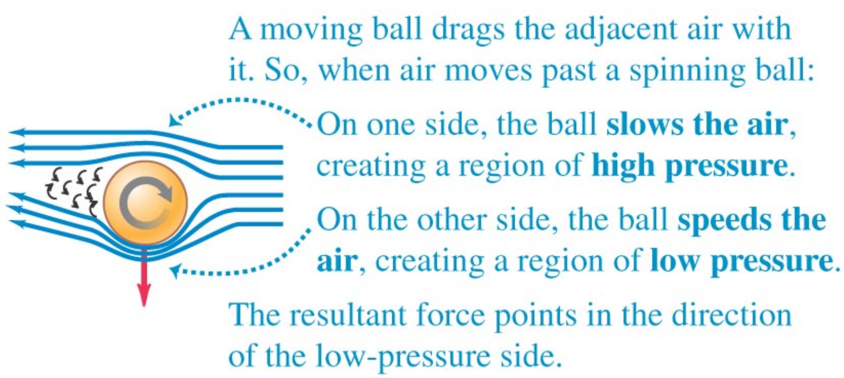
(a) Motion of air relative to a nonspinning ball



(b) Motion of a spinning ball



(c) Force generated when a spinning ball moves through air



(e) Spin causing a curve ball to be deflected sideways