# Chapter 7: Symbolic Translation

## *Introduction*

By now you should have an appreciation for the practical nature of formal symbolic analysis. In addition to saving a lot of time by being able to see the essence of an argument, symbolic analysis is also valuable when arguments and inference situations are complicated and a way is needed to carefully follow the details of a reasoning trail. Much of our technological society is based on the results of people sitting down at desks and following, in one form or another, long logic trails. The computer programmer who analyzes a portion of the millions of lines of symbolic code that guide the space shuttle's computers, the engineer who follows the physical implications of a new engine design, the network programmer who examines Internet traffic for maximum efficiency, and the astrophysicist who from a few facts attempts to reason back 14 billion years to the details of the origin of our universe, are all following the same tradition of Eratosthenes: they take premises and assumptions and "play" with them in a disciplined way to see where they will lead. And like Eratosthenes, all use a form of symbolic reasoning, because it is much too difficult to follow the complex trail of reasoning if expressed in ordinary language.

The goal is the same. The trails must be valid or they will be worthless, or in cases such as the space shuttle, even dangerous. Thus, symbolic reasoning is used because even though we may have an understanding of what to look for in ascertaining a valid argument, it is often difficult to see if complicated arguments stay on track and meet the validity standard. For instance, consider the following discussion between two friends over some of the beliefs contained in the Christian religion. John considers himself a Christian and Dan, an agnostic, is challenging John on what Dan considers to be a major inconsistency in the literal or fundamentalist version of Christianity.

Dan: So God exists and is compassionate and forgiving?

John: Yes.

Dan:

But your God is also omnipotent, all-powerful, and there are rewards and punishments for our behavior. For many Christians this means that we go somewhere when we die, either to a heaven or a hell.

John:

Yes, although there are many versions of this, all Christians believe that ultimately God is in charge so to speak and that He has created a situation where there are consequences for our thoughts and actions; that we create one way or another a spiritual heaven or hell for ourselves. Christians are united in the belief that there would be no basis to morality otherwise.

Dan:

Well, let me prove to you that if you are a good Christian, concerned with justice, compassion, and forgiveness, you should not believe in any place called 'hell.' If God's existence is necessary for the foundation for morality, would you agree, hypothetically, that if God does not exist, then there will be neither a heaven nor a hell for us when we die?

John:

Yes, I would agree hypothetically. As I have said, for a Christian, reward and punishment, good and bad, right and wrong have meaning only if there is a God.

Dan:

And you also agree that although human beings suffer greatly while on this Earth—that whether we are rich or poor, intelligent, gifted, well-educated, or intellectually dull, mentally disabled, or uneducated—we all suffer in one way or another, experiencing pain, doubt, absurd misfortune, and disappointment?

John:

Yes, but for a Christian the absurdity is only apparent; everything that one experiences is meaningful from God's perspective.

Dan:

Yes, exactly. So all human suffering that exists must contribute in some fashion to fulfilling God's purpose?

John:

Yes, all suffering is part of His plan no matter how absurd and unfair it may seem, because God is good and His compassion is limitless.

Dan:

Ah, but you see here is my problem. If God is supposed to be good, infinitely compassionate and forgiving, then if there is to be a kind of human suffering that is eternal—and this by definition is what hell is, an eternal, endless suffering—then this cannot contribute to fulfilling a compassionate and forgiving God's purpose. If there is a hell for some human beings when they die that includes human suffering and eternal suffering, and if eternal suffering is inconsistent with a Christian conception of a forgiving and compassionate God, it follows for a Christian that a literal place of eternal suffering, a hell, cannot exist!

Dan's basic argument is that the literal interpretation of the concept of an eternal hell is inconsistent with the basic postulates of the Christian religion. So, if a good Christian wants to believe in a good, infinitely compassionate and forgiving God, he or she must reject the conception of a literal eternal hell. Here is a formalization of Dan's argument in terms of premises and a conclusion.

**Premises:**

1. If God does not exist, then there will be neither a heaven nor a hell for us when we die.

2. If He does exist, then there should be human suffering only if this suffering contributes to fulfilling God's purpose.

3. However, if there is to be human suffering and eternal suffering, then this cannot contribute to fulfilling God's purpose (because God is supposed to be good, forgiving, and compassionate).

4. There will be human suffering and eternal suffering, if there is a hell for us when we die.

**Conclusion:** It follows that there will not be a hell for us when we die.

Can you follow Dan's argument? Regardless of your religious orientation, which would influence your judgment on the truth or falsity of the premises, can you use the skills we have learned thus far to evaluate the reasoning of the argument? For the average person it is not at all clear whether we would have to accept the conclusion if we accepted the premises. The reasoning of this argument may "sound good," seem to flow and stay on track, but there are many arguments that psychologically give the appearance of staying on track that are not valid.

Dan's argument is valid, although it may not be sound. (A Christian could object to the acceptability of one of the premises – most likely the third premise). But how can I prove this to you—that you should accept the conclusion that a literal eternal hell is impossible if the premises are accepted? It is difficult to keep the implications of all the premises in mind to see if we are locked into the conclusion. The average person will forget the implications of the first premise by the time he or she reads the second or third premise.

Like mathematics, symbolic logic was invented to enable us to follow trails that would be practically impossible (or at least take a very long time) using our normal language-based reasoning tools. As in the case of mathematics, logicians have discovered that our common sense can be systematized symbolically and then valid mechanical techniques used to follow difficult trails without getting lost in complex distracting and irrelevant details. In other words, we can take the smallest, most obviously valid and agreed-upon pieces of our common sense and methodically and objectively check or produce a trail of complex reasoning. This discovery, combined with advances in electronics and philosophy gave birth to our present computer revolution. Computers are essentially symbolic logic machines that know how to do only one thing, stay on track with a vengeance, following the trail of an initial starting point with a single-minded purpose, similar to a Komodo dragon following a meal.[1] Our goal for the next several chapters will be to learn how to be symbolic Komodo dragons, to work on arguments like the one above and understand why they are valid, and learn to appreciate the process of symbolic reasoning.

## Logical Connectives

Unfortunately, even though symbolic logic is just organized common sense, the first step in the learning process is usually the most difficult for students. We must learn to ***translate*** arguments from our normal language into a symbolic notation. We will approach the learning of this translation process as if learning another language. Although the vocabulary of this new language is small, as when learning any language, lots of practice is necessary. Do not expect to be an expert right away.

The new language you will learn is part of the first stage of symbolic logic. This first stage is often called ***propositional logic***, because it deals with the manipulation of the logical implications of linked propositions or statements, such as in "Either John passes the final or he will not pass the course.” The parts of this statement—"John passes the final" and "John passes the course"—are statements that are linked by the words ***or*** and ***not***. These linking words are called ***logical connectives***, and we will see that the validity of propositional inferences depends necessarily on the arrangement of propositions by logical connectives. For instance, once you become proficient at manipulating logical connectives symbolically, you will see that from this statement about John's situation it logically follows that passing the final is a necessary condition for John passing the course. Or, put another way that it is not possible for John to pass the course and not pass the final. But it does not follow that passing the final is sufficient for John passing the course, or, put another way, it is still possible for John to pass the final, but not pass the course. (Stay calm, it is easier to follow these implications in symbols.)

---

[1] The Komodo dragon is a giant lizard that reaches the length of a midsize car and lives on the isolated islands of Komodo, Gillimontang, and Rintja, part of the southeastern edge of the Indonesian archipelago. As a primitive reptile, it is famous for its ability of relentless pursuit of usually much faster prey. According to native stories, some will stay on track of their prey for days or weeks until the prey dies of hopeless fright!

Because propositional logic is only the first level of symbolic logic, and because we will only be interested in manipulating and analyzing inferences of whole statements, the new language to be learned is very simple. In a sense, there are only five key vocabulary terms for the whole language.[2] The five new terms are symbolic representations of five logical connectives as follows:

| not | and | or | if..., then... | if and only if |
|---|---|---|---|---|
| ~ | • | v | ⊃ | ≡ |

Any statement that contains one or more of these connectives we will call a ***compound statement***. A ***simple statement*** will be any complete statement in ordinary language that does not contain any logical connectives. Simple statements will be represented symbolically by capital letters, such as:

A = ***Alice*** is going to the party.

C = The students passed the essential ***competency*** exam.

F = John passed the ***final*** exam.

B = George W. Bush was the 43rd president of the United States of America.

W = The United States has pledged to ***withdraw*** all of its landbased nuclear weapons from the Korean peninsula.

There is nothing absolute about which capital letter is used to stand for each simple statement. The capital letters are merely abbreviations. Consistency, convenience, and agreement are the only requirements. Because we are interested in analyzing the implications of connected simple statements, if we agreed we could have used a U rather than a W for the last statement above. Notice that 'simple' does not mean short. The last sentence is a relatively long sentence, but it is technically a simple sentence in propositional logic because it contains no logical connectives.

With these preliminaries sketched, we are now ready to ***translate*** our first compound statement from ordinary language to a symbolic notation. If A stands for "Alice is going to the party" and B stands for "Barbara is going to the party," how would we translate the compound statement: "Alice and Barbara are both going to the party"? If you answered

A • B

[2] This is certainly simpler than learning a foreign language, in which case more than five new words would be introduced on the first day of class. Our symbolic logic notation will be very simple because we will be learning to map for the most part only the ***syntax*** of our language.

then you are well on your way to learning symbolic translation.

This first step may seem overly simple to you, but for psychological, motivational, and pedagogical reasons it is important to reflect here that the essential ability of mathematical and logical analysis is no more difficult than being able to understand A • B, of being able to see the simple pieces that make up a complex representation or reasoning trail. Remember that in the reasoning of Eratosthenes (Chapter 3) and the example of counting the number of atoms in the entire universe (Chapter 4), an apparent complex trail was just a series of commonsense steps. Similarly, soon we will be dealing with symbolic statements such as:

$$\{\sim[(A \lor B) \supset \sim C] \equiv (D \bullet G)\} \supset \sim[(A \bullet D) \lor \sim B]$$

For most students there is a tendency to suffer from "sensory overload" when confronted by such complex statements. So, it is very important to remember that in a sense all such complex statements are actually just a bunch of A • Bs in disguise, and that a calm, disciplined analysis will show all complex statements are made of simple parts.

Keeping this in mind, let's continue with some basics. Here is how the other logical connectives would be used in basic compound statements.

Alice is going to the party ***or*** Barbara is going to the party.

**A v B**

***If*** Alice is going to the party, ***then*** Barbara is going to the party.

**A ⊃ B**

Alice is going to the party ***if and only if*** Barbara is going to the party.

**A ≡ B**

Alice is ***not*** going to the party.

**~A**

For the most part, language contains rich nuances that propositional logic ignores and also some others that it does address but that we will be ignoring in the beginning. Let's elaborate on the simple mechanics of these translations and touch on a few of the complexities.

Statements such as the last statement above, which contain a ***not***, we will call ***negations***. There is nothing absolute about the placement of the **(~)**, or negation, symbol in the above translation. We could have placed the negation symbol after the **A**. But it is important to have one way to translate negations so that we can communicate symbolically. As Humpty Dumpty points out to Alice in Lewis Carroll's ***Through the Looking Glass***, there is nothing absolute about the words we use to describe things; using the words we do is a matter of convention. A table, for instance, does not have a sign on it saying, "Call me *table*." If we all agreed or were part of a culture that reared its children this way, we could call a table a *dog*, and a dog a *table*. We would not think that this was strange because we would be accustomed to it. But to communicate we do need a standard way of referring to objects. Likewise, in symbolic logic we need a standard way of translating. So we will adopt the basic rule that when a statement involves a negation, we will put the negation in front of the capital letter that stands for the same statement if it were not negated. If **F** is to be used for "John passed the final exam" then "John did ***not*** pass the final exam" would be translated as **~F**.[3]

Compound statements that use ***and*** are called ***conjunctions***. Sometimes conjunctions will not explicitly state the word *and*, as in "Even though Alice is going to the party, so is Barbara." From a propositional point of view, we would still translate this statement as A • B, because the bottom-line claim is that they are both going, even though semantically the use of the phrase *even though* implies something different than a straightforward, unqualified *and*. A similar situation occurs when we reflect on the semantic and possible cultural differences between "Mary became pregnant and married Sai" and "Mary married Sai and became pregnant." In some cultural contexts, say the 1950's in the United States, there would be a very big difference in the implications attached to the meanings of these two statements. The first would have implied a very embarrassing situation for Mary and Sai, whereas the latter would have been cause for celebration. Propositional logic ignores this semantic difference and reflects only the minimum description that Mary is married and pregnant. In propositional logic we are not interested in capturing the full meaning of statements, because a considerable amount of logical analysis can be accomplished by simplifying.[4]

Logicians refer to compound statements that use ***or*** as ***disjunctions***. Sometimes a disjunction will use the word *either* as in "Either Alice or Barbara is going to the party," but this will not change the translation. Sometimes *or* statements will imply the use of a "strong" or ***exclusive*** *or* as in "With your dinner tonight you may have cream of asparagus soup or crab salad." At other times *or* statements will imply a "weak" or ***inclusive*** sense, such as in "(Well I'm not sure, but I think) Either Alice or Barbara is going to the party." In the first case, we know that the context implies that we can't have both the soup and the salad, unless we want to pay extra. In the second case, we know that there is a possibility that both Alice and Barbara may be going to the party. The claim is made only that at least one is going, leaving open the possibility that both may

---

[3] Incidentally, Lewis Carroll's real name was Charles Lutwidge Dodgson (1832-1898), and he was a professor of mathematics and logic at Oxford University. There is much more than excellent children's stories involved in his ***The Adventures of Alice in Wonderland*** and ***Through the Looking Glass***. Political criticism and theories on the nature of logic and language lurk behind the scenes.

[4] In this context, it is important to remember the message of the Huxley quote at the beginning of Chapter 2 and the discussion of its implications for logic. Although we will be "cutting up" our normally rich experience into a logical skeleton, our task will be no different than creating a map. We will be "mapping" reasoning, and although maps should not be confused with reality, they can be used as useful guides.

go.  An exclusive *or* makes the claim that one thing or another is true, but not both; an inclusive *or* makes the claim that one thing or another is true, possibly both.  After we become more comfortable in translating, we will have to address these differences in the use of *or*.

Statements that use or imply an ***if...then*** hypothetical situation are called ***conditionals***. Notice that although two words are used, only one symbol is used ( $\supset$ ).  This symbol looks like a horseshoe turned on its side.  The open part of the horseshoe will always face left.  In a straightforward *if...then* statement, that part of the statement that immediately follows the *if* is called the ***antecedent*** and is placed on the left side of the horseshoe when translated.  The part of the statement that follows the *then* is called the ***consequent*** and placed on the right hand side of the horseshoe.  So in the translation $A \supset B$, A is the antecedent and B is the consequent.

Statements with an ***if and only if*** phrase are called ***biconditionals***.  This phrase is used a lot in contractual situations like, "An employee gets a day off during the week if and only if the employee works on Saturday."  Beginning logic students usually have a difficult time feeling fully comfortable with *if and only if* phrases because in everyday communication we don't say, "I'll help you with your homework tonight if and only if you buy me a soda."  However, such statements are found in logic, science, law, diplomacy, and any field where precise communication is very important.  In 1979 the government of Iran told the U. S. government, "The U. S. hostages will be freed *only if* the U. S. returned the Shah of Iran and his assets to the Iranian government."  (The Shah was living in the U. S. at the time and his considerable assets were in U. S. banks.)  It was important for U. S. leaders to know the difference between this offer and "The U. S. hostages will be freed *if and only if* the Shah and his assets are returned."  The Iranian statement implied no guarantee that the hostages would be released, even if the Shah and his assets were returned.  On the other hand, the use of *if and only if* there would have implied a guarantee.

Perhaps if we spoke this way more often in everyday exchanges there might be less quarreling.  Consider the following statements made by a mother to her son.

(1) "You do not go out tonight with your friends unless you clean your room."

(2) "You do go out tonight with your friends if you clean your room."

(3) "You go out tonight with your friends only if you clean your room."

(4) "You go out tonight with your friends if and only if you clean your room."

There are major differences in the meanings and logical implications of these statements. The simplest things in life are potentially much more complex, rich, and interesting than they seem, and there will always be many points of view on any issue.  Logic, properly understood as a set of tools to deal with complexity, is not intended to destroy the richness of life.  It would not be possible to do this anyway; reality and the interactions within it will always "overflow" beyond the boundaries of any logical analysis. Logic is intended only as a disciplined way of testing trails and points of view. So let's see what points of view logic can reveal in this situation.

Statement #1 is vague. Depending on the context, *unless* sometimes means *or* and sometimes *if and only if*. If the mother made this statement intending the *or*-meaning, then she would be saying essentially, "Either you clean your room or you do not go out tonight." This statement in turn is equivalent to "If you do go out tonight, then you must clean your room." Now, if the son has had a logic course, he will realize that his mother is only specifying a necessary condition for his freedom to go out with his friends. He will realize that statement (1) is saying the same thing as (3), but what he would rather hear is (2). Here's why.

If his mother says, "You do go out tonight with your friends, if you clean your room," she is telling her son, and committing herself to the position, that cleaning his room is ***sufficient*** for his going out with his friends, that this is "all he has to do." Thus, this statement leaves open the possibility that the son does not have to show the proper respect or do anything else deemed appropriate by the mother. For this reason most experienced mothers would not say this. They instead say or intend something along the lines of (1) or (3). These statements are saying, "If you do go out tonight, then you must clean your room," that it is ***necessary*** for the son to clean his room, but his going out is not guaranteed by his cleaning the room. He must clean his room to have a "chance" to go out, but there may be other conditions as well. At this point, if both the mother and the son have had a logic course, they would realize that the appropriate statement to agree on would be the very specific "You do go out tonight if and only if you clean your room." Here's why.

In stating the sufficient condition (2), although there is a clear consequence implied if the son does clean his room—he is allowed to go out—there is no clear consequence if he does not clean his room! From this statement, if he does not clean his room, it would be invalid to infer that he should not be allowed to go out. And obviously, it would also be invalid to conclude that he should be allowed to go out. If he does not clean his room, the situation as to what should happen next is vague. Perhaps the son tells his mother, "But, mom there is no time to clean my room, and Billy's father is taking us to the championship football game, and it's the last time I will see Billy because his family is moving." If the mother decides to let him go out with his friends, she is ***not*** changing her mind, because the only position she has committed herself to is to let her son go out if he cleans his room. She has made no commitment as to what should happen if the son does not clean his room, because saying that "You do go out if you clean your room" is not the same as saying "If you don't clean your room, you do not go out tonight." To summarize this formally:

\

**Valid**

If you clean your room, you do go out tonight with your friends.
You clean your room.
Therefore, you do go out tonight with your friends.

**Invalid**

If you clean your room, you do go out tonight with your friends.
You do not clean your room.
Therefore, you do not go out tonight with your friends.

**Invalid**

If you clean your room, you do go out tonight with your friends.
You do not clean your room.
Therefore, you do go out tonight with your friends.

Similarly, in stating the necessary condition (3), although there is a clear consequence if the son does not clean his room—he does not go out—as we have seen, there is no clear consequence if he does clean his room. After he has cleaned his room, his mother is not committed to letting him go out and may consistently impose other conditions at that point. Summarizing:

**Valid**

You do go out tonight with your friends only if you clean your room.
You do not clean your room.
Therefore, you do not go out tonight with your friends.

**Invalid**

You do go out tonight with your friends only if you clean your room.
You do clean your room.
Therefore, you do go out tonight with your friends.

**Invalid**

You do go out tonight with your friends only if you clean your room.
You do clean your room.
Therefore, you do not go out tonight with your friends.

Thus, from the mother's point of view, if she has really had it with her son's messy room and wants the condition of the room being cleaned to be absolute, the sufficient condition statement is not fair—it is too noncommittal as to what happens if the son does not clean the room. (It is also unfair for another reason. The son could break furniture in the living room or dishes in the kitchen in protest, but if he cleaned his room his mother is committed to letting him go out with his friends.) On the other hand, from the son's point of view, the necessary condition is unfair because it is also not strong enough—he could clean his room and still not be allowed to go out if his mother was in a bad mood and at the last minute wanted something else done. Hence the appropriateness of the very specific contractual "You do go out tonight with your friends ***if and only if*** you clean your room." From this it follows that if the son does not clean his room he cannot go out, and if he does clean his room, he can go out.

**Valid**

You do go out tonight with your friends if and only if you clean your room.
You clean your room.
Therefore, you do go out tonight with your friends.

**Valid**

You do go out tonight with your friends if and only if you clean your room.
You do not clean your room.
Therefore, you do not go out tonight with your friends.

Now if you feel a little dizzy after all this, you are beginning to understand why symbolic logic was invented. It has taken me a couple of pages to describe the various logical implications of the mother-son contracts, and you probably had moments when you had to stop and think about what I was saying. Later, with proficiency in symbolic logic the above logical implications can be presented in a couple of lines and you will be able to understand them in seconds.

But one thing at a time. Let's return to the task of translating ordinary statements into symbolic logic. What follows is a dictionary of examples of how common sentences would be translated into symbolic logic. Because one of the first stages of learning is imitation, we can use this dictionary as a model to imitate when translating similar English sentences even though initially you may not be fully comfortable with what you are doing or why the translations end up the way they do. At this point notice how each italicized connective gets translated and then (don't think too much yet!) attempt to mimic this process by doing the exercises at the end of the dictionary. For instance, item 11 in the dictionary uses a *not both* phrase and is translated ~(J • K), so we mimic this by translating item (4) in Exercises I as ~(A • G) and (9) in Exercises II as ~(S • D).

## *Usage Dictionary of Logical Connectives*

1. John passed the final exam *and* the course. (F, C)

F • C

2. Either John passed the final exam *or* he passed the course. (F, C)

F **v** C

3. *If* John passes the final exam, *then* he will pass the course. (F, C)

F ⊃ C

4. John will pass the course *if and only if* he passes the final exam. (C, F)

C ≡ F

5. John passed the course *but not* the final exam. (C, F)

C • ~F

6. John did *not* pass the course *but* he did pass the final exam. (C, F)

~C • F

7. John passed the course *even though* he did *not* pass the final exam. (C, F)

C • ~F

8. *Even though* he did *not* pass the final exam, John passed the course. (F, C)

~F • C

9. John did *not* pass *both* the final exam *and* the course. (F, C)

~(F • C)

10. John did *not* pass the final *and* he did *not* pass the course. (F, C)

~F • ~C

11. Johnson *and* Kaneshiro will *not both* be hired. (J, K)

~(J • K)

12. Johnson *and* Kaneshiro will *both not* be hired. (J, K)

~J • ~K

13. John either passes the final exam *or* he does *not* pass the course. (F, C)

F v ~C

14. Either John did *not* pass the final exam *or* he did *not* pass the course.  (F, C)

~F v ~C

15. John passed *neither* the final exam *nor* the course.  (F, C)

~(F v C)     or     ~F • ~C

16. John will take the bus to school *unless* his girl friend drives him in her car.  (B, D)

B v D     or     ~D ⊃ B

17. John will pass the course, *if* he passes the final exam.  (C, F)

F ⊃ C

18. John will pass the course *only if* he passes the final exam.  (C, F)

C ⊃ F

19. *If only* John passes the final exam, he will pass the course.  (F, C)

F ⊃ C

20. *Provided that* John passes the final exam, he will pass the course.  (F, C)

F ⊃ C

21. John will pass the course, *provided that* he passes the final exam.  (C, F)

F ⊃ C

22. Passing the final exam is a *necessary condition* for passing the course.  (F, C)

C ⊃ F

23. Passing the final exam is a *sufficient condition* for passing the course.  (F, C)

F ⊃ C

24. *If* John does *not* pass the final exam, *then* he will *not* pass the course.  (F, C)

~F ⊃ ~C

25. *It is not true* that *if* John passes the final exam, *then* he will pass the course. (F, C)

$\sim(F \supset C)$

## Exercises I

Now use the above examples as models for translating the following abbreviated statements into symbolic notation.

1. A but not B.

2.*If not A, then B.

3. Z only if not B.

4. Not both A and G.

5. P, if not G.

6. Either not P or not D.

7. Neither R nor H.

8. S unless not P.

9. *Not Z if and only if Y.

10. A necessary condition for P is not Y.

11. A sufficient condition for P is not Y.

12. If only K, then B.

13. Z if and only if not J.

14. A, provided that not Z.

15. Not P, even though A.

In the above exercises you should have tried simply to imitate the dictionary. Many students will miss number 14 the first time around. Here is how to get #14 correct without thinking too much. Number 21 in the dictionary would be the example to mimic because 21 has "provided that" in the middle of a sentence. This example shows that "provided that" is translated the same as #17 when "if" is in the middle of a sentence.

Both 17 and 21 are telling us that "if" and "provided that" are translated as regular "if, then" statements and that what follows "if" or "provided that" will be an antecedent. So, because “not Z” follows the “provided that” phrase, we put ~Z first (in the antecedent position) and end up with $\sim Z \supset A$. $A \supset \sim Z$ would be incorrect.

Here are some notes that you can add to the right hand margin of the dictionary that summarizes key points made below and will help you translate correctly without thinking too much.

#16 "unless" = "or"

#17 "if" = antecedent

#18 "only if" = consequent

#19 "if only" = antecedent

#s 20 & 21 "provided that" = "if" = antecedent

#22 "necessary condition" = consequent

#23 "sufficient condition" = antecedent

The note for #16 is a reminder that the easiest way to translate "unless" is to interpret it as an "or" statement. The note for #17 is a reminder that when you see "if" without any "only" modifier, the sentence should be translated as a regular "if, then" statement, and what follows the "if" will be the antecedent. The note for #18 is a reminder that an "only if" statement is special and what follows an "only if" in a statement will be translated as a consequent. The notes of #s19, 20, and 21 are a reminder that "if only" and "provided that" are the same as "if.” The note for #22 is a reminder that whatever a necessary condition is in a statement, it will be translated as a consequent. And, the note for #23 is a reminder that a sufficient condition will be translated as an antecedent.

However, as in most learning, we want to be able to obtain a deeper level of understanding. In translating, we want to know why the translations end up the way they do. Why, for instance, in the dictionary, are (18) and (19) translated differently? The implicit claim of this dictionary is that the translations must be this way to capture faithfully the meaning of the original statements. So, before we continue with more complex translations, let's elaborate on the dictionary examples.

Examples (5) to (8) show that we will use the ( • ) symbol to translate any conjunctive expression such as *but* and *even though*. Other English conjunctive expressions translated with ( • ) would be *however*, *moreover*, *although*, *yet*, and *whereas* when these words are used as conjunctive words within a sentence. The latter is often used with a semicolon (;), as in "If Johnson does not get the promotion, Smith will not be able to finish the Japan project; whereas, if Johnson does get the promotion, Kaneshiro will not be able to finish the Singapore project." $(\sim J \supset \sim S) \bullet (J \supset \sim K)$.

The examples in the dictionary show that we will adopt the following convention when translating the negation sign (~). If there are no parentheses, then the negation sign will refer to only the letter immediately following it. So $\sim F \bullet C$ would be a correct translation for the statement, "John did not pass the final, but he did pass the course." Whereas, $\sim(F \bullet C)$ would be a translation of the statement, "John did not pass *both* the final exam and the course." Note that if there is a negation outside parentheses, then the negation is being applied to the *entire statement inside the parentheses*. $\sim(F \bullet C) \neq \sim F \bullet \sim C$.

Let's elaborate.

In looking at the translations of (11) and (12) in the dictionary, many students will think that these statements must have the same meaning. But the negation in (11) is not referring to each letter individually inside the parentheses; it is negating the entire statement $J \bullet K$. So (11) is saying that at least one of the men will not be hired, leaving open the possibility that at least one will; whereas (12) is much stronger stating that neither of the men will be hired. Consider the difference between (9) and (10). Number (10) is very precise in what it is claiming. If (10) is true, then we know that John did not pass the final and he did not pass the course. On the other hand, (9) is less precise. It only claims that John did not pass both the final and the course, leaving open the possibility that he passed one or the other.

The significant difference in the meaning between ***not both*** and ***both not*** shows how easy it is for us to misread the implications of language and make logical mistakes. By now you should be thoroughly aware of how important it is to avoid such misreading. If you are not aware of the potential pitfalls, then it will be easy, as the examples in Chapter 1 show, for someone to take advantage of you. For very good reasons language is rich and complex. But there are times when tools are needed to surgically trace our way through this richness.

In June 2010, New York Times columnist Bob Herbert expressed his frustration with the Obama administration's approach to Afghanistan—that the war was a war of necessity, that we will do whatever is necessary to succeed, and that the withdrawal of U. S. troops "will begin on schedule, like a Greyhound leaving the terminal, a year from now." He argued that <u>not both of these policies can work</u>, because they conflict.

Notice that Herbert was not claiming that both of these policies are misguided. He was not saying that both are not workable. He was saying that only one could be workable, that the policies could not work together. So there was an important difference between saying "not both" and "both not."

Consider how much is implied in the simple statement, "Alice and Barbara are *not both* coming to your party." Suppose I am the one giving a party, and I have invited both Alice and Barbara. My friend, knowing this, comes to me before the party and tells me that unfortunately Alice and Barbara have had a monstrous fight and have vowed never to be seen together again on the face of this earth. He tells me that he is sure that they will *not both* be coming to my party. If his statement is true, what will I be sure of? Will I know that Alice is not coming? No. Will I know that Barbara is not coming? No. Will I know that neither of them is coming? No. Will I know that Alice is coming? No. Will I know that Barbara is coming? No. All that I am sure of is: If Alice is planning to come to the party and Barbara knows this, then Barbara will not come; if Barbara is planning to come to the party and Alice knows this, then Alice will not come; and finally, if Alice thinks Barbara is coming and Barbara thinks Alice is coming, then neither of them will come! All of this meaning is packed into the phrase *not both*. Either one will not come or the other will not come, possibly both will not come.

"Possibly *both* will *not* come"! Suppose in a different set of circumstances my friend tells me that "Alice and Barbara will *both not* be coming to your party." Although the same little words are used—*both* and *not*—the different word order makes now a very different statement, implying a very different set of circumstances. Suppose my friend tells me, "I don't know what you did, but both Alice and Barbara don't like you, so I can guarantee that they will *both not* be coming to your party." This statement is much more precise than the previous not-both example. If it is true, then I will be sure that Alice is not coming to my party and Barbara is not coming to my party. So whereas the *not both* statement implies the possibility of neither coming, the *both not* statement implies that this result is guaranteed. So changing the order of the words (both not ... not both) is a very big deal.

If you find the unsuspected complexity of these examples overwhelming at this point, consider that we can distinguish between a full ***understanding*** of the dictionary and a practical ***mimicking*** of the dictionary. Because our first goal is to simply mimic the dictionary, all you have to remember is this:

**not both** = ~(**_**•**_**)

**both not** = ~**_** • ~ **_**

You should eventually be able to understand why the translations are this way. In fact, you should already understand this, considering that we are supposed to learn how to use these words at a very young age. But for most of this chapter, all you need to do is mimic the dictionary.

Similar considerations apply in using negations with *or* statements. Note the difference between (14) and (15) in the dictionary. These statements are not saying the same thing: ~(F v C) ≠ (~F v ~C). Number (15) is much stronger than (14). If (15) is true, then we know that John did not pass the final and he did not pass the course; whereas, in (14) we only know that he did not pass at least one. When we negate an entire *or* statement as in ~(**_**v**_**)**,** we are stating that the usual claim of the *or* statement (that at least one thing happened) is not true. Notice that ***neither...nor*** statements have the same translations as

***both...not*** statements. With a little reflection we also see that (14) could be expressed using *not both*, as is (9). So to summarize and for future reference,

**not both** = **either not...or not...** = ~(__• __) = ~__ v ~ __

**both not** = **neither... nor...** = ~ __• ~__ = ~(__v __)

BUT

**not both** ≠ **both not** ~(__• __) ≠ ~ __• ~ __

**neither... nor...** ≠ **either not...or not...** ~(__ v __) ≠ ~__ v ~ __

Although the context of *unless* statements sometimes indicates that an *if and only if* meaning is intended, until clarification is given that this stronger meaning is intended it is best to always start with a minimum *or* interpretation. The easiest procedure to adopt at a mimic stage of translating is to simply replace *unless* with *or* and translate accordingly. Thus, the meaning of (16) can also be captured by "Either John will take the bus *or* his girl friend will drive him in her car." Note that this *either-or* statement is equivalent to "If his girl friend does not drive him in her car, John will take the bus to school." Thus, either translation shown in (16) can be used for translating the minimum interpretation of *unless* statements.

Numbers (17), (18), and (19) in the dictionary are very important and are most often confused, both by students doing translations and in general by the average person drawing invalid inferences when these phrases are part of the premises in arguments. Number (17) is an *if...then* statement, but it shows that sometimes for emphasis we will state the consequent first and then the antecedent. The essential meaning of (17) would not be changed if it were restated, "If John passes the final exam, then he will pass the course." Number (18), however, shows that the order of the conditional is reversed when *only* occurs in front of an *if*. Consider this statement:

*A person is pregnant only if that person is female*.

If we were to follow the procedure adopted in (17) and make whatever follows the *if* an antecedent we would end up with **F ⊃ P**. But this incorrectly claims that being female is all that is needed to be pregnant; whereas, the intention of the original statement is that being female is a condition for pregnancy only, and that, as we all know after a little sex education, some male sperm is also needed. So the correct translation of this statement would be **P ⊃ F**.[5]

[5] If you are aware of the intricacies of stem cell research and advances in biotechnology, you know that it is possible to create a baby without any male sperm. It may even be possible some day for a male to have a baby via in vitro fertilization and cesarean birth.

Number (19) shows that sometimes we use *only* to emphasize the importance of the antecedent. But placed after the *if* in this statement, the *only* does not change the indication that passing the final is the antecedent. The difference between (18) and (19) is very important. If a professor said (18), he or she would be implying that it is absolutely necessary to pass the final exam to pass the course, that if John does not pass the final he will not pass the course. But this leaves open the possibility that there are other course requirements that John must meet, such as quizzes and other exams. On the other hand, (19) implies that passing the final exam is all that John needs to do to pass the course.

Consider the difference between Publisher's Clearing House telling you that you have won (this time) 10 million dollars **only if** you return your entry package, and that you have won **if only** you return the entry package. Saying *only if* amounts to nothing more than you will have a chance along with millions of other people if you return the package. Saying *if only* would mean that you are guaranteed the $10 million by simply returning the entry package. Publisher's Clearing House knows that most people will be tricked into thinking they have won, returning the entry package, and probably subscribing to a few magazines. The management of PCH knows that most people will confuse the meanings of *only if* and *if only*.

Here is an easy way of implementing these differences from a mimic point of view. Whatever statement (simple or compound) follows an *only if* that is not part of an *if and only if,* translate that statement as a consequent. Otherwise what follows an *if* will be translated as an antecedent. To illustrate, consider the following statements and their translations:

The number of jobs in marine maintenance will increase ***only if*** the number of boat slips increases. J only if S,

$$\mathbf{J \supset S}$$

The number of jobs in marine maintenance will increase ***if*** the number of boat slips increases. J if S,

$$\mathbf{S \supset J}$$

The number of jobs in marine maintenance will increase ***if and only if*** the number of boat slips increases. J if and only if S,

$$\mathbf{J \equiv S}$$

Note, however, that sometimes there may be some intervening words between the *only* and the *if* as in "These proceedings ***only*** can be legally concluded ***if*** there is an agreement on the financial arrangements." This statement must be read carefully to see that *only if* is intended and hence, (P only if A) $\mathbf{P \supset A}$.

One way of avoiding the tricky nature of *only if* and *if* statements is to use the very precise designation of a necessary or sufficient condition, (22) and (23). When a professor tells her class at the beginning of the semester that her curriculum is organized in such a way that the course will be passed only if the final exam is passed, she means that passing the final exam is a necessary condition for passing the course, but that it is not necessarily sufficient—that passing the final is an absolute condition, but that there are other course requirements as well. On the other hand, if just before the final exam a student asked the professor what his standing was in the course and the professor responded with "Well, if you pass the final you will pass the course," the professor would be implying that given the student's performance to date passing the final at this point would be sufficient for passing the course.

In terms of the mimic rule for translating necessary and sufficient conditions, ***necessary conditions*** will always be translated as consequents, and ***sufficient conditions*** will always be translated as antecedents. As easy as this rule is to remember, take care to identify what is specified as a necessary or sufficient condition in a statement. For instance, if we were to reword (22) to read "A necessary condition for passing the course is passing the final exam," note that although the words *passing the course* occur immediately after the words *necessary condition*, the condition phrase refers to passing the final. Students will often mistranslate this statement as $\mathbf{F \supset C}$ simply because the words *passing the course* appear closer to the words *necessary condition*. The intention of the words *for* and *is* must be recognized.

We can now see that another way of understanding *if and only if* is to see that its use specifies a necessary (only if) and sufficient (if) condition. So the mother's statement to her son, "You go out tonight with your friends if and only if you clean your room" could be translated as either $\mathbf{O \equiv R}$ or $\mathbf{(O \supset R) \bullet (R \supset O)}$.

Finally, the same care in using negations with parentheses in combination with *and* and *or* statements applies to *if...then* statements. Number (24) is a very different statement from (25). It would be a mistake to think that $\mathbf{\sim F \supset \sim C}$ equals $\mathbf{\sim(F \supset C)}$. As we have seen, a negation outside parentheses cannot simply be pushed inside and applied to the parts inside the parentheses without altering the connective. Number (24) says that not passing the final is sufficient for not passing the course; whereas, (25) is denying that passing the final is a sufficient condition for passing the course. Number (25) says that it is possible for John to pass the final and still not pass the course $\mathbf{(F \bullet \sim C)} = \mathbf{\sim(F \supset C)}$; whereas, (24) says that if he does not pass the final he is guaranteed to fail the course, that it is not possible for him to not pass the final and still pass the course, $\mathbf{\sim(\sim F \bullet C) = (\sim F \supset \sim C)}$. Number (25) refers to a consequence related to passing the final and (24) refers to a consequence related to not passing the final.

Easier to understand, consider also the difference between $\mathbf{\sim(F \supset C)}$ and $\mathbf{(\sim F \supset C)}$. An instructor might make the first statement, indicating that there is more to his or her course than just passing the final. Suppose at the beginning of the semester a transfer student asks an instructor if he can "challenge" her course. The student claims to have taken the course previously at another college and asks if he can just take the final exam to prove that he knows the material. The instructor might say, “No, in my class you would have to pass more than just the final exam; you would also have to pass the midterm exam. Essentially the instructor would be saying **~(F ⊃ C)**; it is not true that passing the final exam is sufficient for passing the course. But an instructor would never make the second statement – "If you don't pass the final, you will pass the course," **~F ⊃ C**! Obviously, the parentheses matter big time. **~(F⊃ C) ≠ (~F⊃ C)**

If this makes you dizzy again, note the mimic way of capturing the meaning of *if...then* statements mixed with negations. In (24), in the statement the negation occurs after the *if*. Thus the negation refers only to the antecedent. In (25), the negation occurs before the *if*, so it refers to the entire *if...then* statement. The statement, "*If* we do *not* have a winning team this year, the manager's contract will not be renewed," would be translated as **~W ⊃ ~R**. Whereas, the statement, "It is **not** true that *if* we have a winning team this year, the manager's contract will be renewed," would be translated as **~(W ⊃ R)**.

## Exercises II

Now use the dictionary as a model for translating the following full statements into symbolic notation.

1. The economy will perform poorly again next year, and the Republicans will have a difficult time at the ballot box. (E, R)

2. This man must either be drunk or have a brain tumor. (D, T)

3. If the economy performs poorly again next year, then the Republicans will have a difficult time at the ballot box. (E, R)

4. *If the economy does not perform poorly again next year, then the Republicans will not have a difficult time at the ballot box. (E, R)

5. Neither the economy nor the Republican chances at the ballot box will improve next year. (E, R)

6. Children will be promoted to the next grade only if they pass the essential competency test. (G, C)

7. Passing the essential competency test is a necessary condition for promoting children to the next grade. (C, G)

8. If you don't clean your room, then you don't go to the dance tonight. (R, D)

9. *You cannot both study tonight and go to the dance. (S, D)

10. The economic situation and Obama’s chances for Democrats at the ballot box will both not improve next year. (E, O)

11. You have won $10 million only if you return the magazine subscription page. (W, R)

12. You have won $10 million if only you return the magazine subscription page. (W, R)

13. If the security situation in Iraq does not improve, we can't have credible elections. (S, C)

14. Dividends will be tax-free under the new Bush tax plan if and only if the company that pays them has paid enough taxes. (D, T)

15. Israeli soldier on the Israeli invasion of the West Bank, "We did not shoot civilians unless we had to." (S = We did shoot civilians; C = We had to shoot civilians.)

## *Complex Translations, the Use of Parentheses, and Arguments*

We now must consider a method for mapping more complex statements. As we proceed, keep in mind again that complex wholes are made up of simple parts. To avoid sensory overload, maintain a mental calmness and discipline and you will see the combination of simple parts behind the scene of what appears to be a perplexing jumble of symbols.

In translating complex statements, we use parentheses in much the same way that we use punctuation in ordinary language. Consider the difference between the following two statements, noting the position of the commas in the originals and the use of parentheses in the translations.

1. If the president implements his tax program, then the deficits will continue to increase and the economy will not improve. (T, D, E)

   $T \supset (D \bullet \sim E)$

2. If the president implements his tax program then deficits will continue to increase, and the economy will not improve. (T, D, E)

   $(T \supset D) \bullet \sim E$

These are very different statements. Statement (1) claims that increasing deficits *and* a poor economy will be consequences of the president's tax program. Statement (2) claims that although the deficits will be a consequence of the president's tax program, the economy will perform poorly regardless of what the president does with taxes.[6] Someone might claim (1) who believed that the president's tax program will be bad for the deficits and the economy; whereas, (2) might be claimed by someone who believed that the economy's poor performance will result from circumstances other than the president's tax program. To understand the difference in meaning, we see that (1) is an *if...then* statement; whereas, (2) is an *and* statement. To map this difference we use parentheses around D • ~E in (1) to show the ( ⊃ ) as the major connective, and parentheses around the T ⊃ D in (2) to show the ( • ) as the major connective.

Thus, the general rule to remember in translating complex statements is to identify the major connective first, then decide what parts need parentheses to set off the major connective. The ***major connective*** is the distinguishing or basic connective of the statement. For instance, in translating,

3.

If a student does not achieve at least a 2.0 grade point average or does not complete at least 50% of all credits attempted during a semester, then the student will be on probation. (A, C, P)

we identify the basic statement as a conditional and write down the (⊃) symbol first. We know that an *if...then* statement has two parts and the (⊃) symbol has two sides. To keep things as simple as possible and to build up your confidence as you translate, next identify the simplest part and translate it, placing the translation on the appropriate side. In this case, the simplest part is the consequent—"the student will be on probation." Using the letter **P**, we now have

**⊃ P**

Next take the more complex part of the statement and translate it separately, identifying the connective of this part and letters for simple statements. In the example, the minor connective is an *or* statement—"a student does not achieve at least a 2.0 grade point average or does not complete at least 50% of all credits attempted during a semester." So, we would have

**~A v ~C**

Now we combine this with ⊃ **P**, to get

**~A v ~C ⊃ P**

Finally, to show that the (⊃) is the major connective, we put parentheses around the **~A v ~C**, so that our final translation is

---

6 Grammatically, the meaning of (2) might be better stated as "The economy will not improve, but if the president implements his tax program then deficits will continue to increase."

**$(\sim A \lor \sim C) \supset P$**

Without the parentheses, the translation **$\sim A \lor \sim C \supset P$** is ambiguous; it could mean:

$(\sim A \lor \sim C) \supset P$

If a student does not achieve at least a 2.0 grade point average or does not complete at least 50% of all credits attempted during a semester, then the student will be on probation.

Or:

$\sim A \lor (\sim C \supset P)$

Either a student does not achieve at least a 2.0 grade point average or if a student does not complete at least 50% of all credits attempted during a semester then the student will be on probation.

Believe it or not this last statement is equivalent to "If a student *both (!?) achieves* at least a 2.0 grade point average and does not complete at least 50% of all credits attempted during a semester, then the student will be on probation." Clearly this is not the statement intended.[7]

Before we try another set of exercises, some comments are in order regarding the translation of negations. How would we translate the following statement?

4.

It is impossible for the president to be reelected next year, even though his foreign policy record is excellent.

As we have seen in our simple propositional language, the choice of a capital letter to represent a simple statement is relatively arbitrary. However, if you used a G, and I used an A for the simple statement "A student achieves a 2.0 grade point average" we would have a hard time evaluating each other's translation. So consistency is a constraint: We need to decide on a letter and maintain the use of that letter throughout a complex statement or argument. We could decide to translate (4) as **$I \bullet F$**. However, if this statement were part of an argumentative exchange where the statement "It is possible for the president to be reelected next year" occurred, we would now have a problem. If we use *I* to stand for "It is impossible for the president to be reelected," I can't use *P* for "It is possible for the president to be reelected next year," because the proper opposition would not be captured. A better approach, and the one we will adopt, is to have ***a capital letter always represent a non-negated expression***. Thus, statements with expressions such as

[7] If you think this statement is farfetched and would not be asserted by anyone, consider how often professors read such statements from student writing caused by the lack of proper punctuation.

*impossible*, *uncommon*, and *illegal* are best translated as ~P, ~C, and ~L respectively. A better translation for (4) would be ~**P • F**.

However, this does not mean that any statement with a negative connotation should involve a ( ~ ) symbol when translated. For instance, in general, the simple statement "The economy will perform poorly next year" should not be translated as **~E** because we would then have to use **~ ~E** for the statement "The economy will not perform poorly next year." Such translations are unnecessarily complicated. However, there is no absolute rule other than the general goal to be as simple as possible while capturing the essence of any logical relationships. Sometimes it will be relatively arbitrary what simple statement will have the (~) symbol when translated. Given an argumentative exchange with simple statements that contained the words *impartial* and *biased*, a decision would definitely have to be made on the use of the (~) symbol to capture the logical relationship. But it is a toss-up whether we translated the simple statement that contained *impartial* as **~B** and the simple statement that contained *biased* as **B** or the statement with *impartial* as **I** and the statement with *biased* as **~I**. The important point is that we would need to agree and then stay consistent.[8]

Next, although this will be rare, there are times when the complexity of statements requires that in addition to parentheses **( )**, we must also use brackets **[ ]** and sometimes even braces **{ }**. Suppose someone claimed that **(1)** above, **T ⊃ (D • ~E),** is not true. To translate this claim we would need to negate **T ⊃ (D • ~E)**. In doing so we would need to put brackets **[ ]** around the entire statement and place the ( ~ ) symbol outside the brackets as follows:

5. ~[T ⊃ (D • ~E)]

In the United States it is not unusual to hear football announcers discussing very complicated hypothetical playoff scenarios towards the end of the regular football season. They say things like,

6.

"If Chicago wins a playoff spot only if San Francisco and Los Angeles are both eliminated from the playoffs, then Green Bay is eliminated, provided that Chicago beats Philadelphia by more points than Atlanta did." (C, S, L, G, B)

C = Chicago wins a playoff spot.

S = San Francisco wins a playoff spot.

L = Los Angeles wins a playoff spot.

---

[8] If the argumentative exchange also involved a simple statement with the phrase *not biased*, then it would be best to translate the simple statement with *impartial* as **~B**. Otherwise, if *impartial* were translated as **I** and *biased* as **~I**, then *not biased* would be translated as **~~I**, and this is unnecessarily complicated.

G = Green Bay wins a playoff spot.

B = Chicago beats Philadelphia by more points than Atlanta did.

This statement would be translated as

$$B \supset \{[C \supset (\sim S \bullet \sim L)] \supset \sim G\}$$

No doubt at this stage this seems like a very difficult translation. But we ought to be able to capture what the average football fan can understand. For practice, see if you can identify and write out the compound parts that make up (6).

Finally, although we have concentrated in this chapter on translating **statements**, because our goal is to analyze **arguments**, we must learn how to translate arguments. Consider the following argument:

7.

If a star is 13 billion years old, then the universe cannot be 11 billion years old. This is so, because stars make up galaxies, and galaxies make up the universe. Moreover, if stars make up galaxies, and galaxies make up the universe, then it is not possible both for a star to be 13 billion years old and the universe 11 billion years old.

T = A star is 13 billion years old.

E = The universe is 11 billion years old.

S = Stars make up galaxies.

G = Galaxies make up the universe.

As in our previous argument structuring exercises, you should identify the conclusion first. The phrase "This is so, because" indicates that the first statement is the conclusion. Next, we identify the number of statements other than the conclusion. Usually, each of these statements indicates a premise. So, notice that since "Moreover" is not functioning as a conjunctive word within the last sentence, it is best not to connect this sentence with the previous one by ( • ). Here is a translation then of the above argument: (Note that the symbols **/** ∴ are used to designate the conclusion and it is positioned adjacent to the last premise.)

1. $S \bullet G$
2. $(S \bullet G) \supset \sim(T \bullet E)$ $\quad / \therefore\ T \supset \sim E$

As noted previously, do not expect to be an expert at translating right away. You would not expect to be an expert at Japanese, Russian, or Chinese after only one introductory chapter on either of these languages. Continue to practice translating a little at a time while we develop additional symbolic techniques of analysis in the following chapters. In addition to the exercises below, you can continue to practice translating additional arguments—see Chapter 8, Exercise IV, and Chapters 9 and 10, Translations and Formal Proofs.

## *Summary of key concepts in C7*

Here is a summary of the terminology for this Chapter.

**Negations** -- statements with 'not' (~)

**Conjunctions** -- statements with 'and' ( • )

**Disjunctions** -- statements with 'or' ( v )

**Conditionals** -- statements with 'if/then' (⊃)

**Biconditionals** -- statements with 'if and only if' ( ≡ )

**Antecedent** -- that part of a conditional statement in front of the (⊃) symbol

A ⊃ B

**Consequent** -- that part of a conditional statement in back of the (⊃) symbol

A ⊃ B

/∴ -- symbol that we use for the conclusion of an **argument**.

**Major connective** -- the distinguishing or basic connective of the statement. For instance, for this statement

(S • G) ⊃ ~(T • E)

the ( ⊃ ) is the major connective.

In this statement

~[(S • G) ⊃ ~(T • E)]

the first (~) is the major connective.

## Exercises III

The following translations are more complex, requiring parentheses ( ) and in a few cases [ ] for punctuation. Hint: Translate each part separately, then recombine into a whole using parentheses to show the major logical connective.

1. If not A, then neither B nor C.

2. *X only if not both P and Z.

3. P and Z, if not D.

4. Either D or not both P and Z.

5. If Alice is going to the party, then Barbara and Carol will also go. (A, B, C)

6. If Johnson gets the job, then neither Smith nor Kaneshiro will be hired. (J, S, K)

7. *Either Alice is not going to the party or Barbara and Carol are both not going. (A, B, C)

8. The economy will not improve next year, but if the Obama decides to run again he still has his foreign affairs record to sustain his popularity with the voters.

   I = The economy will improve next year.
   R = Obama decides to run again.
   F = The president still has his foreign affairs record to sustain his popularity with the voters.

9. If the labor contract is renewed at the present salary levels, then there will either be an illegal strike or a work slowdown.

   R = The labor contract is renewed at the present salary levels.
   L = There will be a legal strike.
   W = There will be a work slowdown.

10. If I find something in the laundry that shouldn't be there, someone is going to be in big trouble unless it is money.

    L = I find something in the laundry that should be there.
    T = Someone is going to be in big trouble.
    M = I find money in the laundry.

11. Syria will recognize Israel only if Israel gives back all the Arab lands it captured in the 1967 Mideast war and accepts past U.N. resolutions.

R = Syria will recognize Israel.
G = Israel gives back all the Arab lands it captured in the 1967 Mideast war.
A = Israel accepts past U.N. resolutions.

12. If the basketball team has an exciting team and a winning team this year, then coach Little's contract will be renewed.

E = The basketball team has an exciting team this year.
W = The basketball team has a winning team this year.
L = Coach Little's contract will be renewed.

13. North Korea will not allow inspection of its nuclear research facility unless the U. S. nuclear involvement in the peninsula is stopped and international teams carry out simultaneous inspection of South Korean weapons sites.

A = North Korea will allow inspection of its nuclear research facility.
S = U.S. nuclear involvement in the peninsula is stopped.
C = International teams carry out simultaneous inspection of South Korean weapons sites.

14. *If the U. S. and Israel both don't support Gemayal's Christian government in Lebanon, then Jumblatt will not remain in the Syrian camp for long.

U = The U.S. supports Gemayal's Christian government in Lebanon.
I = Israel supports Gemayal's Christian government in Lebanon.
J = Jumblatt will remain in the Syrian camp for long.

15. The State of Hawaii will preserve marriage between one man and one woman, provided that it does not deprive any person of civil rights on the basis of sex and the voters approve the Constitutional amendment this November.

P = The State of Hawaii will preserve marriage between one man and one woman.
D = It does deprive a person of civil rights on the basis of sex.
V = Voters approve the Constitutional amendment this November.

16. Enforcing the No Child Left Behind requirement is a necessary condition, but not a sufficient condition for us to meet the steep educational challenges of the twenty-first century.

E = We will enforce the components of the No Child Left Behind requirement.

M = We will meet the steep educational challenges of the twenty-first century.

17. Military action should be taken against Iraq only if Iraq refuses to cooperate, and if the Security Council of the United Nations approves such action.

    M = Military action should be taken against Iraq.
    R = Iraq refuses to cooperate.
    A = The Security Council of the United Nations approves taking military action against Iraq.

18. It is not true that having an argument with true premises is a sufficient condition for a valid argument. (T, V)

19. From an article commenting on the little known Japanese nuclear bomb program in the 1940's. "If Japan considers American hands to have been soiled by the atomic bomb then Japanese hands are equally dirty, although most Japanese are unaware."

    S = Japan considers American hands to have been soiled by the atomic bomb.
    D = Japanese hands are equally dirty.
    A = Japanese are aware that Japanese hands are equally dirty.

20. Political commentary on the 1992 elections: "The 1992 election choices will be serious if and only if political parties debate plans for escaping depression and militarism, and heed the people who have long been deprived of competent public services.

    C = The 1992 election choices will be serious.
    D = Political parties debate plans for escaping depression.
    M = Political parties debate plans for escaping militarism.
    H = Political parties heed the people who have long been deprived of competent public services.

    Note: [ ] should be used in this translation.

21. The United States has pledged to withdraw all of its land based nuclear weapons from the Korean peninsula, although it would still be able to strike North Korean targets with nuclear missiles launched from submarines and will still have a substantial naval presence in Japan.

    P = The United States has pledged to withdraw all of its land based nuclear

weapons from the Korean peninsula.
S = The United States would still be able to strike North Korean targets with nuclear missiles launched from submarines.
N = The United States will still have a substantial naval presence in Japan.

22. An explanation for why the Kansas City Chiefs were out of the football playoffs. "The Chiefs beat the 49ers and Buffalo, however, the Raiders beat the 49ers and Buffalo both by more points."

A = The Chiefs beat the 49ers.
B = The Chiefs beat Buffalo.
C = The Raiders beat the 49ers by more points.
D = The Raiders beat Buffalo by more points.

23. The Russian President Putin is not prepared to accept whatever the Bush administration offers in terms of verification of new nuclear weapons levels unless it is put on paper and accepted by other nuclear powers.

A = The Russian President Putin is prepared to accept whatever the Bush administration offers in terms of verification of new nuclear weapons.

P = Whatever the Bush administration offers in terms of verification of new nuclear weapons levels is put on paper.

O = Whatever the Bush administration offers in terms of verification of new nuclear weapons levels is accepted by other nuclear powers.

24. Supreme Court ruling: Obscene material is illegal only if it is disseminated and not just possessed, unless there is an overwhelming societal interest in protecting children.

L = Obscene material is legal.
D = Obscene material is disseminated.
P = Obscene material is just possessed.
S = There is an overwhelming societal interest in protecting children.

Note: [ ] should be used in this translation.

25. *Translate the Hell-does-not-exit argument at the beginning of this chapter. Number each premise and remember to indicate the conclusion with the conclusion symbol ( /∴). Use these capital letters for the simple statements.

G = God exists.

A = Heaven exists.
B = Hell exists.
H = Human suffering exists.
E = Eternal suffering exists.
C = Human suffering contributes to fulfilling God's purpose.

## Exercises IV

Some of the following translations require a rephrasing into the connectives "*and, or, if ... then*, *if and only if*."

1. An adequate condition for being excused from the final is having a quiz average of over 90%. (F, Q)

2. (Snoopy) "The commanding officer only offers me a root beer when there's a dangerous mission to be flown." (R, D)

3. *(Ecology poster) "Without you it won't get done." (H = you help, J = the job gets done)

4. "It is not true there was not a threat in relation to weapons of mass destruction (in Iraq)." Toni Blair, 7/6/2004

   T = There was a threat in relation to weapons of mass destruction (in Iraq).

5. There is only one possibility for a negotiated settlement: Konner must resign from the negotiation team. (S, K)

6. Sports announcer Dan Derdorf at the Minnesota Metrodome, during a Bears and Minnesota football game. "It's noisy here even when it is quiet." (Q = quiet, ~Q = noisy)

7. U. S. Rep. Charles Rangel, D-N.Y. commenting on the plight of Haitian refugee treatment: "There's no question, if they were not poor, if they were not black, that we would find some compassion to let these people in."

   P = If the refugees were poor.
   B = If the refugees were black.

C = We could find some compassion to let these refugees in to our country.

8. Australian Court decision ruling against a man charged with the rape of his wife. The man had argued that sexual intercourse with his wife was a marital right. "If it was ever the common law that by marriage (a woman) gave irrevocable consent to sexual intercourse with her husband, it is no longer the common law." (C)

9. *Putting political issues aside and rescinding some of the adverse tax legislation produced in 1986 and 1989 is the only way we can avoid being economically killed by the Japanese and enslaved by the new European Common Market.

   P = We put political issues aside.
   R = We rescind some of the adverse tax legislation produced in 1986 and 1989.
   K = We are economically killed by the Japanese.
   E = We are economically enslaved by the new European Common Market.

10. The CIA's assessment is that Iraq is unlikely to use biological or chemical weapons against the United States unless we attack Iraq and Saddam concludes he has nothing to lose.

   B = The CIA's assessment is that Iraq is likely to use biological weapons.
   C = The CIA's assessment is that Iraq is likely to use chemical weapons.
   A = We attack Iraq.
   S = Saddam concludes he has something to lose.

11. Should the economy improve and consumer confidence return, the Republicans will do better at the ballot box provided that they get a handle on the embarrassment of the right wing.

   I = The economy improves.
   C = Consumer confidence returns.
   B = The Republicans do a better job at the ballot box.
   H = The Republicans get a handle on the embarrassment of the right wing.

12. A doctor in the Netherlands will be allowed to perform an assisted suicide only when the following conditions are met: the doctor is convinced that the patient's request is well considered; the patient's suffering is unbearable; and finally, the doctor consults with another independent doctor who also examines the patient.

   A = A doctor in the Netherlands will be allowed to perform an assisted suicide.
   C = The doctor is convinced that the patient's request is well considered.
   B = The patient's suffering is bearable.

I = The doctor consults with another independent doctor who also examines the patient.

Note: [ ] should be used in this translation.

13. If Arafat cannot control his territory, it is in anarchy and Israel must subdue his territory; whereas, if Arafat can control his territory but refuses, then he has earned expulsion under the principle America cites in expelling the Taliban from power.

    C = Arafat can control his territory.
    A = Arafat's territory is in anarchy.
    S = Israel must subdue his territory.
    R = Arafat refuses to control his territory.
    E = Arafat has earned expulsion under the principle America cites in expelling the Taliban from power.

    [ ] should be used in this translation.

14. If American Jews really care about Israel, if Arab leaders really care about Palestinians, and if Iraq hawks really want to get rid of Saddam, then these groups must lobby President Bush to station U.S. troops around Israel.

    J = American Jews really care about Israel.
    A = Arab leaders really care about Palestinians.
    I = Iraq hawks really want to get ride of Saddam.
    L = These groups must lobby President Bush to station U.S. troops around Israel.

    [ ] should be used in this translation.

15. Rain and humidity are not uncommon in Hawaii.

    R = Rain is common in Hawaii.
    H = Humidity is common is Hawaii.

## Exercises V

Writing Exercises.

1. Write a short essay explaining why the statement, "An employee gets a day off during the week if and only if the employee works on a Saturday," is fair to both employees and an employer. Hint: Think of the different meanings of $O \equiv W$, $O \supset$

W, and $W \supset O$. Remember that $O \equiv W$ is equivalent to $(O \supset W) \bullet (W \supset O)$.

2. If **W** stands for "We have a winning team this year," and **R** stands for "The managers contract will be renewed," write out an English equivalent for each of the following. Then explain how each differs in meaning. What is (1) saying? How does what it differ from (2) in meaning? And so on.

   1. $W \supset R$

   2. $\sim W \supset \sim R$

   3. $\sim(W \supset R)$

   4. $\sim(\sim W \supset \sim R)$

3. Write out an explanation for the difference between (1) $F \supset (C \bullet G)$ and (2) $(F \supset C) \bullet G$.

   **F** = "John passes the final exam."
   **C** = "John passes the course."
   **G** = "John's GPA is high enough for eligibility for the Dean's list."

   How would (2) best be expressed in English?

4. Explain in writing why the statement, "It is not true that being female is a sufficient condition for being pregnant," $\sim(F \supset P)$, is not equivalent to the statement, "If a person is not female, then that person is not pregnant," $\sim F \supset \sim P$.

5. If **T** stands for "An argument has true premises," and **V** stands for "An argument is valid," write out English equivalents for the following:

   1. $\sim(T \supset V)$

   2. $\sim T \supset \sim V$

   3. $V \supset T$

   4. $\sim(V \supset T)$

   Based on what you have learned from Chapter 1 concerning the concept of validity, which statements are true and which are false? Explain.

**Answers to Starred Exercises:**

I.

2. $\sim A \supset B$

9. $\sim Z \equiv Y$

II.

4. $\sim E \supset \sim R$

9. $\sim(S \bullet D)$

III.

2. $X \supset \sim(P \bullet Z)$

7. $\sim A \lor (\sim B \bullet \sim C)$

14. $(\sim U \bullet \sim I) \supset \sim J$

25.

1. $\sim G \supset \sim(A \lor B)$
2. $G \supset (H \supset C)$
3. $(H \bullet E) \supset \sim C$
4. $B \supset (H \bullet E)$ $/\therefore$ $\sim B$

IV.

3. $\sim H \supset \sim J$ or $J \supset H$

9. $(\sim K \bullet \sim E) \supset (P \bullet R)$ —The phrase "only way" indicates a necessary condition.