Chapter 11

Angular Momentum

P. Lam 7_25_2012

Learning Goals for Chapter 11

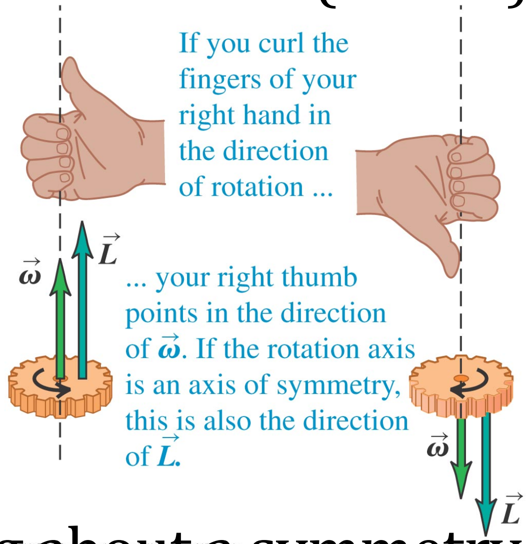
- To study angular momentum and its conservation
- To relate rotational dynamics and angular momentum

Definition of Angular momentum

Analogy between linear and angular variables:

Linear momentum ($\vec{p}=m\vec{v}$) $\rightarrow$ Angular momentum ($\vec{L}=I\vec{\omega}$?)

$$\frac{d\vec{p}}{dt}=\vec{F}\rightarrow\frac{d\vec{L}}{dt}=torque=\vec{r}\times\vec{F}?$$



$\vec{L}=I\vec{\omega}$ only when the rigid object is rotating about a symmetry axis.

Think of the object is a sum of many point particles.

The angular momentum of each point particle is:

$\vec{L}=\vec{r}\times\vec{p}=\vec{r}\times(m\vec{v})$ (definition)

$$\frac{d\vec{L}}{dt}=\frac{d\vec{r}}{dt}\times(m\vec{v})=\vec{r}\times(m\frac{d\vec{v}}{dt})=\vec{r}\times\vec{F}\ \text{(first term=0}\ \because\vec{v}\times\vec{v}=0)$$

Compare linear and rotational dynamics

Linear

$\vec{F} = m\vec{a}$ (if $m = \text{constant}$)

$\vec{p} = m\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For a system of objecys:

If net $\vec{F}_{\text{external}} = 0$

then $\vec{p}_{\text{total}}$ is conserved.

Rotational dynamics of a rigid object

$\vec{\tau} = I\vec{\alpha}$ (if $I = \text{constant}$)

$\vec{L} = I\vec{\omega}$ (valid for rotation about a symmetry axis)

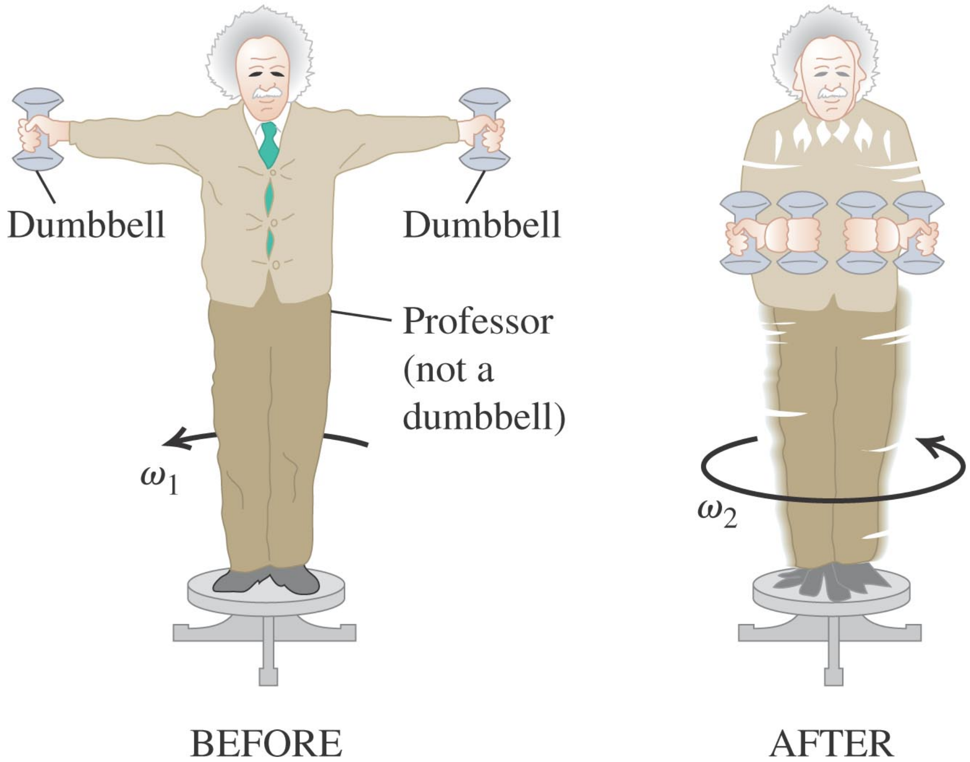
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

If net $\vec{\tau}_{external} = 0$,

then, $\vec{L}_{\text{total}}$ is conserved.

Conservation of Angular momentum

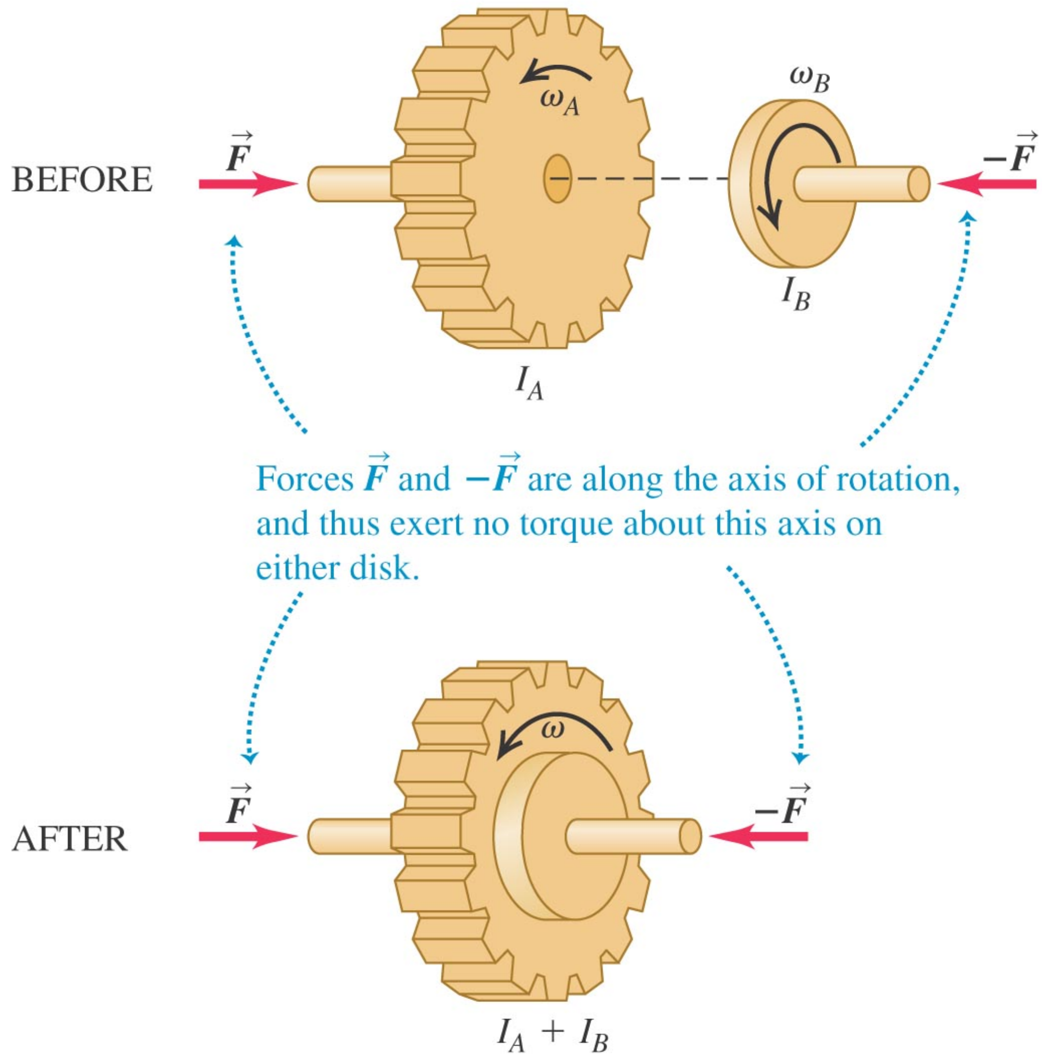
- The frictionless platform ensures the external torque along the z-direction is zero=>L_z is conserved



$$I_1\omega_1 = I_2\omega_2; \quad I \downarrow \omega \uparrow$$

This is how a car's clutch works-conservation of angular momentum

$$I_A\vec{\omega}_A + I_B\vec{\omega}_B = (I_A + I_B)\vec{\omega}$$



Conservation of angular momentum in daily devices:

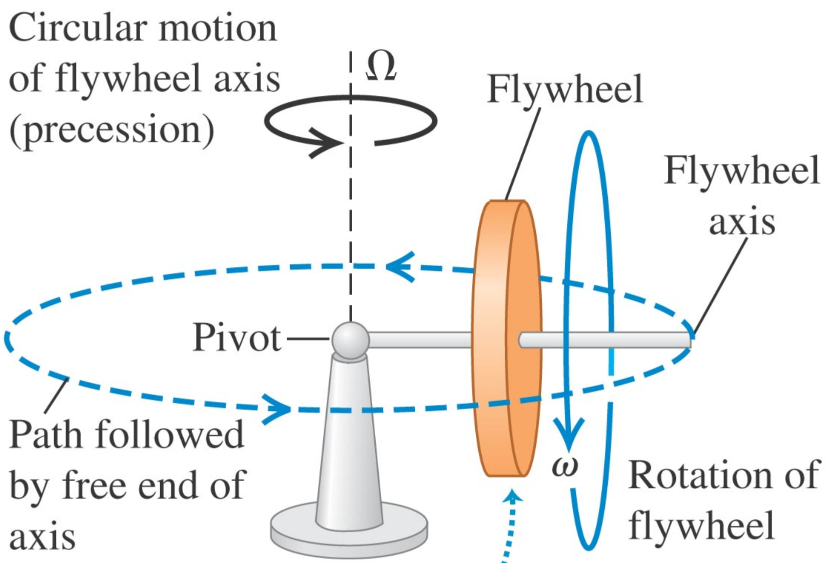
Gyroscope - A fast spinning top whose angular momentum vector points at a fixed direction in space => a navigation device.

Fast rotating bicycle wheels keep the bicycle stable.

What is the purpose of the small propeller at the back of a helicopter?

Gyroscopic precession

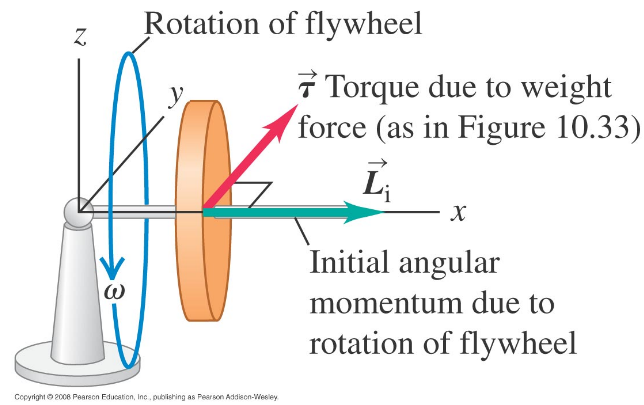
- If external torque =/ =0, there could be precession motion.



When the flywheel and its axis are stationary, they will fall to the table surface. When the flywheel spins, it and its axis "float" in the air while moving in a circle about the pivot.

(a) Rotating flywheel

When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.



(b) View from above

Now the effect of the torque is to cause the angular momentum to precess around the pivot. The gyroscope circles around its pivot without falling.