where $w_{o_i}$ is an additive noise term that accounts for measurement error. We model the error as being normally distributed with $w_{o_i} \sim \mathcal{N}(0, \Sigma_{o_i})$ and $\mathbb{E}[w_{o_i}w_{o_j}^\top] = 0$ for all $i \neq j$.

### C. MLE Optimization

We pose sensor fusion as a maximum likelihood estimate (MLE) optimization problem. In this context, we treat ship/vehicle sample pairs as unknown parameters that we wish to estimate. To begin, we consider the case of

\[
X = \{X_v, X_s\} \quad \text{and} \quad Z = \{Z_r, Z_g, Z_o\}.
\]

Denoting the measurement likelihood as $L(X)$ we have

\[
L(X) = p(Z|X) = p(Z_r|X_v, X_s)p(Z_g|X_s)p(Z_o|X_v),
\]

where the mutual independence of measurements on parameters is explicit. To optimize, we wish to find

\[
\hat{X} = \arg \max_X p(Z|X),
\]

which is equivalent to solving

\[
\hat{X} = \arg \min_X - \ln p(Z|X).
\]

Under the assumed observation models and noise statistics, we can write our objective function, $C(X)$, as

\[
C(X) = - \ln p(Z|X)
\]

\[
= - \frac{1}{2} \sum_{i=0}^{n-1} (z_{t_i} - \|x_{v_i} - x_{s_i}\|)^\top \Sigma_{v_i}^{-1} (z_{t_i} - \|x_{v_i} - x_{s_i}\|)
\]

\[
+ \frac{1}{2} \sum_{i=0}^{n-1} (z_{g_i} - x_{s_i})^\top \Sigma_{g_i}^{-1} (z_{g_i} - x_{s_i})
\]

\[
+ \frac{1}{2} \sum_{i=1}^{n-1} (z_{o_i} - (x_{v_i} - x_{v_{i-1}}))^\top \Sigma_{o_i}^{-1} (z_{o_i} - (x_{v_i} - x_{v_{i-1}})).
\]

(4)

To optimize, we recognize that (4) can be more compactly written as

\[
C(X) = (Z - h_Z(X))^\top \Sigma_Z^{-1} (Z - h_Z(X)),
\]

(5)

where $h_Z(X)$ is the stacked vector of observations and $\Sigma_Z$ is the block-diagonal measurement covariance. In this form, it is clear that our objective function results in a nonlinear weighted least-squares optimization problem. To solve, we employ the Levenberg-Marquardt algorithm [23] starting with an initial guess of dead-reckoned (DR) vehicle position and GPS-measured ship position.

## III. EXPERIMENTAL RESULTS

### A. Experimental Setup

The SeaBED AUV is instrumented with a typical suite of oceanographic navigation sensors including pressure sensor depth, 1200 kHz DVL body-frame velocities, an IXSEA North-seeking 3-axis fiber optic gyro for attitude, and a PPS-capable WHOI Micro-modem [25]. In addition, we integrated our PPSBOARD and a Garmin GPS-16HVS GPS receiver into the vehicle system so that we could conduct OWTT experiments. The Garmin GPS unit outputs a 1 PPS reference signal accurate to within 1 $\mu$s of UTC when it has GPS lock. We use this PPS reference for pre-dive time synchronization of the free running TXCO onboard the AUV while at the surface.

The surface ship is equipped with a PPS-capable Micro-modem, a GPS receiver used for measuring ship transducer position, and a Meinberg GPS-based NTP time server as a stable clock reference. The Meinberg unit outputs a 1 PPS reference signal accurate to within 100 ns and is drift free.

### B. Experimental Results

**1) Experiment 1 — GPS Validation:** During December of 2005, we operated the SeaBED AUV offshore the coast of Woods Hole, MA using the R/V Tioga. For this set of experiments we deployed the AUV in approximately 15 m of water and progranmed the vehicle to swim two 100 m concentric boxes at a forward speed of 0.4 m/s. In this environment,