

Chapter 13

Gravitation

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Learning Goals for Chapter 13

- Understand the mathematical expression of Newton's Law of Gravitation
 - Combine Newton's Law of Gravitation and Newton's Second Law of Motion ($F=ma$) to calculate orbital speed of satellites and planets
 - Use energy method to determine “escape velocity”
 - Use Newton's Law of Gravitation and Newton's Second Law of Motion to explain Kepler's Laws of planetary motion
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Newton's Law of Gravitation

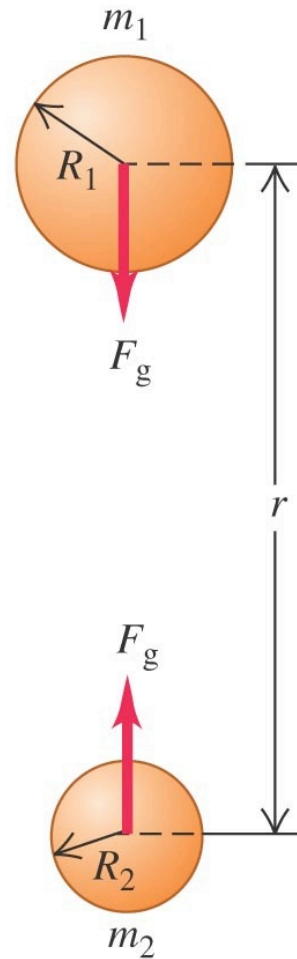
- The gravitational force is always attractive and its magnitude depends on both the masses of the bodies involved and their separations.

$$|\vec{F}_g| = \frac{Gm_1m_2}{r^2}$$

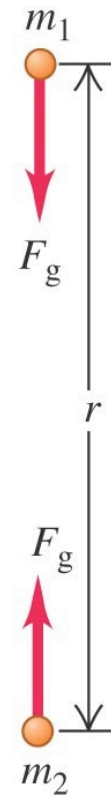
$$G = 6.6742 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

was deduced in

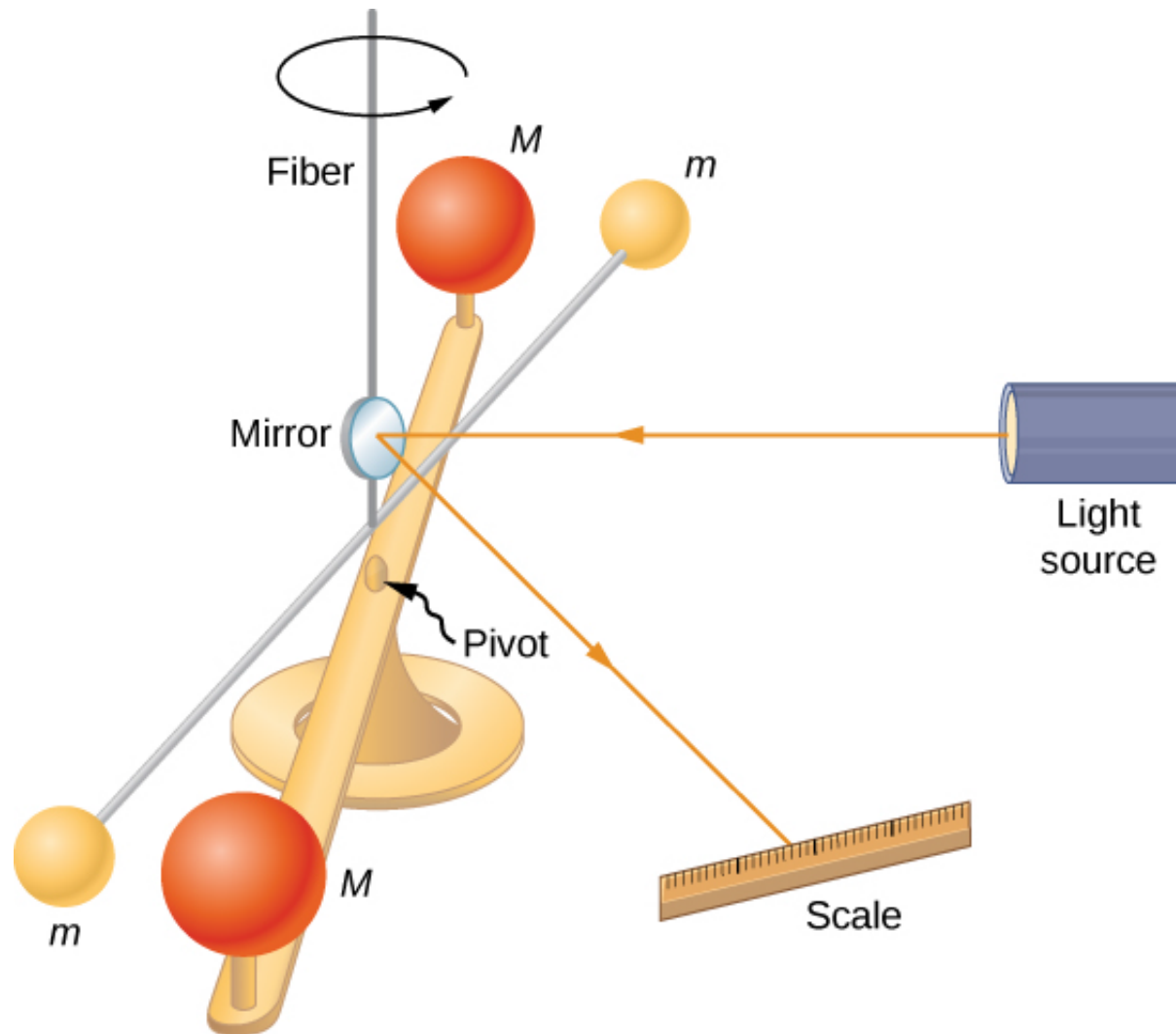
(a) The gravitational force between two spherically symmetric masses m_1 and m_2 ...



(b) ... is the same as if we concentrated all the mass of each sphere at the sphere's center.



Cavendish's experiment to find G – 1997-1998



Deduce Mass of the Earth

We can measure little g

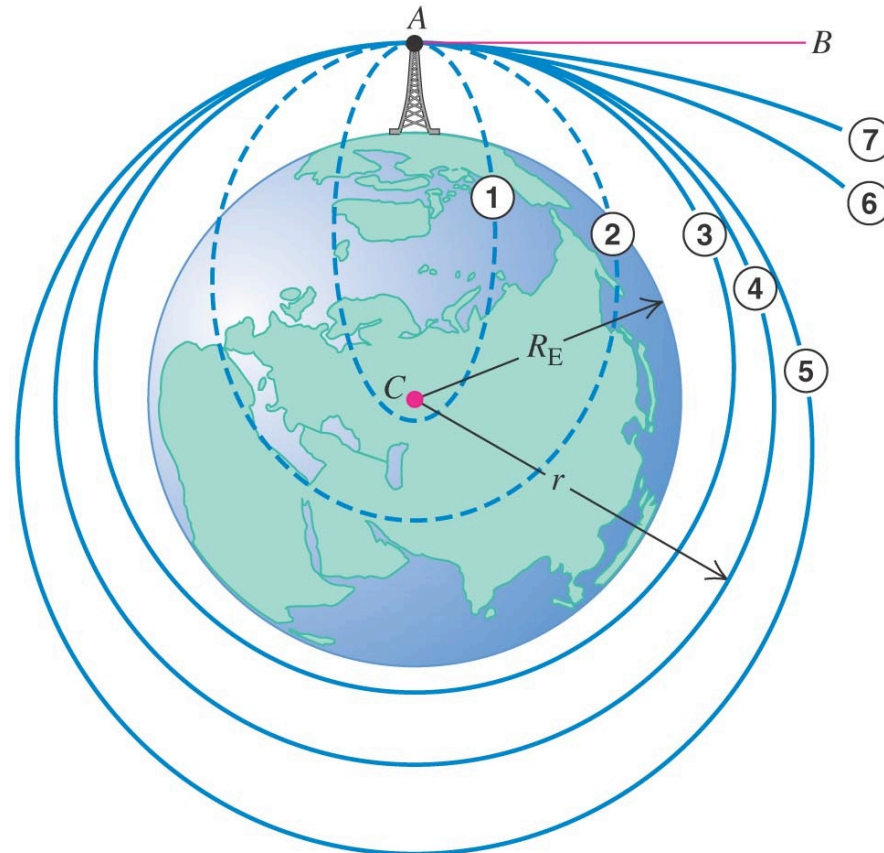
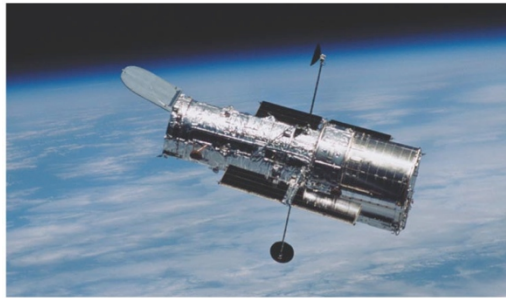
We can measure the radius of the Earth (the Greek did it long time ago, how?)

Once Cavendish determined the Universal Gravitational constant G .

Q: Deduce the mass of the Earth from these information.

(In the later part of this Chapter, you will learn how to determine the mass of the Earth from the orbital period of the moon around the Earth and Earth-moon distance)

Satellite motion = projectile with large enough initial speed

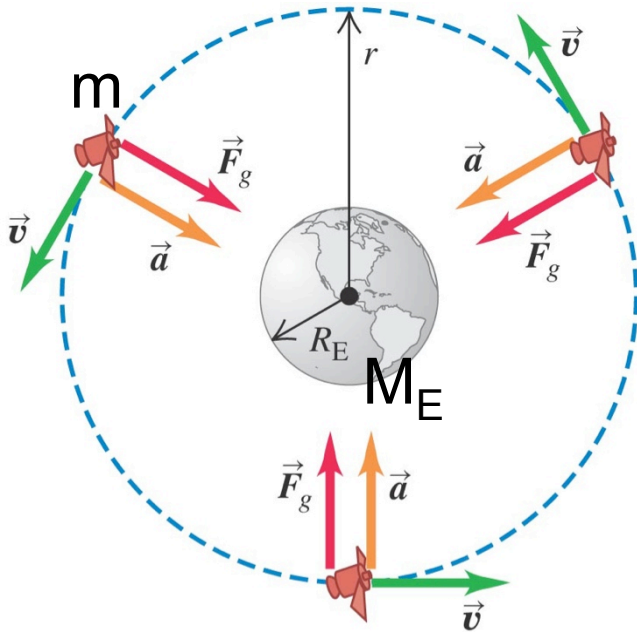


A projectile is launched from A toward B . Trajectories ① through ⑦ show the effect of increasing initial speed.

Orbital motion = projectile motion with very large initial velocity
How large an initial velocity is required?

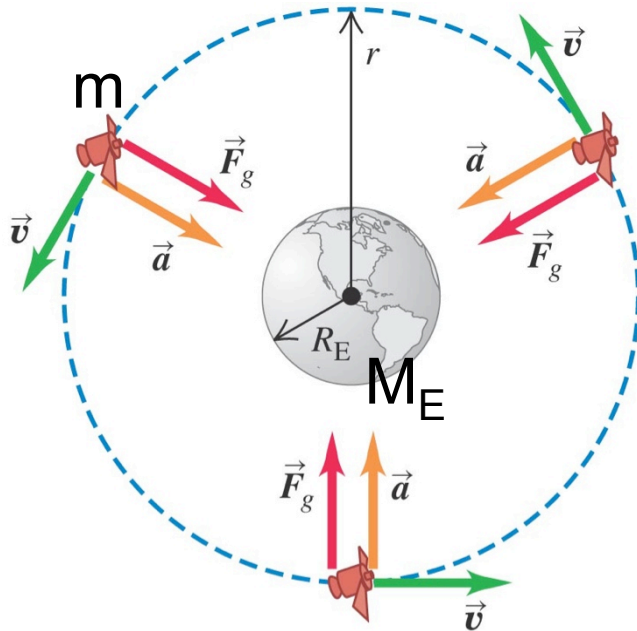
Calculate the orbital speed for a given orbital radius

Apply $F=ma$ and F is given by Newton's Law of gravitation



The satellite is in a circular orbit: Its acceleration \vec{a} is always perpendicular to its velocity \vec{v} , so its speed v is constant.

Calculate satellite orbits - assume circular orbit



The satellite is in a circular orbit: Its acceleration \vec{a} is always perpendicular to its velocity \vec{v} , so its speed v is constant.

(1) Draw free diagram on the satellite

(2) Set up Newton's 2nd Law $F=ma$ equation ($a = v^2 / r$)

$$\Rightarrow F = m \frac{v^2}{r}$$

(3) Apply Newton's Law of Gravitation: $F = \frac{GmM_E}{r^2}$

(4) Combine $\Rightarrow \frac{GmM_E}{r^2} = m \frac{v^2}{r}$

(5) Solve for v in terms of r

$$\Rightarrow v = \sqrt{\frac{GM_E}{r}}$$

For an orbit near the surface of the Earth

$$(r \approx R_E = 6.4 \times 10^6 \text{ m}), \frac{v^2}{R_E} = \frac{GM_E}{R_E^2} = g \approx 9.8 \text{ m/s}^2$$

$$v \approx 8 \times 10^3 \text{ m/s} \sim 18,000 \text{ mi/h}$$

$$\text{Period of orbit} = \frac{2\pi R_E}{v} \approx 5,026 \text{ s} \approx 83.8 \text{ min.}$$

The space shuttle has a mass $\sim 80,000$ kg, what is its kinetic energy while in near-Earth orbit?

Space Shuttle stats:

“The Space Shuttle weighed 165,000 pounds empty. Its external tank weighed 78,100 pounds empty and its two solid rocket boosters weighed 185,000 pounds empty each. Each solid rocket booster held 1.1 million pounds of fuel. The external tank held 143,000 gallons of liquid oxygen (1,359,000 pounds) and 383,000 gallons of liquid hydrogen (226,000 pounds). The fuel weighed almost 20 times more than the Shuttle. At launch, the Shuttle, external tank, solid rocket boosters and all the fuel combined had a total weight of 4.4 million pounds. The Shuttle could also carry a 65,000 payload”

“Cost per launch US\$ 450 million to 1.5 billion (2011)”

Gravitational potential energy

The expression for the gravitational potential energy for a mass at height h from the surface of the Earth:

$U = mgh$ is only valid for an object near the surface of the Earth.

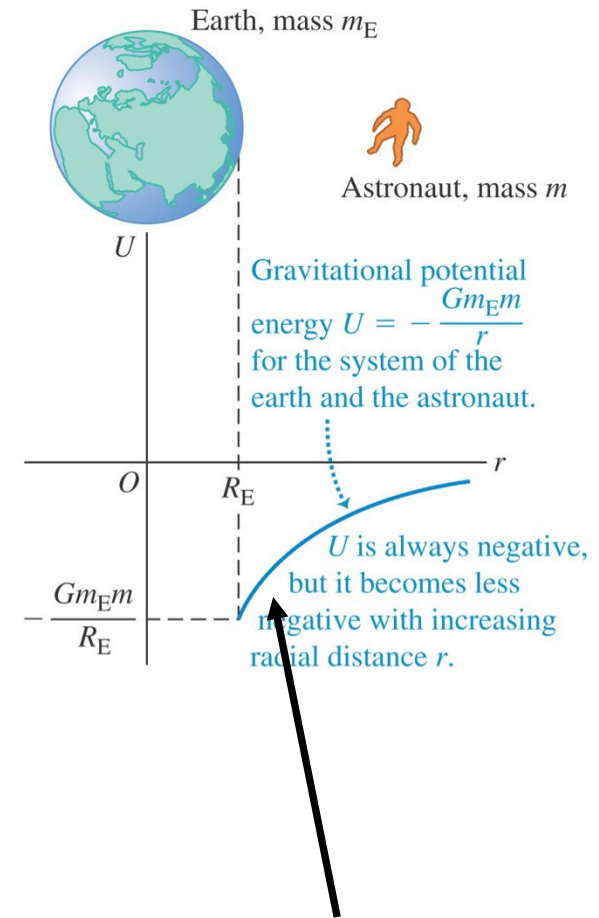
The more general expression is:

$U = -\frac{GmM_E}{r}$ = potential energy for the mass-Earth system separated by a distance of r .

Recall: $F = -dU/dr$

Near the surface of the Earth $r = R_E + h$

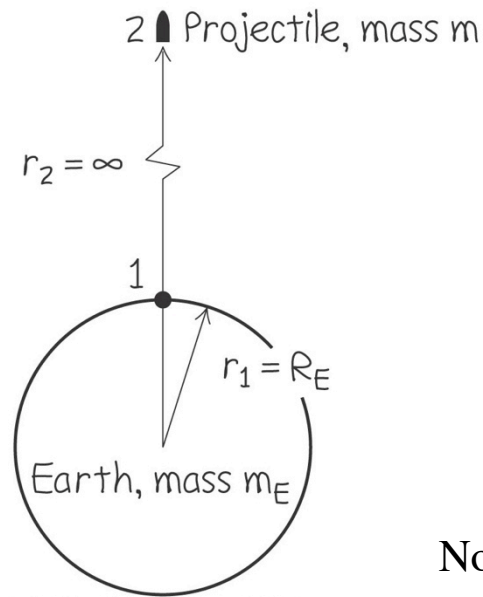
$$U(R_E + h) - U(R_E) = \left(-\frac{GmM_E}{R_E + h} \right) - \left(-\frac{GmM_E}{R_E} \right) \approx m \frac{GM_E}{R_E^2} h = mgh$$



Calculate escape velocity

- What is the minimum initial velocity of a rocket for it to escape from the gravitational effect of the Earth?

(b)



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$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}M_E V_i^2 - \frac{GmM_E}{R_E} = 0 + \frac{1}{2}M_E V_f^2 - \frac{GmM_E}{\infty}$$

$$V_f \cong V_i$$

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = 0$$

$$\Rightarrow v_i = \sqrt{\frac{2GM_E}{R_E}} \approx 25,300 \text{ mi/h}$$

Note : If a star is very dense such that it has a large mass but a small radius, then $v_{\text{escape}} > \text{speed of light!!}$

Such a star is a "black hole".

However, correct treatment of black hole requires Einstein's theory of Gravitation (General Relativity).

Newton's theory of gravitation is only approximate.

Newton's Law of gravitation

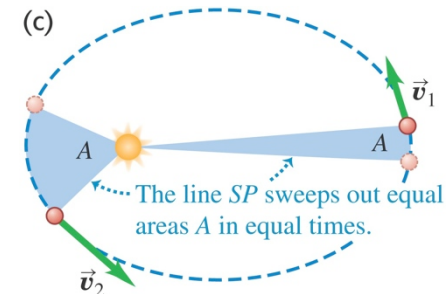
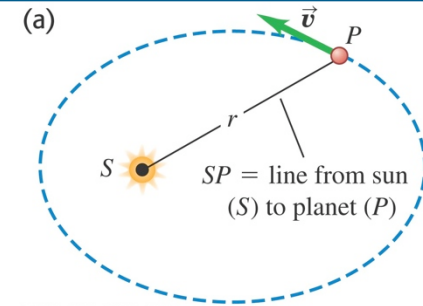
How did Newton come up with the Law of Gravitation?

$$|\vec{F}| = \frac{Gm_1m_2}{r^2}$$

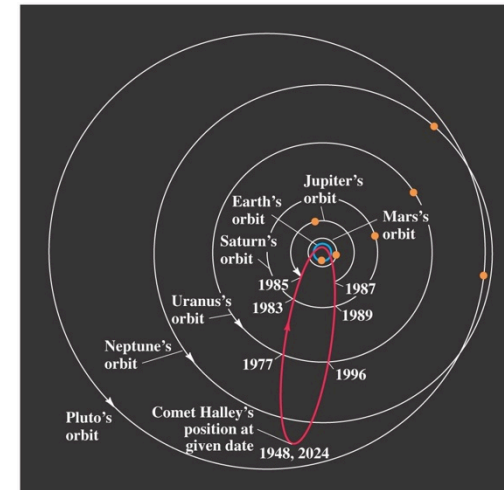
Newton relied on the astronomical data of Kepler.

Kepler's laws for planetary motion

- 1st Law: Each planet moves in an elliptical orbit with the sun at one of the foci
- 2nd Law: A line connecting the sun to a given planet sweeps out equal areas in equal times.
- 3rd Law: The periods of the planets are proportional to the $3/2$ powers of the major axis lengths in their orbits. (Planets further away from the Sun takes longer to go around the Sun)



(a)



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Kepler's 1st Law - elliptical orbits

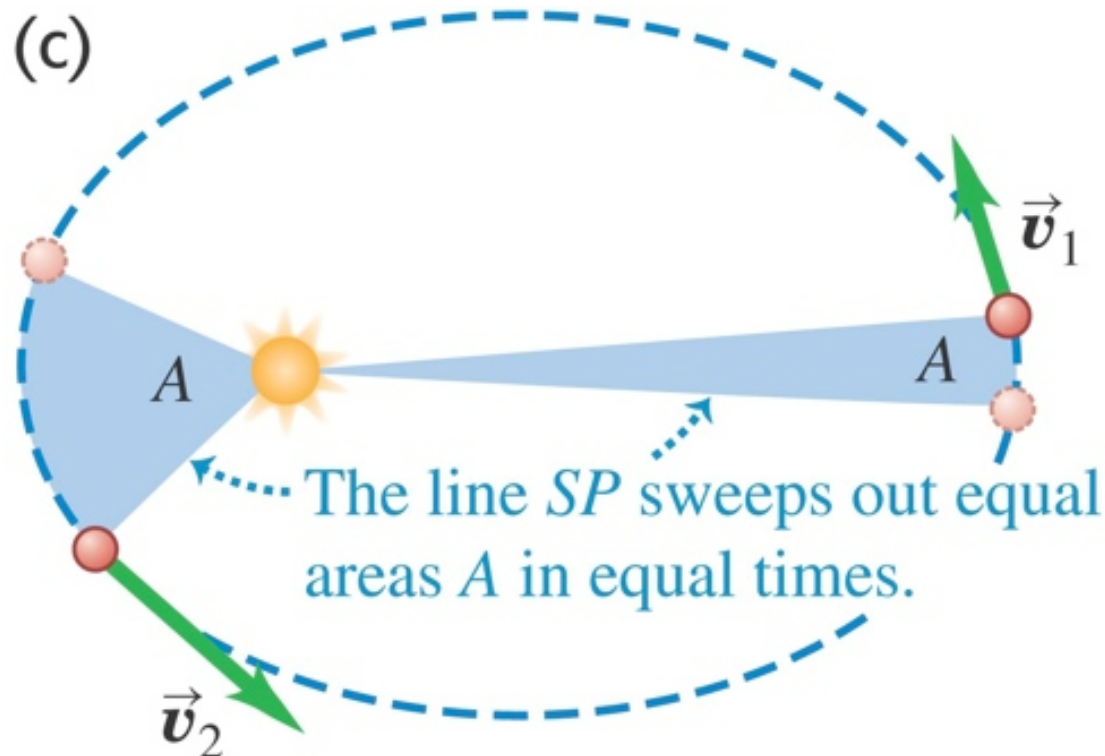
Elliptical orbits are consequence of the $1/r^2$ dependence of the gravitational force.

The proof requires advanced calculus.

Circular orbit is a special case of elliptical orbit.

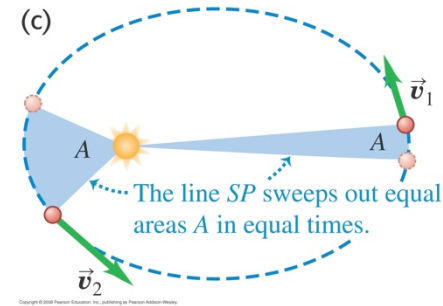
Kepler's 2nd Law

- 2nd Law: A line connecting the sun to a given planet sweeps out equal areas in equal times \Rightarrow **a planet moves faster when it is closer to the sun.**



Kepler's 2nd Law - Conservation of angular momentum

Kepler's 2nd Law : $\frac{dA}{dt} = \text{constant}$



Relate $\frac{dA}{dt}$ to angular momentum (\vec{L}) of the planet about the Sun

$$\frac{dA}{dt} = \frac{1}{2} |\vec{r} \times \vec{v}| = \frac{1}{2m} |\vec{r} \times m\vec{v}| = \frac{|\vec{L}|}{2m}$$

$$\frac{dA}{dt} = \text{constant} \Rightarrow |\vec{L}| = \text{constant} \Rightarrow \text{small } r \text{ means larger } v.$$

$\vec{L} = \text{constant}$ when torque = 0.

Newtons concluded that the gravitational force must be directed along the "radius" from the Sun to the planet.

Since \vec{F} and \vec{r} are along the same direction, $\vec{r} \times \vec{F} = 0$.

Kepler's 3rd Law and Newton's Law of Gravitation

Derived in an earlier slide, repeat here:

$$\text{Newton's 2nd Law: } F=ma = m_p \frac{v^2}{r}$$

$$\text{Newton's Law of gravitation: } F = \frac{Gm_p M_s}{r^2} = m_p \frac{v^2}{r}$$

$$\Rightarrow v^2 = \frac{GM_s}{r}$$

$$\text{Assume circular orbit } \Rightarrow v = \frac{2\pi r}{T}, T=\text{orbital period}$$

$$\Rightarrow \left(\frac{2\pi r}{T} \right)^2 = \frac{GM_s}{r}$$

$$\Rightarrow T = \frac{(2\pi)r^{3/2}}{\sqrt{GM_s}} \propto r^{3/2} \text{ (Kepler's 3rd Law)}$$

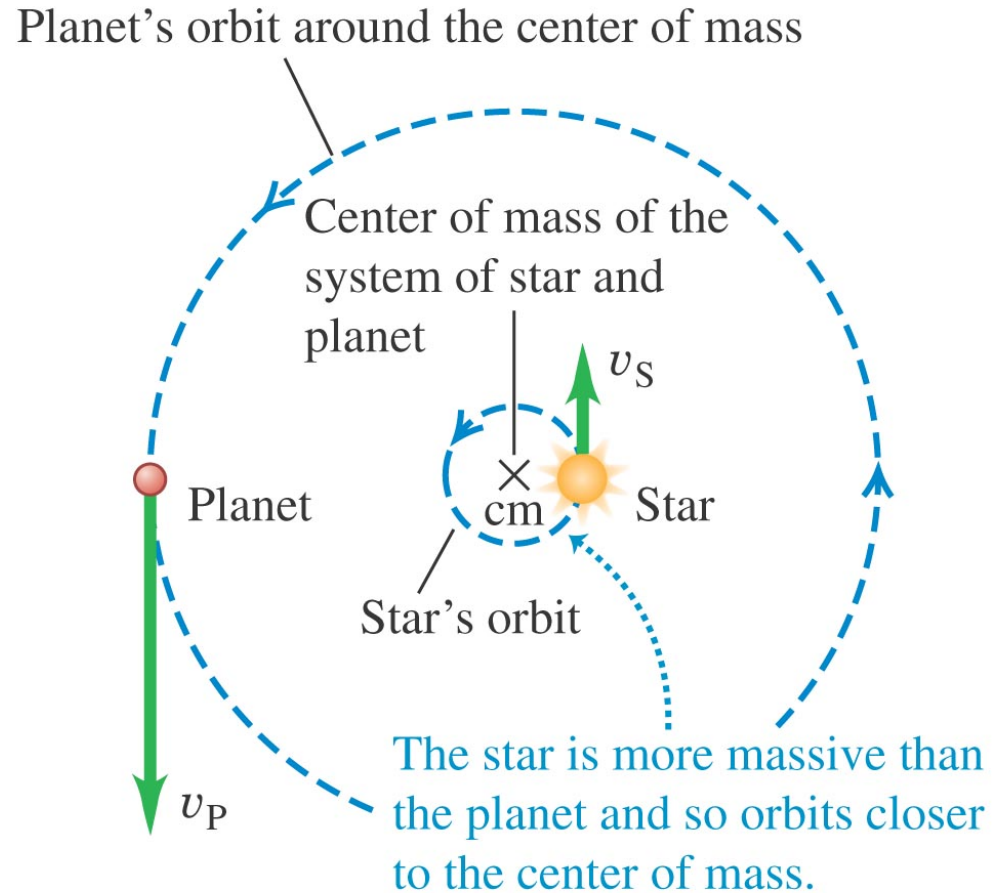
Newton did this backward, he started with Kepler's 3rd Law and deduce that gravitaional force = GMm/r^2 .

Note: We can use Kepler's 3rd law to calculate the mass of the Sun based on the radius and the period of the Earth's orbit around the Sun.

Sun and planet revolve about the center of mass.

We have assumed the Sun was stationary while the planet orbits around the Sun.

In fact, a better way to view their motion is that they both revolve about their center of mass. This will lead to a slight correction to Kepler's 3rd Law.



The planet and star are always on opposite sides of the center of mass.

Review Questions

In a hypothetical solar system, there is a sun and two planets in circular orbits.

Planet A revolves around the sun once per 2 Earth's years.

Planet B revolves around the sun once per 5 Earth's years.

The distance from the sun to Planet A is $3.0 \times 10^{11} m$

Radius of Planet A is $1.0 \times 10^6 m$.

The escape velocity from Planet A is $1.0 \times 10^4 \text{ m/s}$.

Q.1 What is the distance from the sun to Planet B? Which planet is closer to the sun?

Q.2 What is mass of their sun?

Q.3 What is the mass of planet A and the "g" near its surface?

Q.4 Suppose Planet B has twice the mass as A but its radius is 3 times larger. What is the "g" near Planet B surface

Q.5 What is the escape velocity from Planet A including the gravitation effect of the sun (not that Planet B)?
