Chapter 10.B

Dynamics of Rotational Motion

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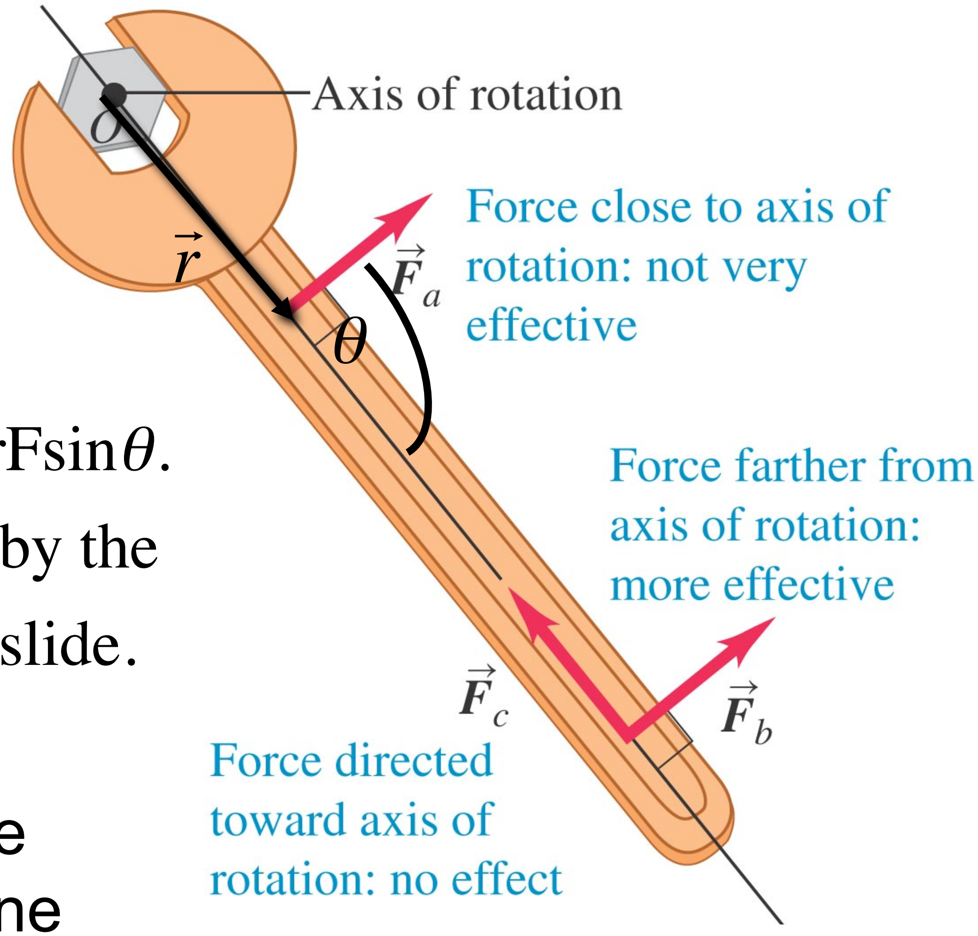
Learning Goals for Chapter 10.2

- To see how torques cause rotational dynamics (just as linear forces cause linear accelerations)
- To calculate work done by a torque
- To study angular momentum and its conservation
- To relate rotational dynamics and angular momentum

Torque

- A force (**F**) applied at a distance (r) and at an angle (θ) will generate a torque (**τ**).

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

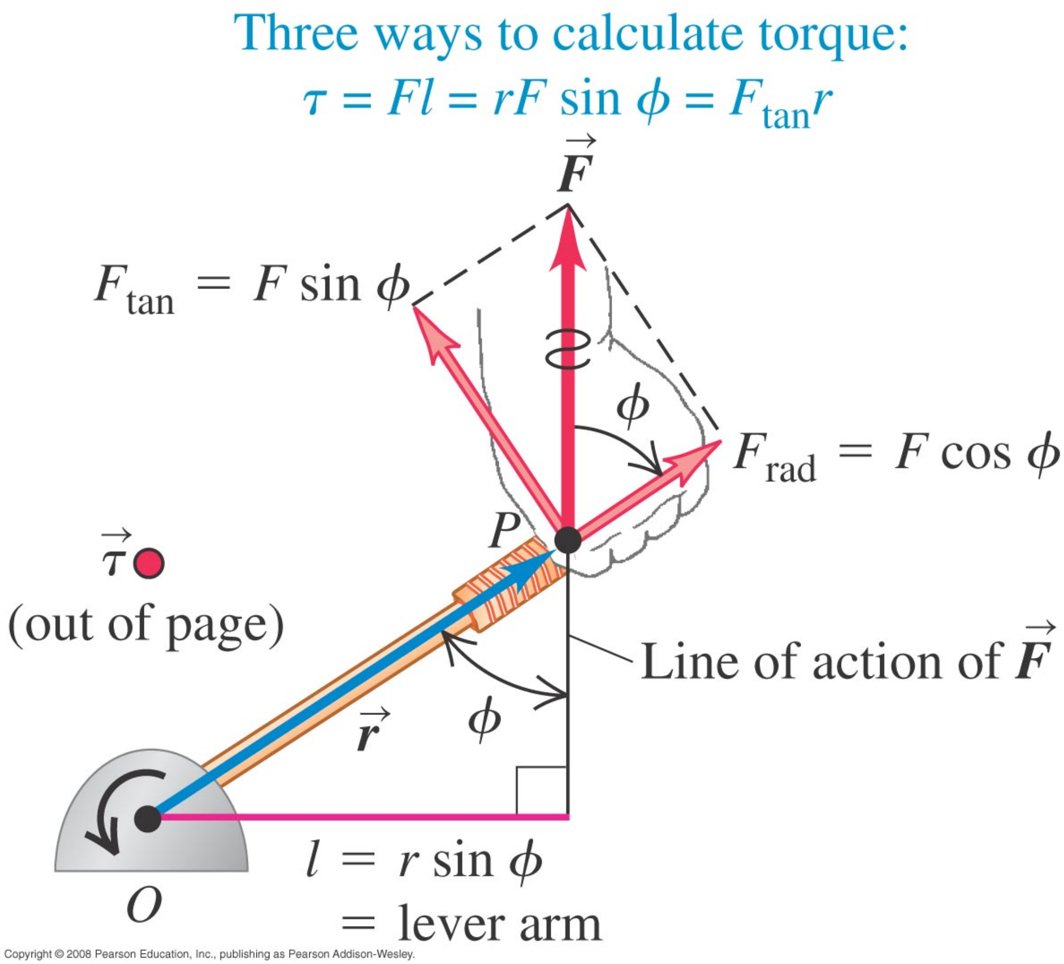


$$\text{Magnitude of torque} = |\vec{\tau}| = \text{rFsin}\theta.$$

Direction of torque is given by the "right - hand rule" - see next slide.

Which force on the figure produces the largest torque *about point O* and which one produces the smallest torque?

Moment force and Lever arm



Compute cross product

using $\hat{i}, \hat{j}, \hat{k}$:

In this example:

$\vec{r} = r\sin\phi\,\hat{i} + r\sin\phi\,\hat{j}$

$\vec{F} = F\hat{j}$

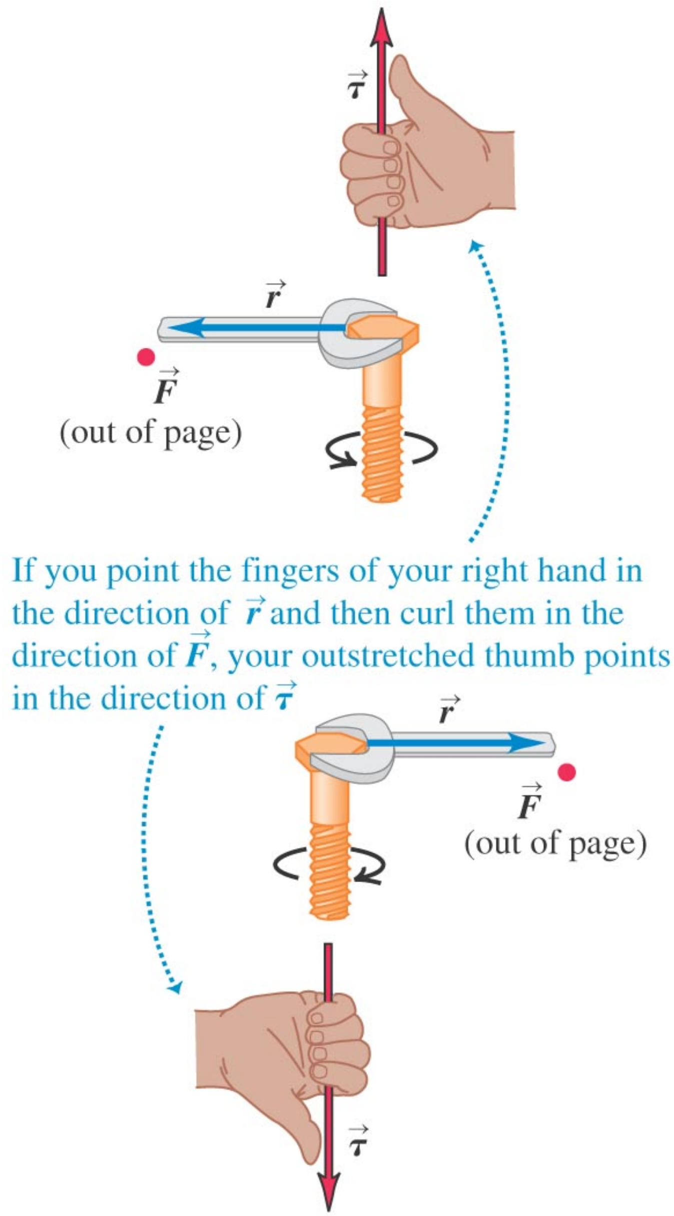
$\vec{r} \times \vec{F} = \left(r\sin\phi\,\hat{i} + r\cos\phi\,\hat{j}\right) \times F\hat{j}$

$= r\sin\phi\,\hat{i} \times F\hat{j} + r\cos\phi\,\hat{j} \times F\hat{j}$

$= rF\sin\phi\,\hat{k} + 0$

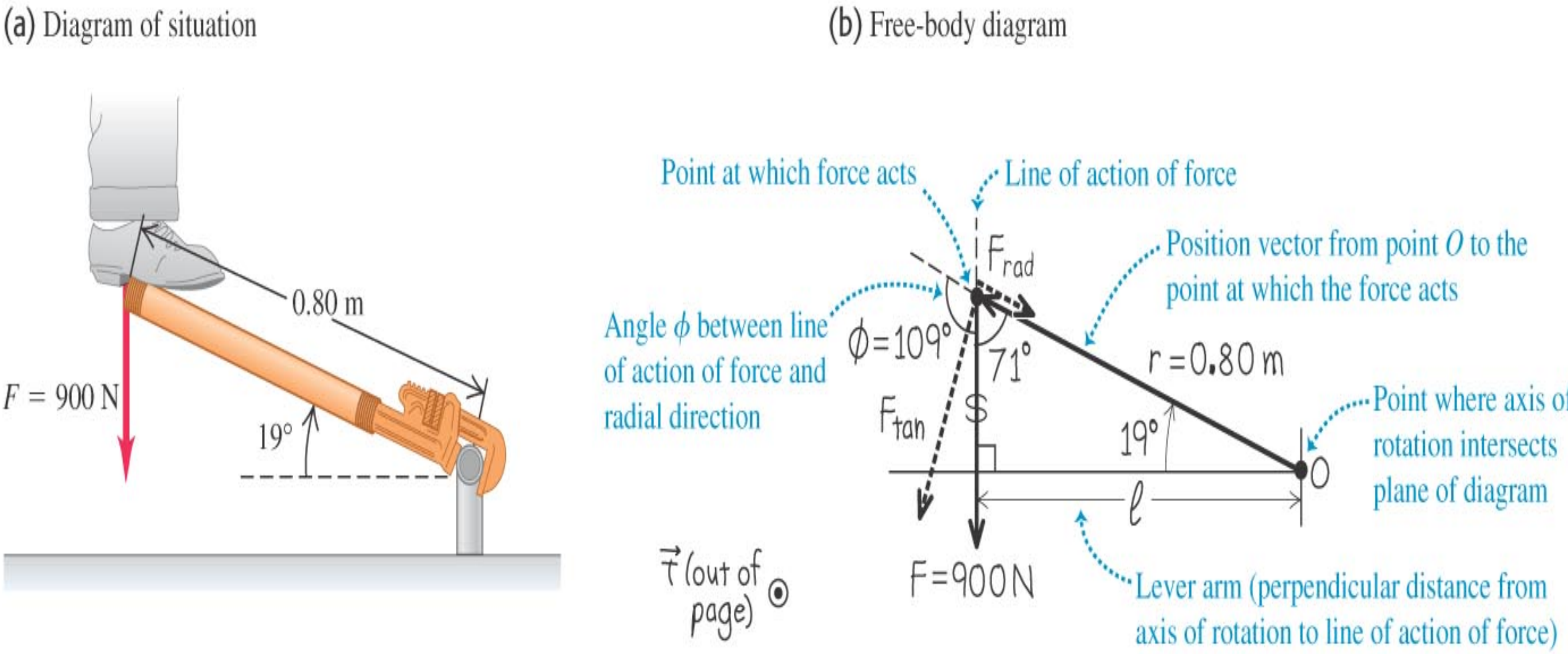
Direction of torque vector

- Mathematically, the direction of torque vector is given by the right-hand rule (RHR) by convention.
- Physical effect of torque is tend to rotate the object counterclockwise or clockwise.



Calculate an applied torque

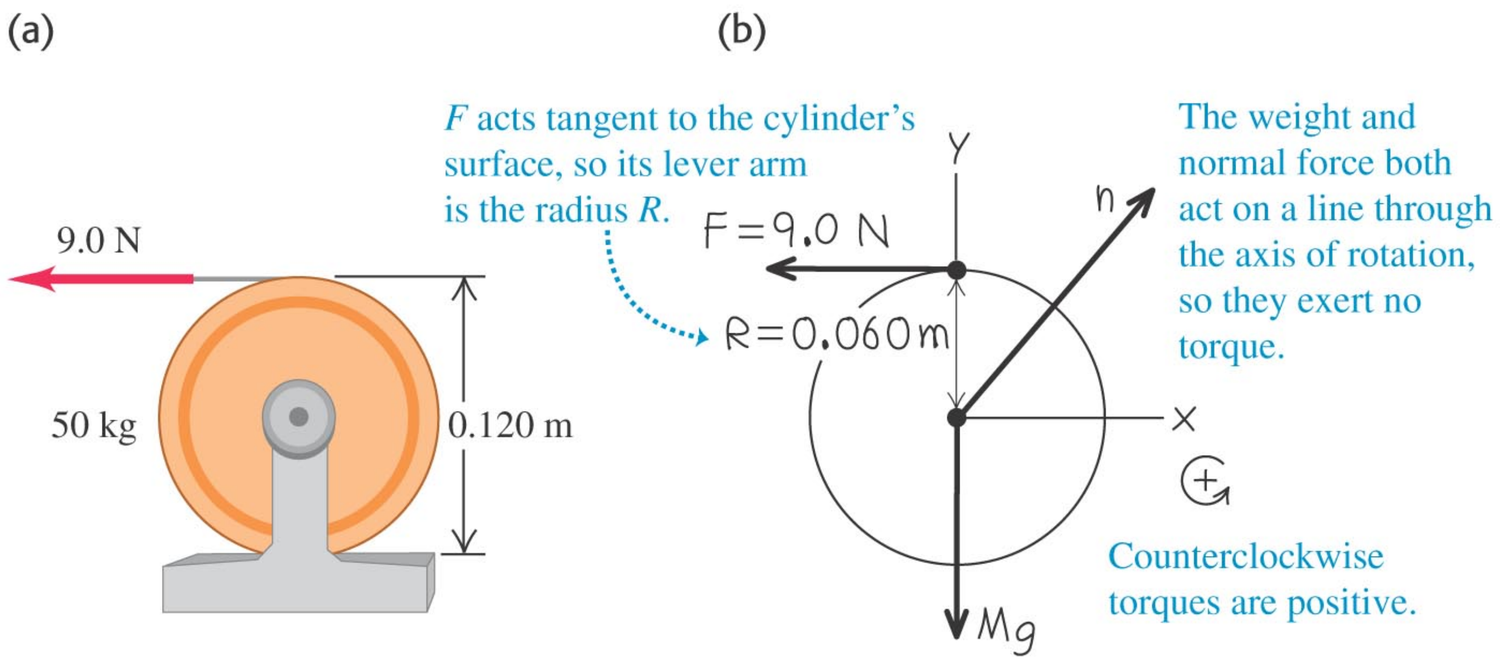
- Consider Example 10.1.
- Refer to Figure 10.5.



Which angle should you use in $|\vec{\tau}| = rF\sin\theta$?

$\mathcal{T} = I\alpha$ is just like $F = ma$

- A 9.0N force is applied to the wheel for 2s and then released.
- What is the angular acceleration as a function of time?
- What is the angular velocity as a function of time? (Given the wheel was initially at rest)



Another look at the unwinding cable

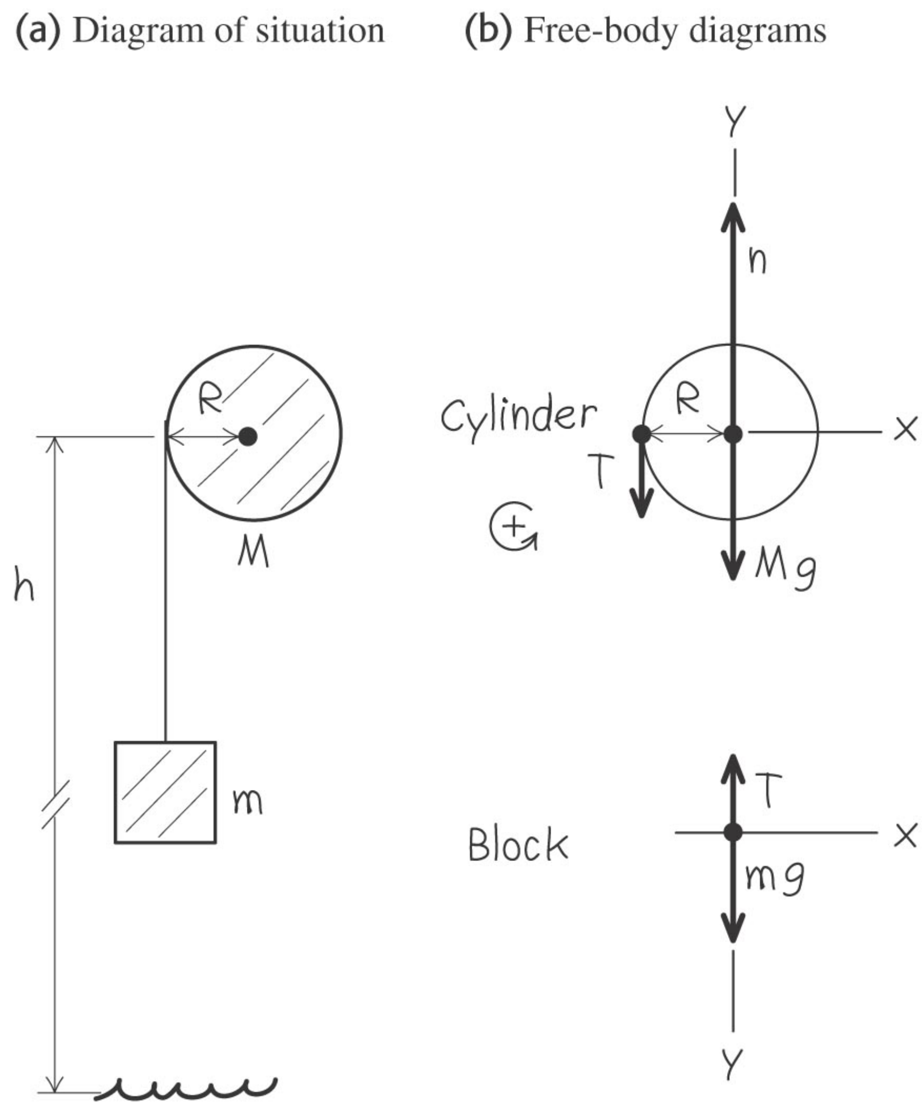
Given the mass (m) and the wheel (M), find the acceleration of m. (Assume no air-resistance or friction at the axle of the wheel, assume the no-slipping)

Concept 1: net $\vec{F} = m\vec{a}$

Concept 2: net $\vec{\tau} = I\vec{\alpha}$

Concept 3: $|\vec{a}| = R|\vec{\alpha}|$

(no slipping condition)



The yo-yo - rolling without slipping

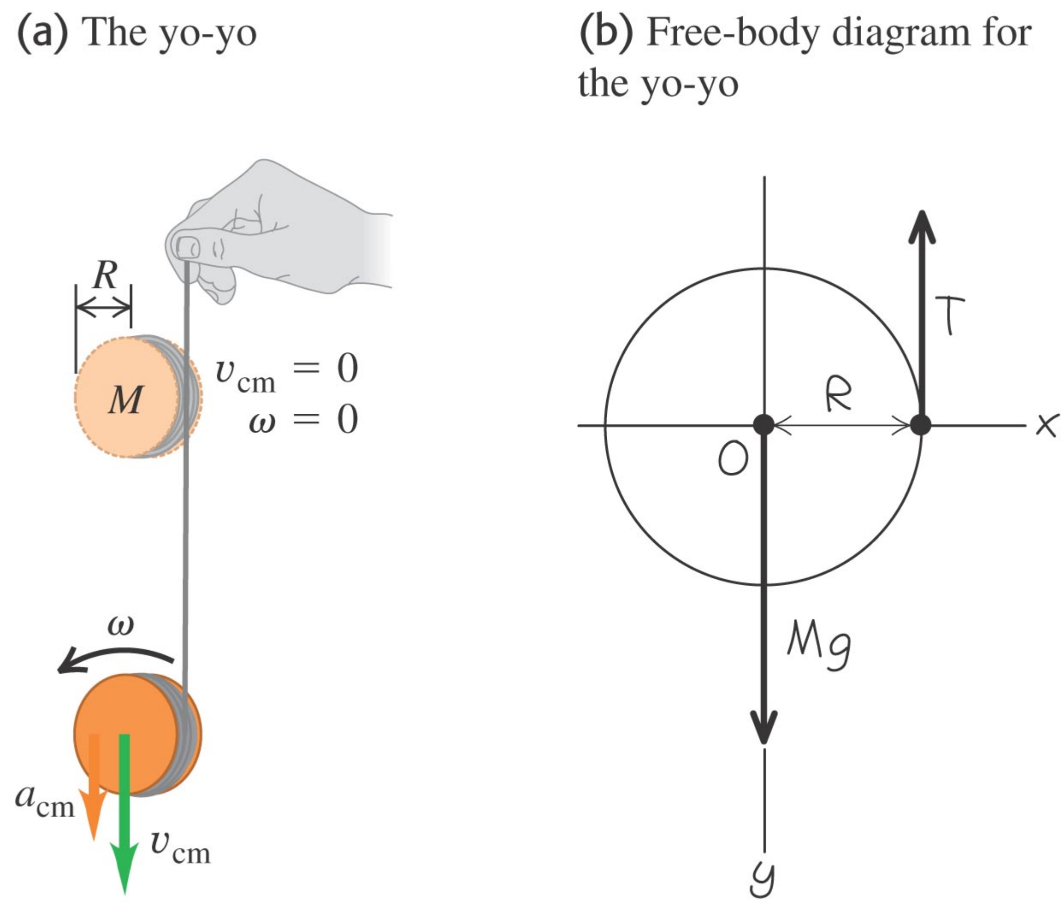
- Calculate the yo-yo's acceleration and then the final v after it has fallen a distance h.

Concept 1 : net $\vec{F} = M\vec{a}$

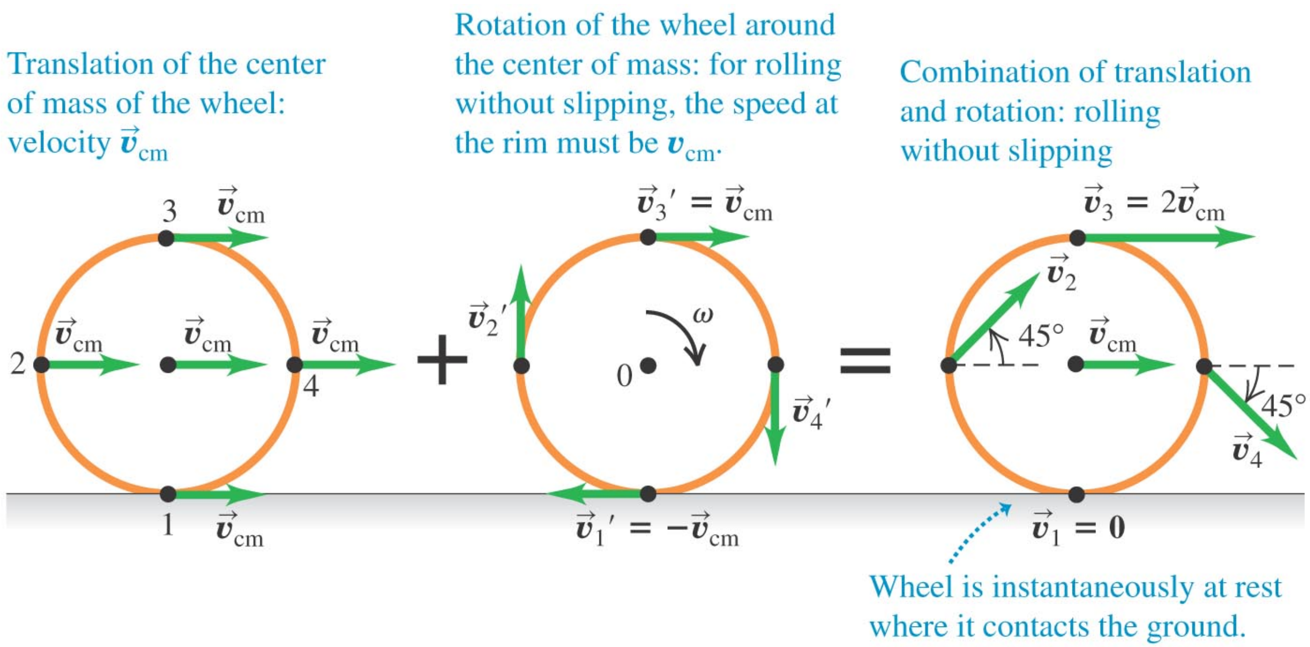
Concept 2 : net $\vec{\tau} = I\vec{\alpha}$

Concept 3 : $|\vec{a}| = R\,|\vec{\alpha}|$

(no slipping condition)



Energy method (translation + rotation)



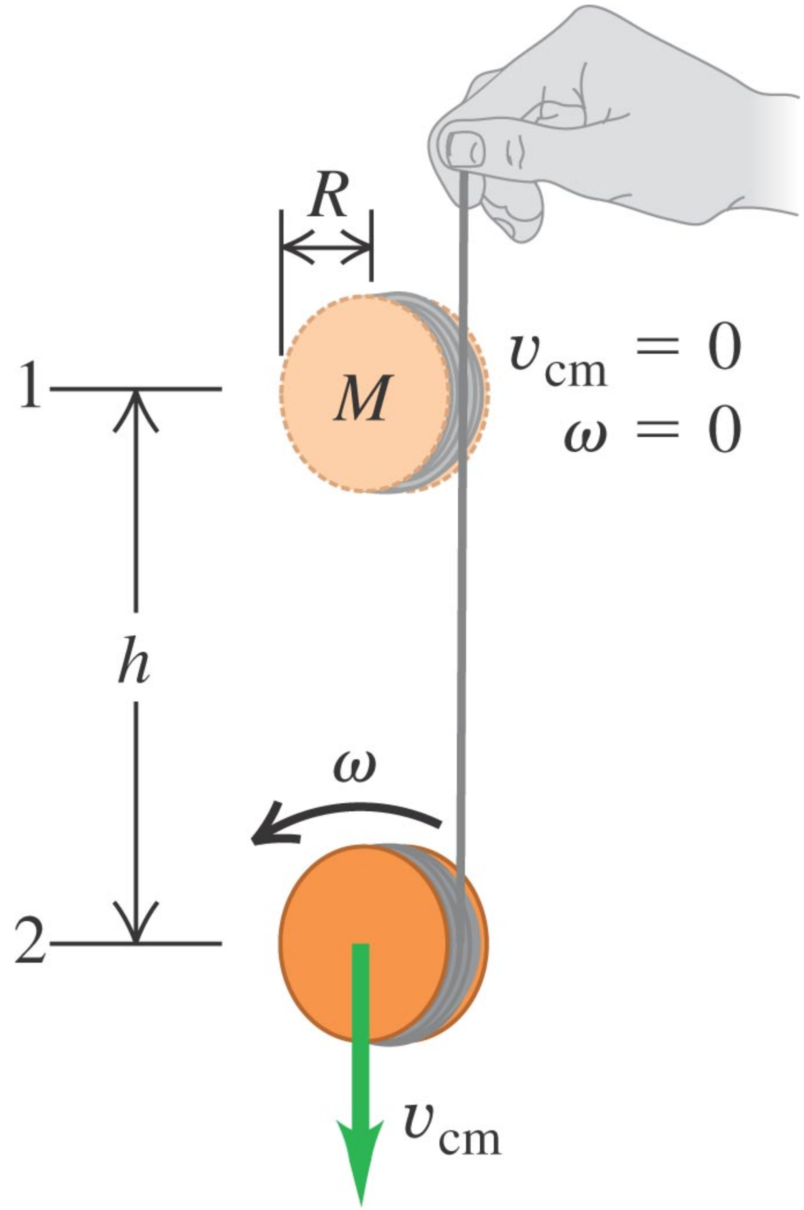
$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

Rolling without slipping condition $\Rightarrow v_{CM} = R\omega$

$$\Rightarrow K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \left(\frac{v_{CM}^2}{R^2} \right)$$

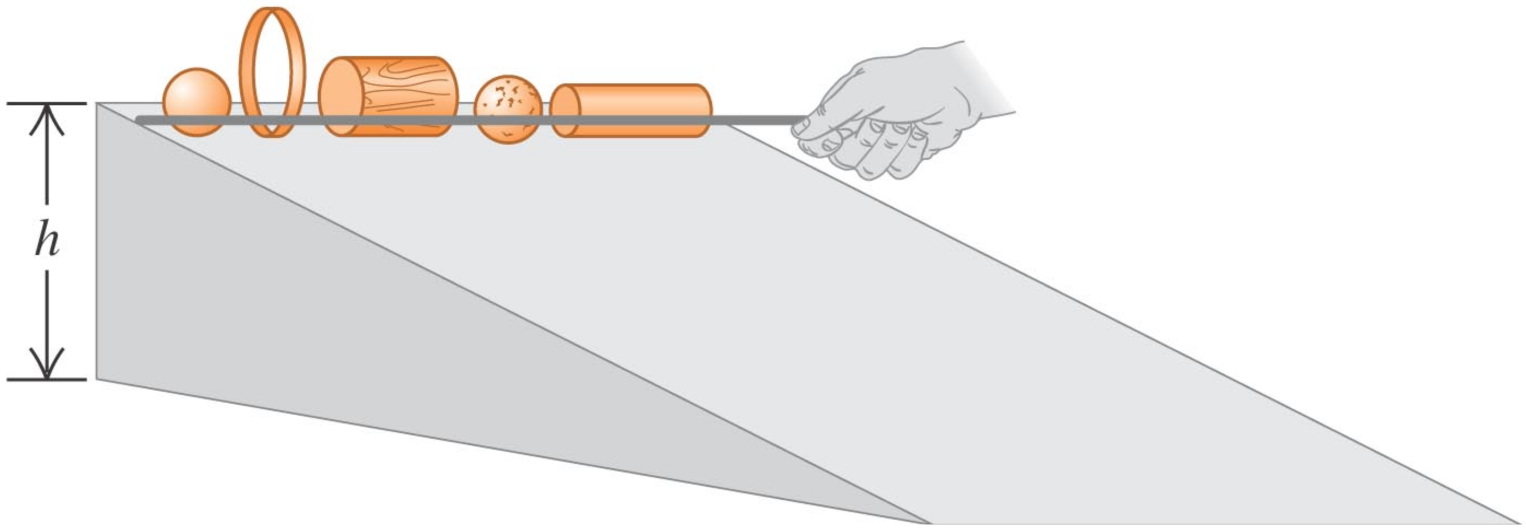
Application of Conservation of Energy

- Find speed of yo-yo after falling a distance h.



The race of objects with different moments of inertia

- Use energy method to determine which object will reach the bottom of the incline first (i.e. which object reach the bottom with the largest speed?)



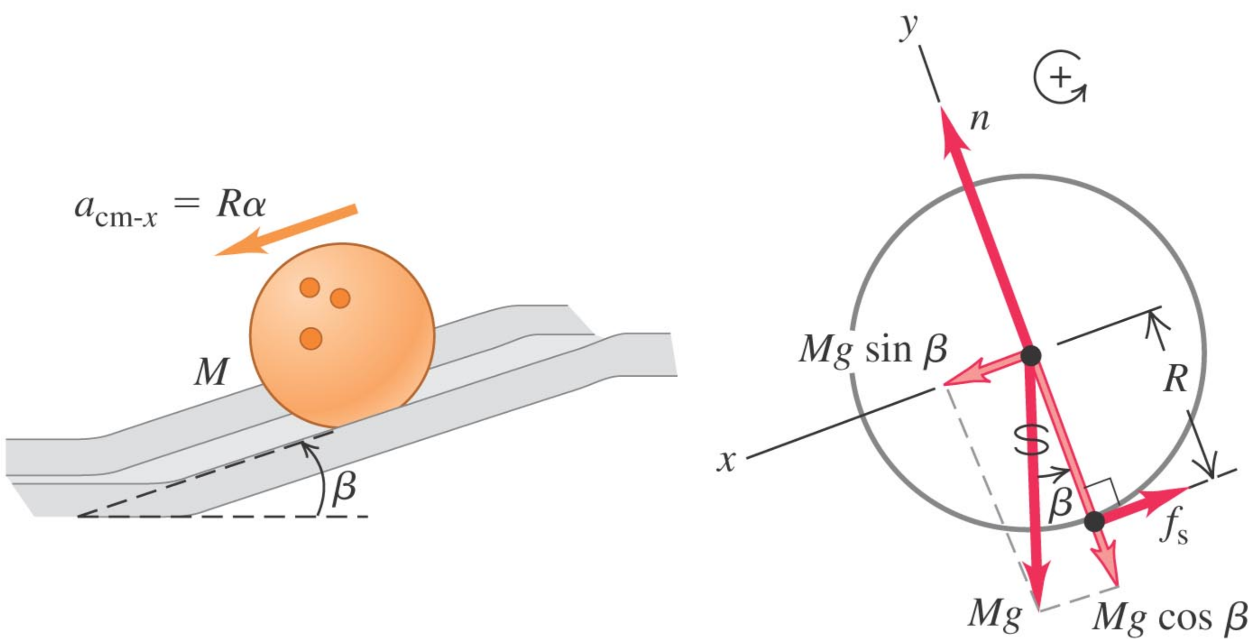
The object with the smallest moment of inertia will spend less energy rotating and hence has more energy for translation. The sphere has the smallest moment of inertia => will have the largest speed.

Why conservation of mechanical energy works when there is friction?

- What is the role of friction?

(a) The bowling ball

(b) Free-body diagram for the bowling ball



Answer: The role of static friction is to do negative work on the center of motion while doing positive work on the rotation motion. Net result, the work done by static friction is zero.