

Chapter 12

Static Equilibrium

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Goals for Chapter 12

- To study the conditions for static equilibrium
- Deduce forces and torque under static equilibrium condition

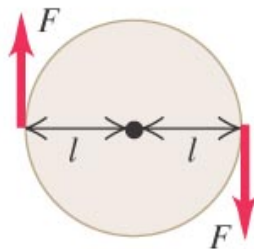
Introduction

- An object in static equilibrium if
 - (1) it has zero linear acceleration (\Rightarrow net force = 0)
 - and
 - (2) it has zero angular acceleration (\Rightarrow net torque = 0)
-

Conditions for static equilibrium

- $a=0 \Rightarrow$ The sum of all forces present in the x , y , and z directions are each equal to zero.
- $\alpha = 0 \Rightarrow$ The sum of all torques for any given point are equal to zero.

(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.



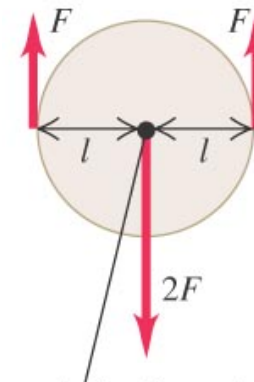
First condition satisfied:

Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied:

There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

(a) This body is in static equilibrium.



Axis of rotation (perpendicular to figure)

Equilibrium conditions:

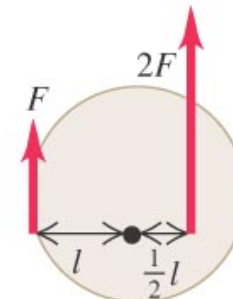
First condition satisfied:

Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied:

Net torque about the axis = 0, so body at rest has no tendency to start rotating.

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.



First condition NOT

satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied:

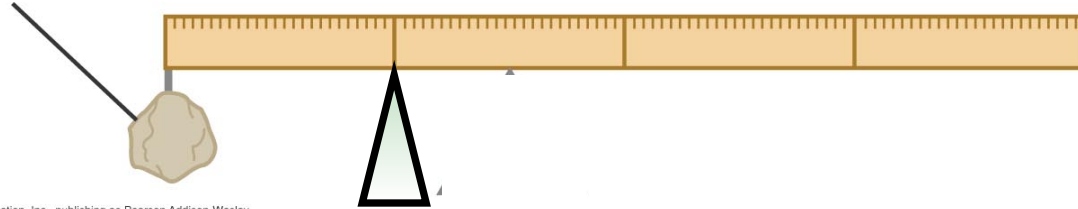
Net torque about the axis = 0 so body at rest has no tendency to start rotating.

Example of static equilibrium

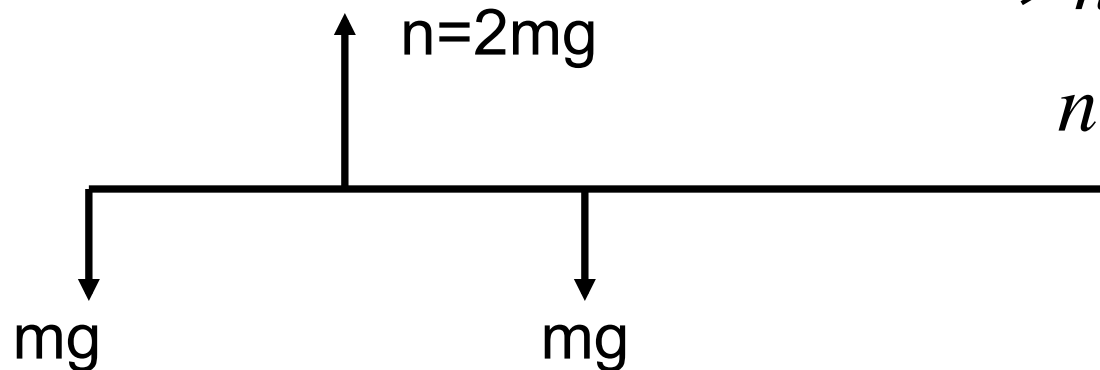
A rock and an uniform meter stick have the same mass.
Prove that placing the mass-stick system over the a support at 0.25m from the left end, the meter stick is in static equilibrium.

Rock, mass m

Meter stick, mass m



Free-body
Diagram for
the meter
stick



$$\Rightarrow net \vec{F} = 0$$
$$net \vec{\tau} = 0$$

Note: We have assumed the “center of gravity” of the meter stick is located at the geometric center of the meter stick.

Center of gravity is the point where one can consider as if all the mass is concentrated.

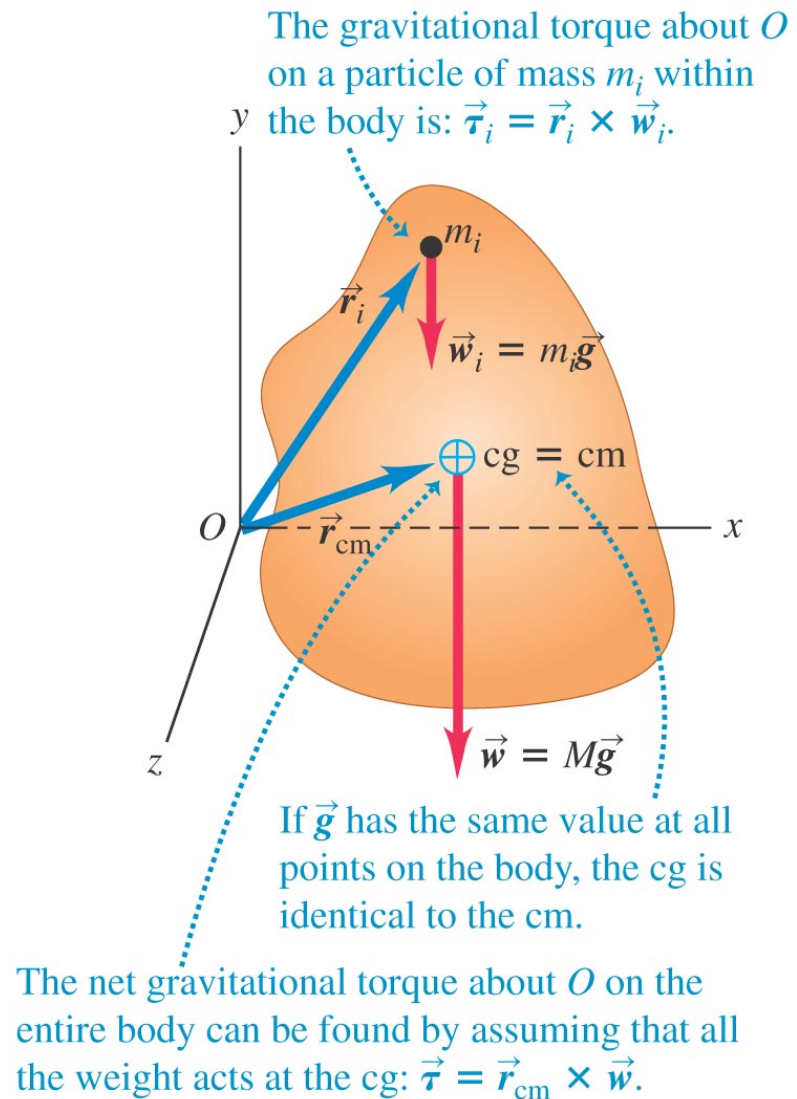
The center of gravity (cg)

If \vec{g} is uniform, then the location of the center of gravity (cg) = the location of center of mass (cm).

If you want to calculate the torque about a point O due to the weight of an object, you can treat as if all the mass is located at the cg.

$$\vec{\tau} = \vec{r}_{cg} \times (m\vec{g})$$

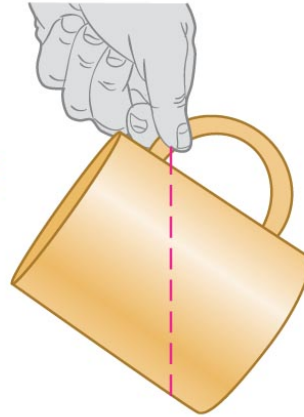
\Rightarrow If we choose O to be at the cg, then the torque due to the weight of the object is zero.



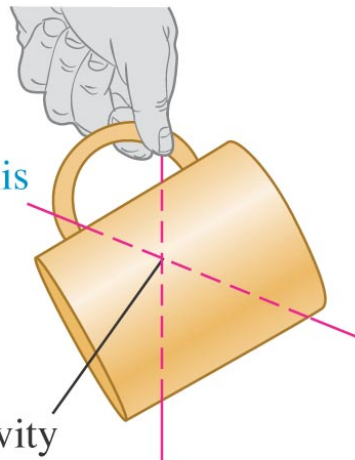
Experimental method of finding the center of gravity

What is the center of gravity of this mug?

① Suspend the mug from any point. A vertical line extending down from the point of suspension passes through the center of gravity.



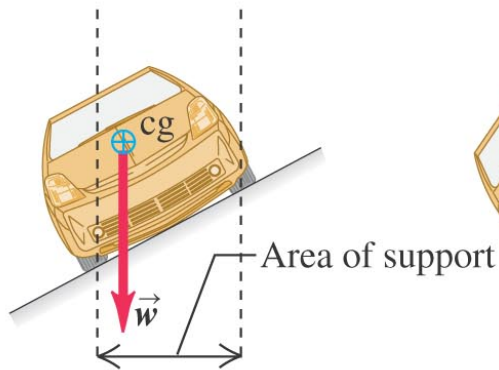
② Now suspend the mug from a different point. A vertical line extending down from this point intersects the first line at the center of gravity (which is inside the mug).



Center of gravity

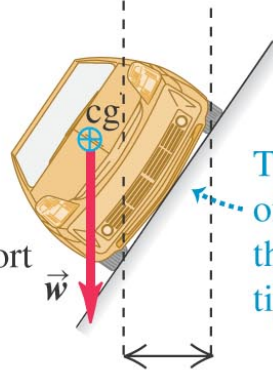
Center of gravity and static equilibrium

(a)



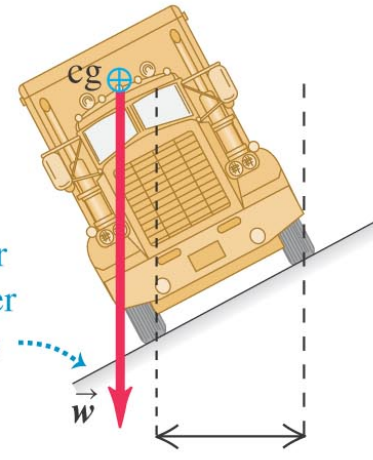
Center of gravity is over the area of support: car is in equilibrium.

(b)



The higher the center of gravity, the smaller the incline needed to tip the vehicle over.

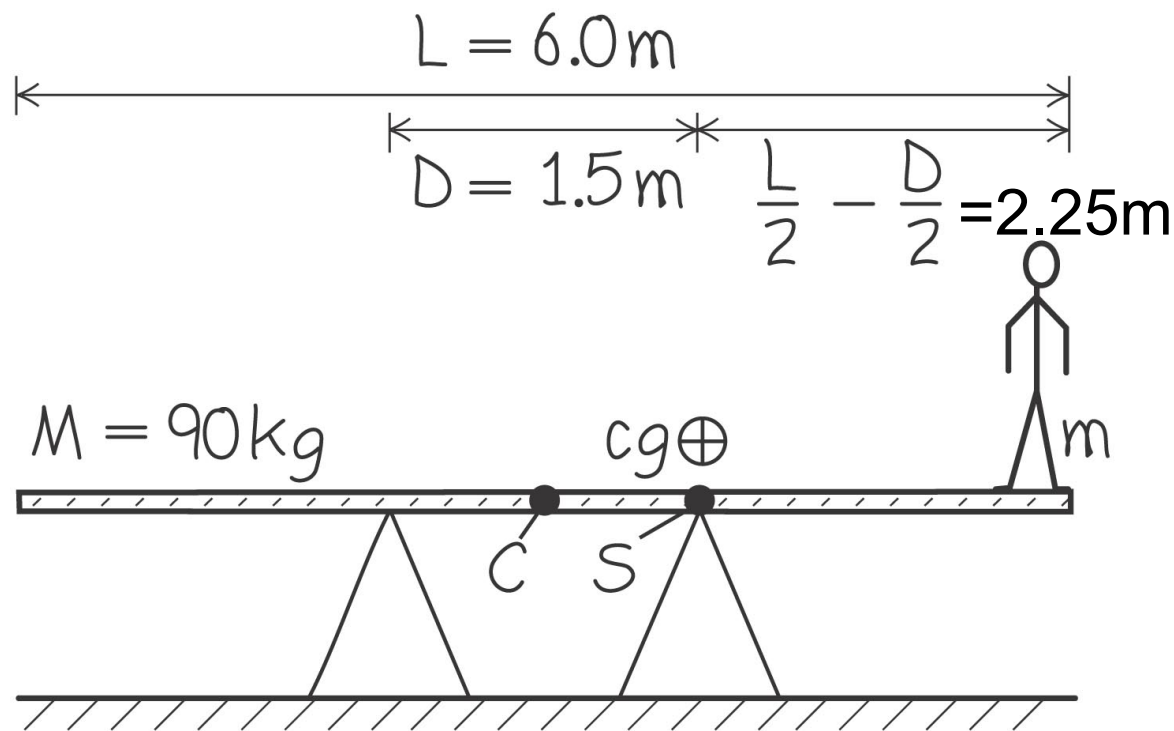
(c)



Center of gravity is outside the area of support: vehicle tips over.

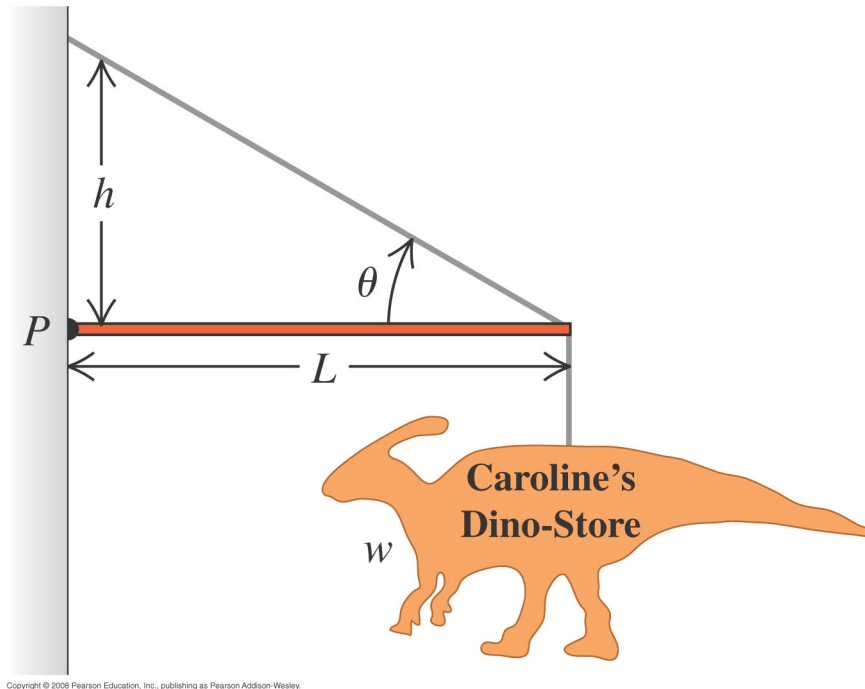
A “teeter-totter”

Given: Mass of an uniform plank (M) = 90kg. Find maximum mass of boy (m) such that the boy-plank system is in equilibrium.



Answer: Want the cg of boy-plank system to be located at S.
=> Net torque due to the weight of plank and boy about S is zero.

Solving rigid-body equilibrium problems



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Given: $w=100$ lb, w of rod= 50 lb,
 $L=1$ m, $\theta = 30^\circ$

Find: (a) Tension (T)
(b) the force vector exerted on
the rod by the hinge at point
 P .

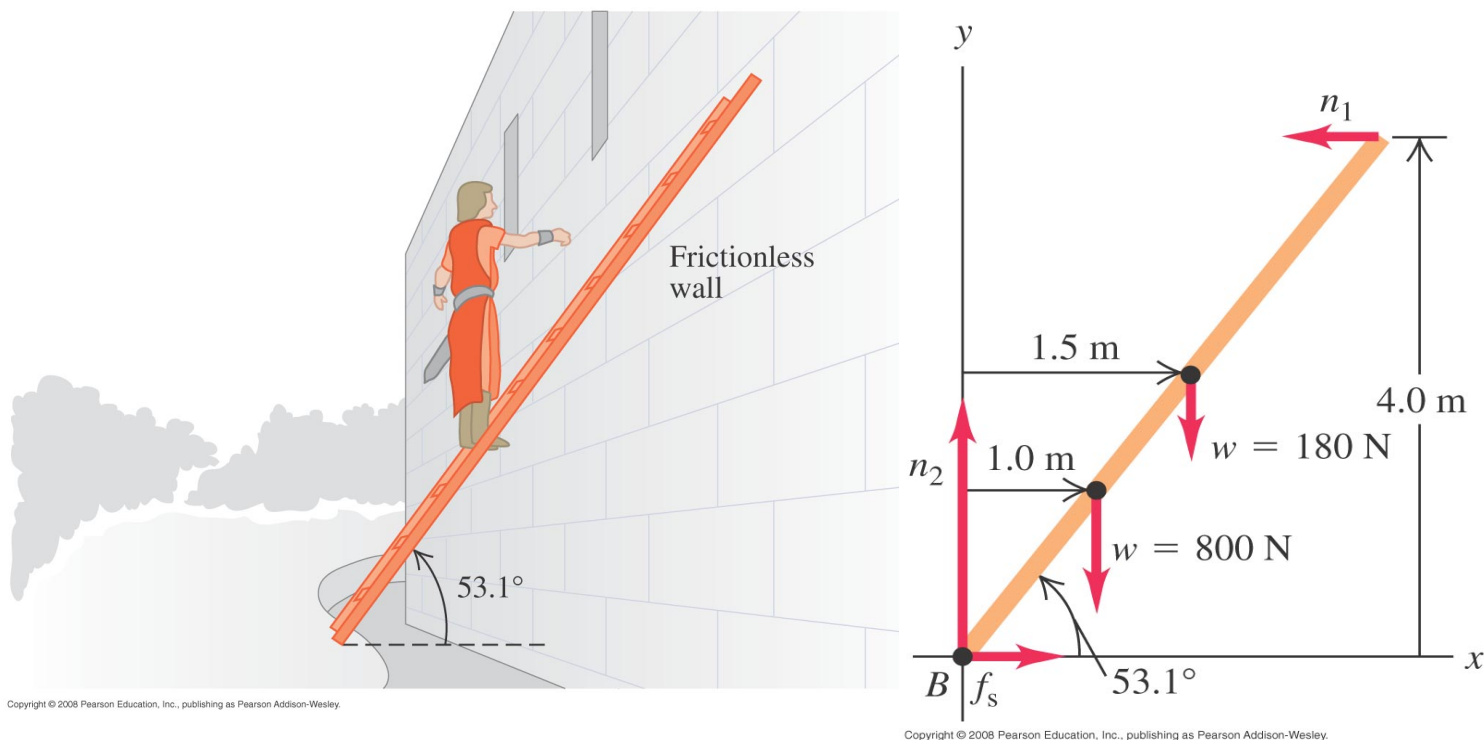
Procedures

- 1) Pick the rod to be the object in equilibrium.
- 2) Draw free-body diagram for the rod.
- 3) Set up net $F=0$ equations.
- 4) Set up net torque= 0 equation (pick a convenient origin)

Another rigid-body equilibrium problem

(b)

(a)



Given: wt. of man=800N, wt. of ladder=180N, length of ladder = 5 m

Man is 1/3 way up the ladder, wall is frictionless.

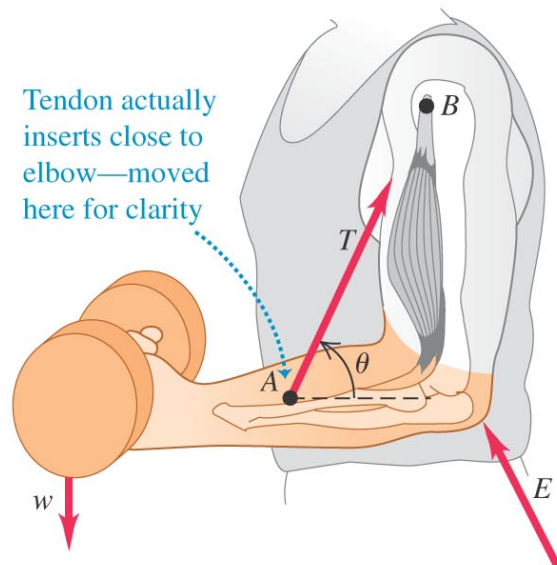
Find: n_1 , n_2 , f_s .

Another static equilibrium problem

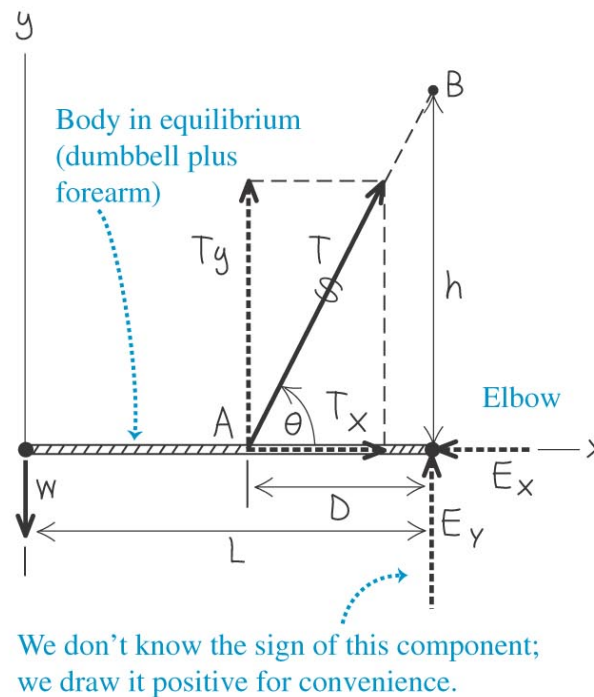
Given: $w=200\text{N}$, $D=0.05\text{m}$, $L=0.30\text{m}$, and $\theta=80^\circ$

Find $|T|$ and $|E|$

(a)



(b)



Answer on P. 362.