# Chapter 10.A

## Rotation of Rigid Bodies

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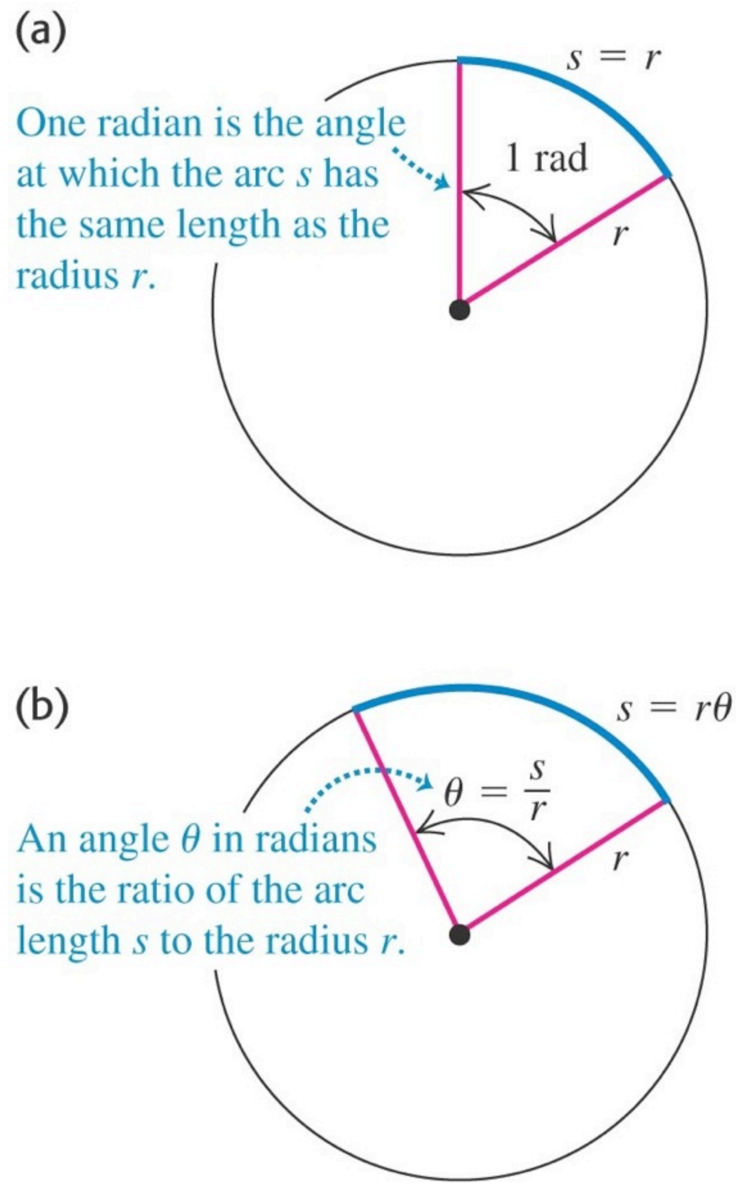
## Learning Goals for Chapter 10.1

- Understand the equations govern rotational kinematics, and know how to apply them.
- Understand the physical meanings of moments of inertia and rotational kinetic energy
- Apply Work-Kinetic Theorem and Conservation of Mechanical Energy for rotational motions.

# Angular motions in revolutions, degrees, and radians

- One complete rotation of 360° is one revolution.
- One complete revolution is 2π radians (definition of radian, see Fig. a). The arc length s = rθ.
- Relating the two, 360° = 2 π radians or 1 radian = 57.3°.



# Angular displacement = the angle being swept out

- We denote angular displacement as $\Delta\Theta$ (theta). It is the angular equivalence of $\Delta x$ or $\Delta y$ in earlier chapters.

How to treat $\Delta\theta$ as a vector?

We use the axis of rotation as the unit vector direction. For example:

Let the axis of rotation be along $\hat{k}$,

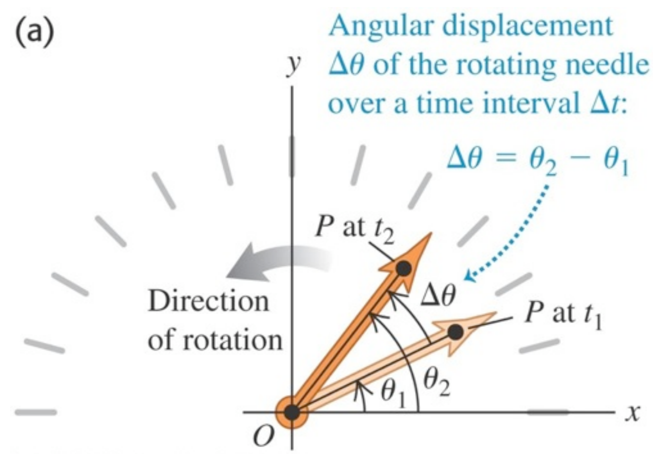
a counterclockwise rotation of 2 radians

about the z-axis is denoted by $\Delta\vec{\theta} = +2\hat{k}$,

a clockwise rotation of 2 radians

about the z-axis is denoted by $\Delta\vec{\theta} = -2\hat{k}$

("*Right – hand – rule*")



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# Angular velocity

Angular velocity (denoted by $\omega$) $\equiv \dfrac{\Delta\vec{\theta}}{\Delta \mathrm{t}}$

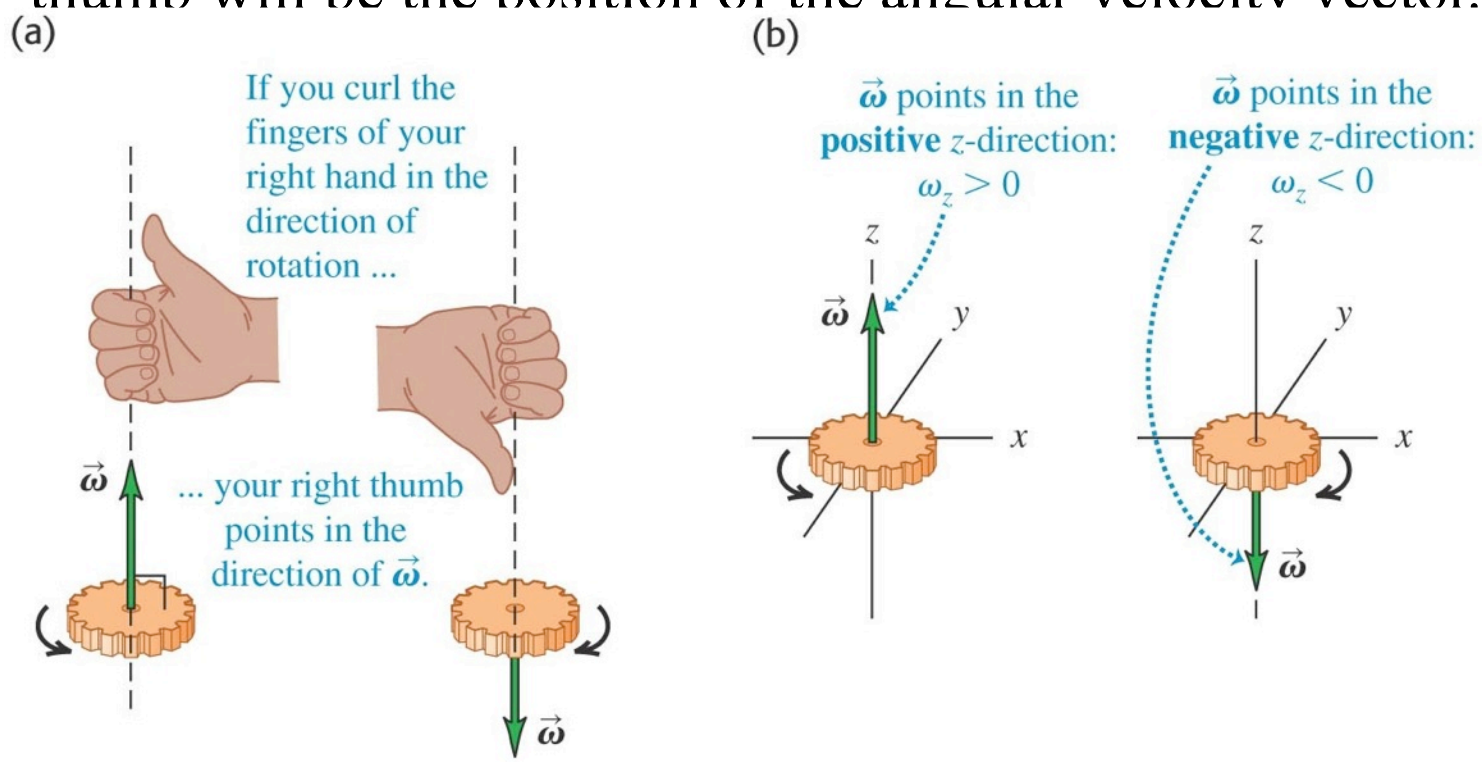
Instantaneous angular velocity, $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$

The direction of $\vec{\omega} - vector$ is denoted by the axis of rotation (positive=counterclockwise; negative=clockwise). The unit of $\vec{\omega}$ is rad/s.

Question: What is the angular velocity of the second-hand of a clock?

# Angular velocity is a vector

- You can visualize the position of the vector by sweeping out the angle with the fingers of your right hand. The position of your thumb will be the position of the angular velocity vector. This is

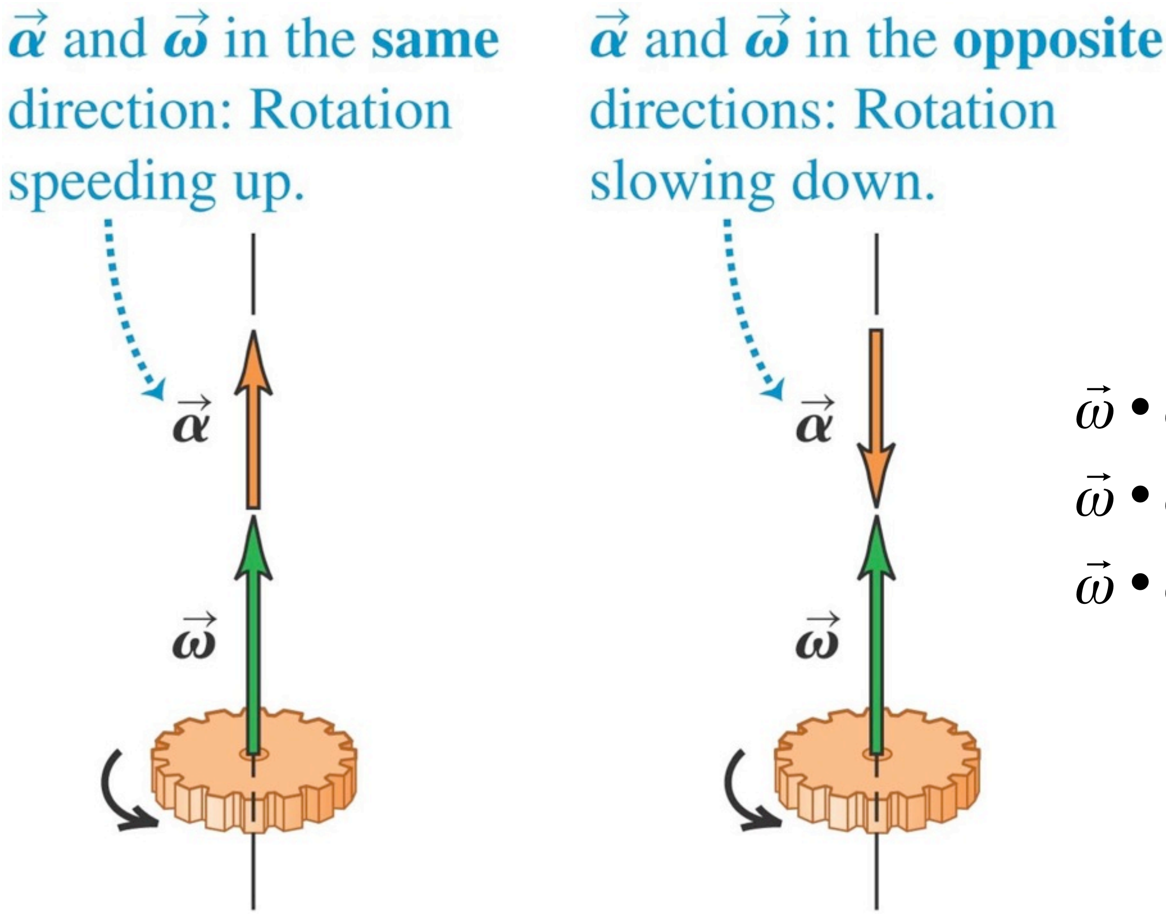



e.g. $\vec{\omega} = +10\dfrac{\text{rad}}{\text{s}}\hat{\text{k}} \Rightarrow$ counterclockwise rotation of $10\dfrac{\text{rad}}{\text{s}}$ about the z - axis.

# Angular acceleration

- The angular acceleration (α) is the change of angular velocity divided by the time interval during which the change occurred. $\vec{\alpha}_{average} = \frac{\Delta\vec{\omega}}{\Delta t}, \quad \vec{\alpha}_{\text{instantaneous}} = \frac{d\vec{\omega}}{dt}$
- The unit for α is radians per second$^2$.
- Question: What is the angular acceleration of the second-hand of a clock?

# Angular acceleration is a vector




$\vec{\omega} \bullet \vec{\alpha} > 0 \Rightarrow$ speeding up

$\vec{\omega} \bullet \vec{\alpha} < 0 \Rightarrow$ slowing up

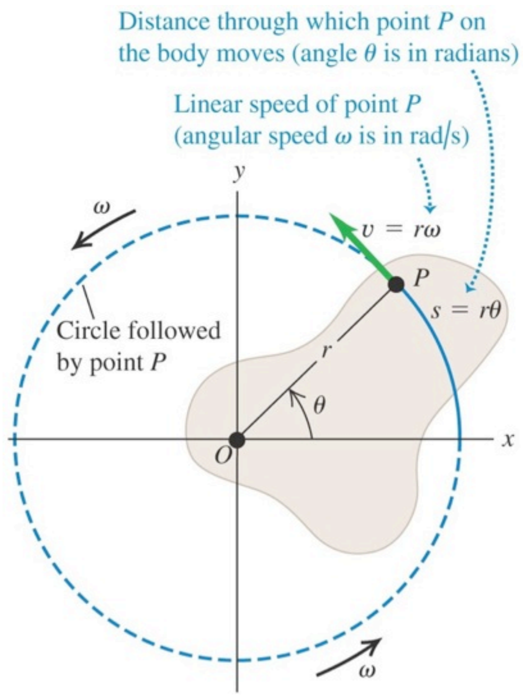
$\vec{\omega} \bullet \vec{\alpha} = 0$ => ?

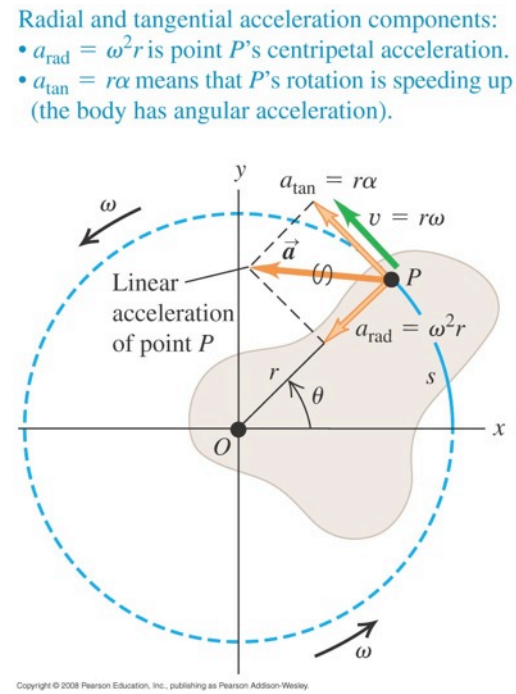
# Compare linear and angular kinematics

**Table 9.1** Comparison of Linear and Angular Motion with Constant Acceleration

| Straight-Line Motion with Constant Linear Acceleration | Fixed-Axis Rotation with Constant Angular Acceleration |
|---|---|
| $a_x = \text{constant}$ | $\alpha_z = \text{constant}$ |
| $v_x = v_{0x} + a_x t$ | $\omega_z = \omega_{0z} + \alpha_z t$ |
| $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$ | $\theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$ |
| $v_x^2 = v_{0x}{}^2 + 2a_x (x - x_0)$ | $\omega_z^2 = \omega_{0z}{}^2 + 2\alpha_z (\theta - \theta_0)$ |
| $x - x_0 = \frac{1}{2}(v_x + v_{0x}) t$ | $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z}) t$ |

# Relate Linear and angular quantities



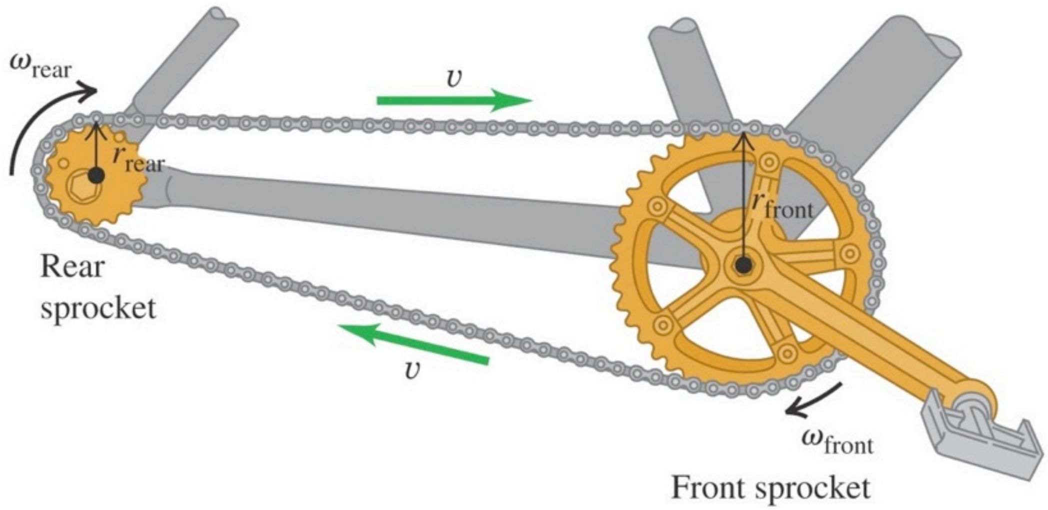

Arc length : $s = r\theta$

Tangential speed : $v_t = r\omega$

Tangential acceleration : $a_t = r\alpha$

Radial acceleration (centripetal acceleration) : $a_r = \dfrac{v_t^2}{r} = \dfrac{(r\omega)^2}{r} = \omega^2 r$

# Relate Linear and angular quantities-example




Suppose you paddle the front wheel at 2 rev/s.

(a) Find $\omega_{front}$

(b) Find speed of the chain

(c) Find $\omega_{rear}$

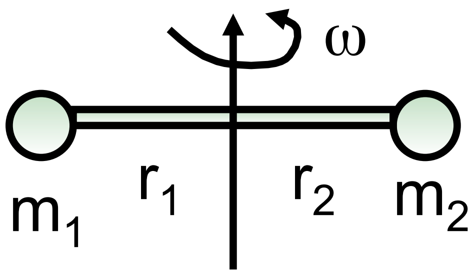
(d) A fly is sitting on the rim of the front sprocket,

what it is centripetal acceleration? is tangential acceleration?

Given: $r_{rear} = 0.05m$, $r_{front} = 0.1m$.

# Rotational inertia and rotational kinetic energy

Consider the rotational kinetic energy of a barbell (**idealized as two "point" masses connected by massless rod**)




$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1\left(r_1\omega\right)^2 + \frac{1}{2}m_2\left(r_1\omega\right)^2 = \frac{1}{2}\left[m_1 r_1^2 + m_2 r_2^2\right]\omega^2$$

$$\left[m_1 r_1^2 + m_2 r_2^2\right] \equiv I = \textit{ moment of inertia about the rotational axis } (\textit{rotational inertia})$$

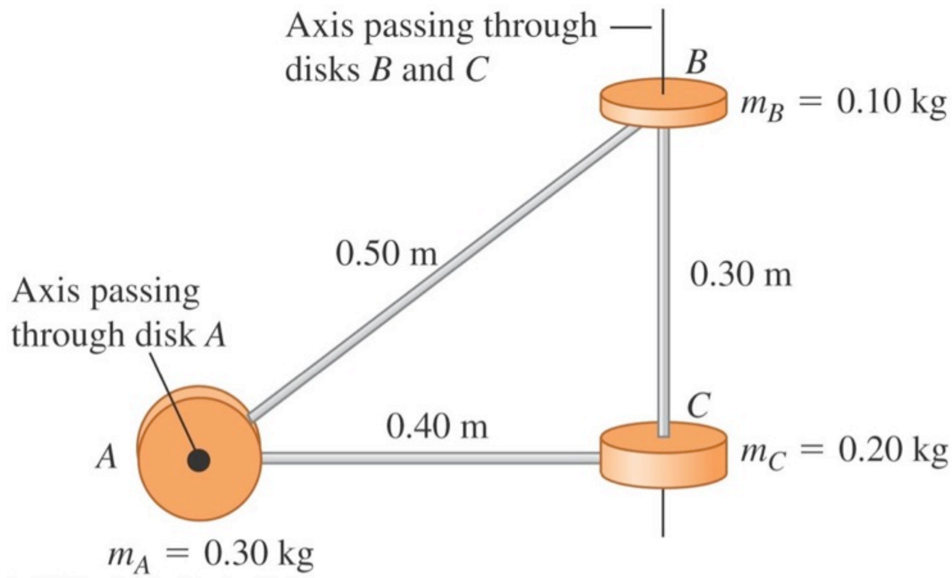
$$\Rightarrow K = \frac{1}{2}I\omega^2$$

Becareful : I $\neq$ Impulse from last chapter.

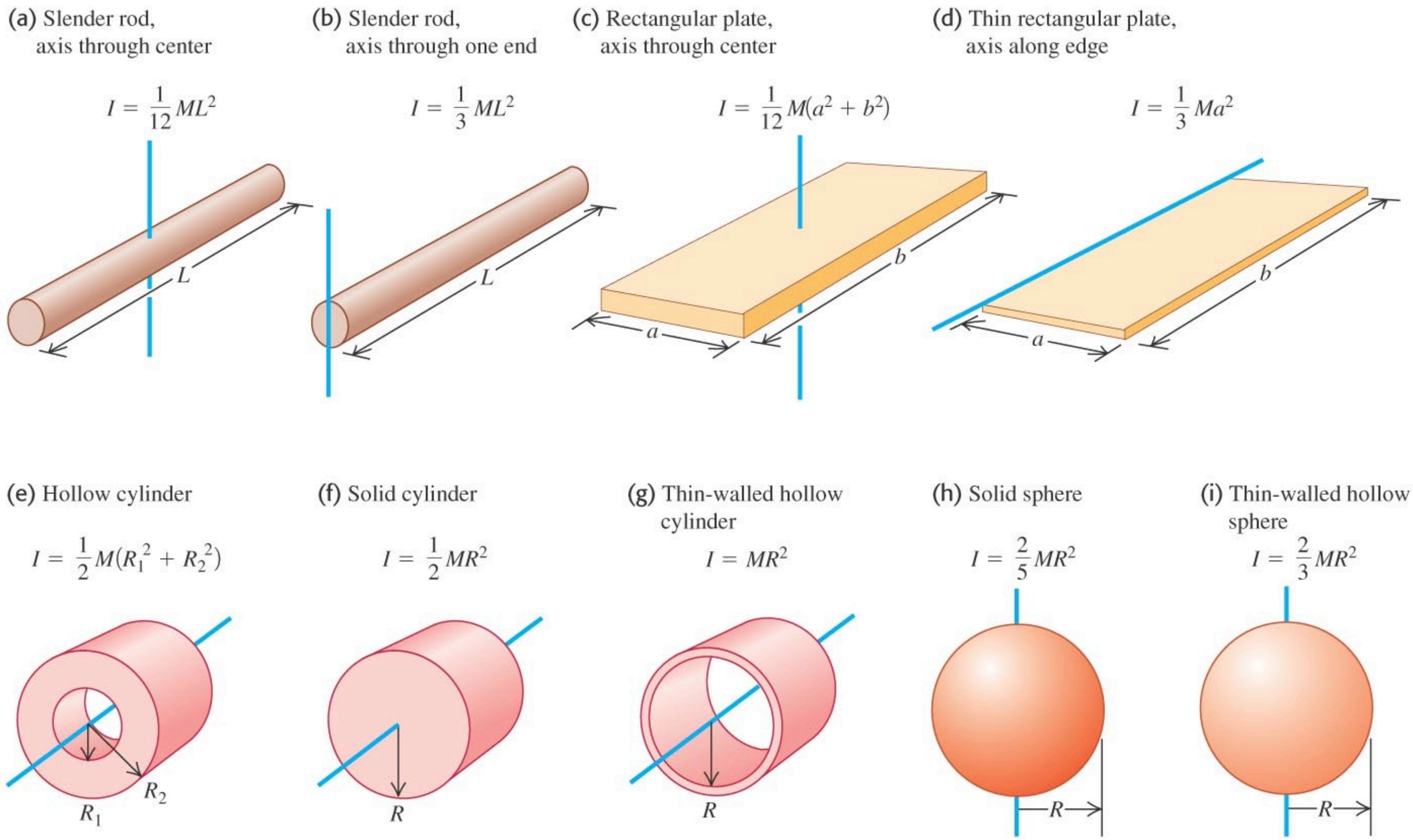
***Note: The further away the masses are from the axis, the greater the moment of inertia.***

# Moment of inertia of a distribution of "point" masses

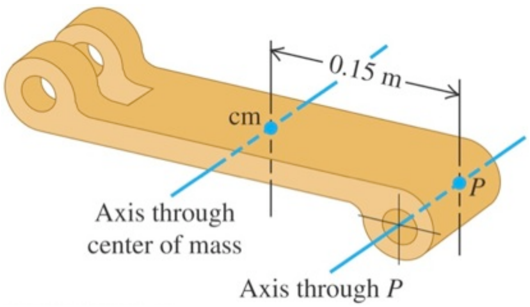
- **For now, treat each disk as a point mass**
- The moment of inertia depends on the distribution of mass about the rotation axis.
- Example: Find the moment of inertia about an axis passing through disk A and compare it with the moment of inertia about an axis passing through disk B and C. Treat each object as point masses for now.



# Moment of inertia for common shapes (about an axis that passes through the center of mass)



# Parallel Axis Theorem




$$I_{\text{axis of rotation}} = I_{cm} + Md^2$$

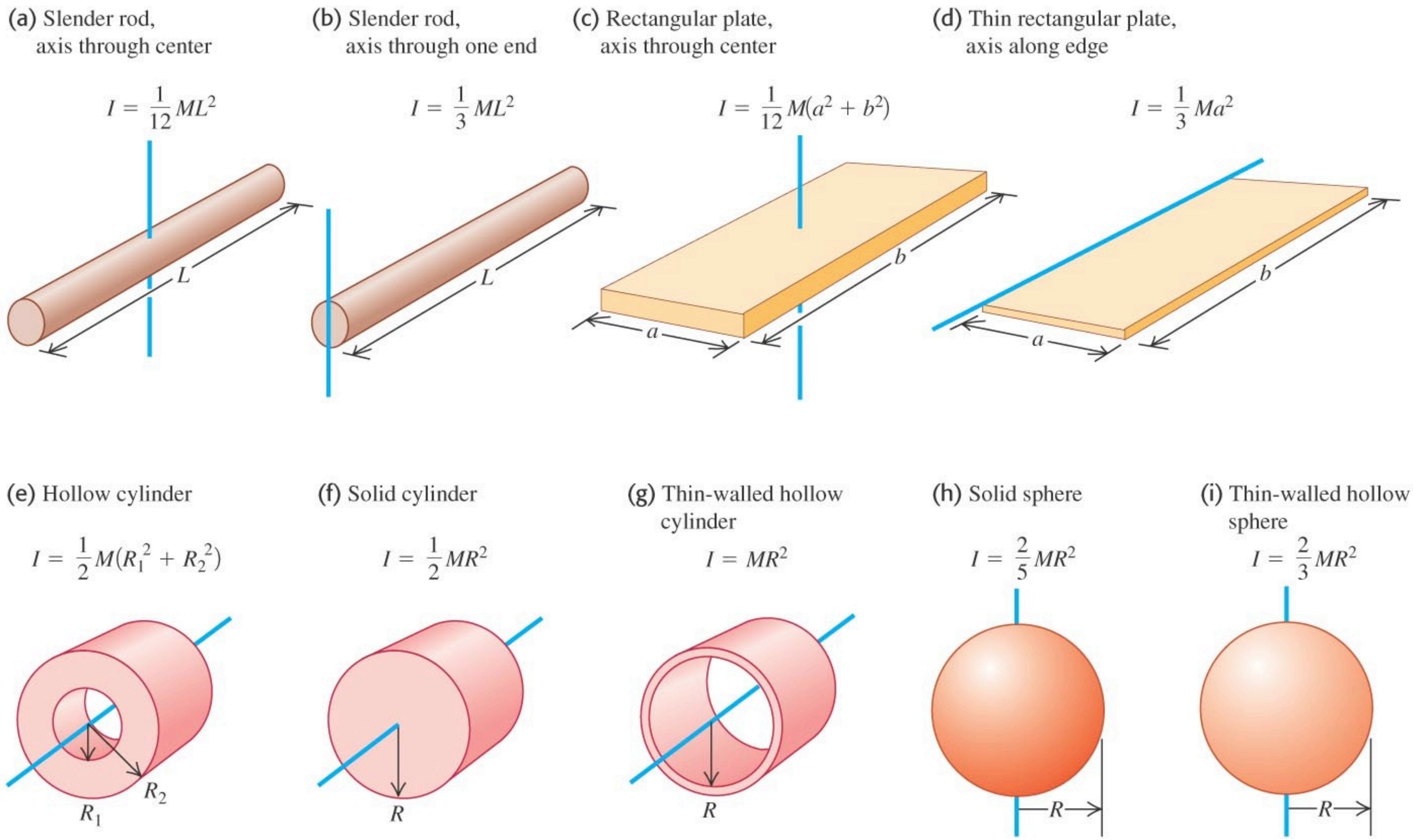
$d$ = distance from axis of rotation to the center of mass.

M = mass of the object

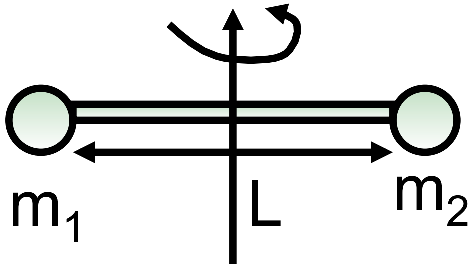
Given: $\mathrm{M} = 3.6\ \mathrm{kg},\ \mathrm{I}_{\mathrm{cm}} = 0.05 kg \bullet m^2$

Find $\mathrm{I}_{\text{axis through P}}$

# Do results (a) and (b) satisfy the “parallel-axis theorem” ?



## Optional problem: Finding the moment of inertia of a "real" barbell




$$\text{Given: } m_1 = m_2 = 5kg \quad \text{radius } R = 0.03\text{m}$$

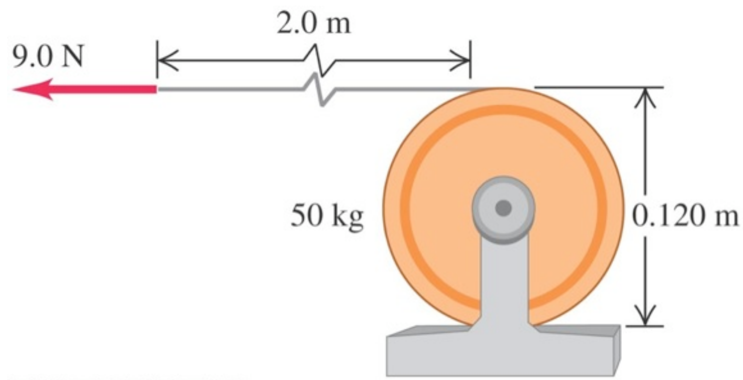
$$m_{rod} = 2\text{kg},\ L = 0.1\text{m}$$

Axis passes through the mid point of the rod.

Find I about this axis.

$$I = I_{rod} + I_1 + I_2 = ?$$

# Work-kinetic energy Theorem for rotational motion




Given: Wheel is initially at rest.

Find the final $\omega$, after a 9.0N of force has acted on the wheel over a distance of 2 m (pulling on the string which wraps around the wheel).

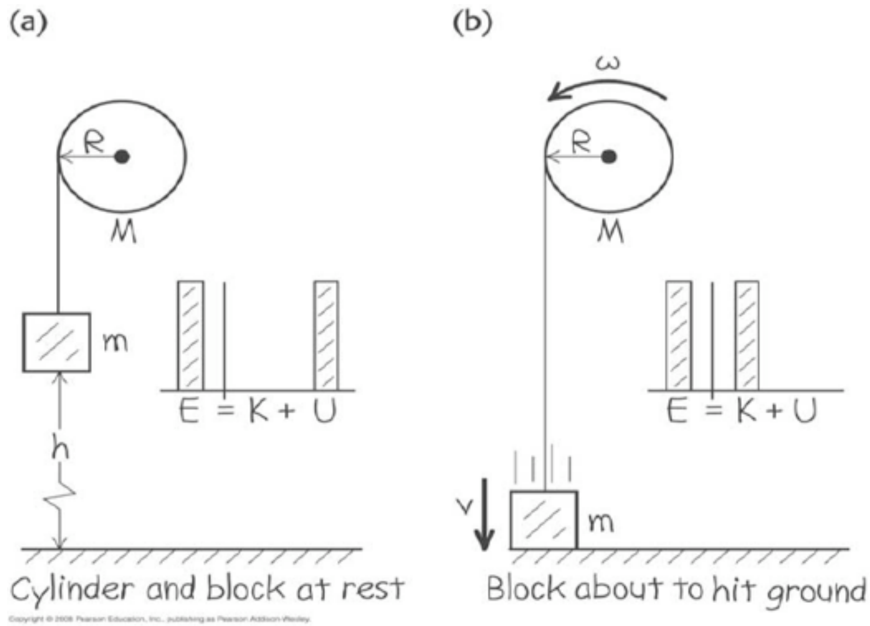
$$W = K_f - K_i = \frac{1}{2} I\omega_f^2 - 0$$

$$(9)(2) = \frac{1}{2}\left[\frac{1}{2}(50)(0.06)^2\right]\omega_f^2$$

$$\Rightarrow \omega_f = 20\frac{rad}{s}$$

*Instead of pulling on it with 9.N force, hang a mass of 0.9kg and let gravity pulls on it. Find* $\omega_f$ after the mass has dropped 2 m. (Assume g=10m/s$^2$)

# Conservation of mechanical energy - translational and rotational motion




Concept 1:

Conservation of Mechanical Energy

$$K_i + U_i = K_f + U_f$$

Concept 2:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Concept 3:

Rolling without slipping

$$R\omega = v$$

$Q$. Find the speed of mass m after it has fallen a distance h.

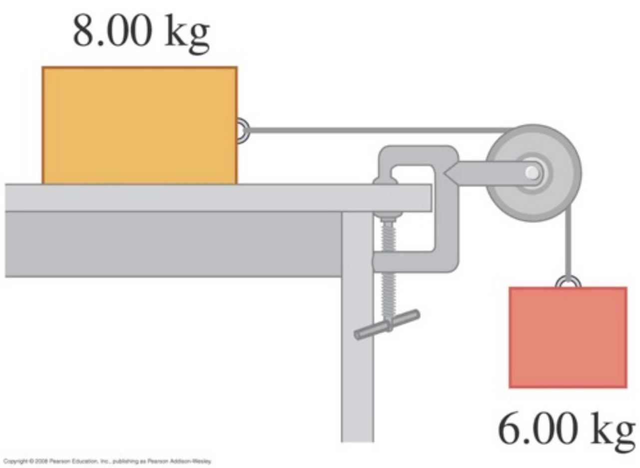
Assume the rolling without slipping for the pulley.

Assume no friction and no air-resisstance.

# Conservation of mechanical energy - translational and rotational motion

Let's revisit this problem but this time we do NOT assume the pulley is massless. Let the mass of the pulley be 1-kg and approximate the pulley as a solid disk of radius r=0.05m.

Assume rolling without slipping, find the speed of the block after the 6-kg mass has fallen 0.5m




Note: We are able to use conservation of energy because the question did not ask about time (and acceleration).

For example, what is the acceleration of the system and how long does it take for the 6kg mass to fall 0.5m?

$\Rightarrow$ Need to know dynamical equations (Newton's second law rotational motion.)