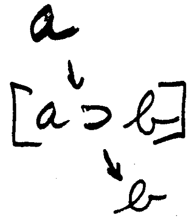
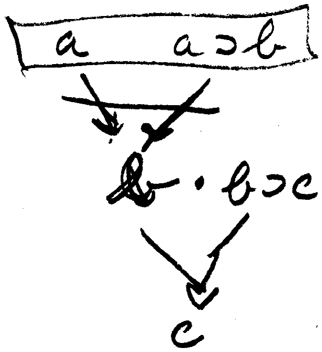
Proof of *2.15 ①

Analysis

11/15/55





(3) $p_1 \supset p_2$

(4) $p_2 \supset p_3$

(6) $p_3 \supset p_4$

(2.15)

(1) $q_1 \supset . \ p_1 \supset p_2$ (2.05) $\vdash$(1)

(2) q_1 $[q \supset \sim(\sim q)]$ (2.12) $\vdash$(2)

Op. 1.11 [(2); (1)] ↓ (3)

(3) $p_1 \supset p_2$

(4) $p_2 \supset p_3$ (2.03) $\vdash$(4)

(5) $q_2 \supset . \ p_3 \supset p_4$ (2.05) $[\sim(\sim p) \supset p]$ $\vdash$(5)

Op. 1.11 $[q_2; (5)]$ ↓ (6)

(6) $[q_2]$ $p_3 \supset p_4$

(7) $p_2 \supset p_3 . \supset :. \ p_1 \supset p_2 . \supset . \ p_1 \supset p_3$ (2.05) $\vdash$(7)

Op. 1.11 [(4); (7)] ↓ (8)

(8) $p_1 \supset p_2 . \supset . \ p_1 \supset p_3$

Op. 1.11 [(3); (8)] ↓ (9)

(9) $p_1 \supset p_3$

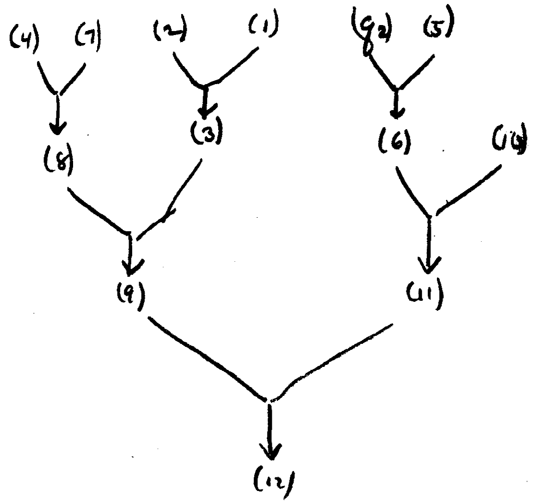
(10) $p_3 \supset p_4 . \supset :. \ p_1 \supset p_3 . \supset . \ p_1 \supset p_4$ (2.05) $\vdash$(10)

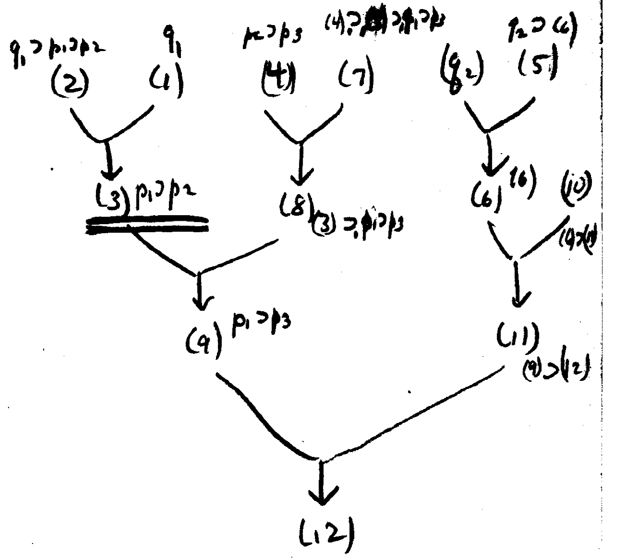
op. 1.11 [(6); (10)] ↓ (11)

(11) $p_1 \supset p_3 . \supset . \ p_1 \supset p_4$

op. 1.11 [(9); (11)] ↓ (12)

(12) $p_1 \supset p_4$





$②$

$$
\begin{tikzcd}[column sep=small]
\boxed{\overset{(3)}{p_1 \supset p_2}} \arrow[dr] & \boxed{\overset{(4)}{p_2 \supset p_3}} \arrow[d] & \overset{(5)}{p_2 \supset p_3 .\supset:. p_1 \supset p_2 .\supset. p_1 \supset p_3} \arrow[dl] & \\
& \underset{(8)}{p_1 \supset p_2 .\supset. p_1 \supset p_3} \arrow[dl] & \boxed{\overset{(6)}{p_3 \supset p_4}} \arrow[d] & \overset{(10)}{p_3 \supset p_4 .\supset:. p_1 \supset p_3 .\supset. p_1 \supset p_4} \arrow[dl] \\
\overset{(9)}{p_1 \supset p_3} \arrow[dr] & & \underset{(11)}{p_1 \supset p_3 .\supset. p_1 \supset p_4} \arrow[dl] & \\
& \overset{(12)}{p_1 \supset p_4} & &
\end{tikzcd}
$$

Taut. 2.05 with Subst. $\left\{\frac{p_1}{p}; \frac{p_2}{q}; \frac{p_3}{r}\right\}$

Taut. 2.05 with subst. $\left\{\frac{p_1}{p}; \frac{p_3}{q}; \frac{p_4}{r}\right\}$

Call this Syll. $[(3), (4), (6)]$

(op. 1.11) $[(4); (2.05\, s_1)] \rightarrow (8)$

(op. 1.11) $[(3); (8)] \rightarrow (9)$

(op. 1.11) $[(6); (2.05\, s_2)] \rightarrow (11)$

(op. 1.11) $[(9); (11)] \rightarrow (12)$

~~Syl. [(3),(4),(6)] .=. θ[(9); (11)] .=. θ [θ((3);(8))]; [θ(6);~~

$$\text{Syl.}[(3),(4),(6)] .=. \theta[(9);(11)] .=. \theta\Big[\big(\theta[(3);(8)]\big); \big(\theta[(6);(S_2[(2.05)])]\big)\Big]$$

$$.=. \theta\Big[\Big(\theta\big[(3);\big(\theta[(4);(S_1[(2.05)])]\big)\big]\Big); \big(\theta[(6);(S_2[(2.05)])]\big)\Big]$$

$$S_1 = S\left\{\frac{p_1}{p}; \frac{p_2}{q}; \frac{p_3}{r}\right\} \qquad S_2 = S\left\{\frac{p_1}{p}; \frac{p_3}{q}; \frac{p_4}{r}\right\}$$

$$Syl.\left[\overset{(3)}{(p_1 \supset p_2)}, \overset{(4)}{(p_2 \supset p_3)}, \overset{(6)}{(p_3 \supset p_4)}\right]$$

$$.=.\ \theta\left[\left(\theta\left[(3); \left(\theta\left[(4); \left(S_1\left[(2.05)\right]\right)\right]\right)\right]\right); \left(\theta\left[(6); \left(S_2\left[(2.05)\right]\right)\right]\right)\right]$$

$$\cancel{S_1 = S\left\{\frac{p_1}{p};\right.}$$

$$.=.\ \theta\left[\theta\left[(p_1 \supset p_2); \theta\left[(p_2 \supset p_3); S_1\left[(2.05)\right]\right]\right]; \theta\left[(p_3 \supset p_4); S_2\left[(2.05)\right]\right]\right]$$

where $S_1 = S\left\{\frac{p_1}{p}; \frac{p_2}{q}; \frac{p_3}{r}\right\}$ $S_2 = S\left\{\frac{p_1}{p}; \frac{p_3}{q}; \frac{p_4}{r}\right\}$

$$\boxed{\begin{array}{l} Syl.\left[(p \supset q), (q \supset r), (r \supset s)\right] \quad \rightarrow p \supset s \\ .=.\ \theta\left[\theta\left[(p \supset q); \theta\left[(q \supset r); (2.05)\right]\right]; \theta\left[(r \supset s); S\left\{\frac{p}{p}, \frac{r}{q}, \frac{s}{r}; (2.05)\right\}\right]\right] \end{array}}$$

$$\boxed{\begin{array}{l} Syl\left[(q \supset r), (p \supset q)\right] \longrightarrow p \supset r \\ .=.\ \theta\left[(p \supset q); \theta\left[(q \supset r); (2.05)\right]\right] \end{array}}$$

$$\cancel{Taut\left[p \vee p\right]} \rightarrow p$$

Hence!

$$Syl\left[(p \supset q), (q \supset r), (r \supset s)\right]$$

$$.=.\ \theta\left[(p \supset s); Syl\left[(q \supset r), (p \supset q)\right]\right]$$

$$.=.\ \theta\left[Syl\left[(q \supset r), (p \supset q)\right]; \theta\left[(r \supset s); S\left\{\frac{p}{p}, \frac{r}{q}, \frac{s}{r}; (2.05)\right\}\right]\right]$$

$$.=.\ \theta\left[\theta\left[(p \supset q); \theta\left[(q \supset r); (2.05)\right]\right]; \theta\left[(r \supset s); S\left\{\frac{p}{p}, \frac{r}{q}, \frac{s}{r}; (2.05)\right\}\right]\right]$$